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Abstract

We construct a model of bubbles where an asset can be used as collateral primarily due to higher-order uncertainty: while both a lender and a borrower know that the intrinsic value of the asset is low, they may still believe that a "greater fool" exists who will purchase it at a much higher price. In environments where agents suffer hold-up problems, we show that such bubbles can raise investment and, under certain conditions, lead to inefficient overinvestment, even when all agents know that the asset's intrinsic value is low or even zero. Using this framework, we also examine the impacts of macroprudential policies, as well as other regulatory measures such as interest rate hikes and the resolution of uncertainty.

Keywords: collateral; higher-order uncertainty; speculative bubbles; hold-up problems

1 Introduction

While collateral and secured loans generally facilitate transactions, they also raise public concerns—namely, that they fuel *asset bubbles* or induce overinvestment. When lenders extend credit based on collateral, borrowers may take excessive risks or inefficiently allocate

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substantial resources to risky projects, threatening economic stability and social welfare. The prevailing explanation for these concerns is that the intrinsic value of collateral is often perceived to be too high relative to its true value. This typically arises when some agents, such as lenders, miscalculate an asset value or hold misguided beliefs.

This paper argues that such first-order uncertainty regarding the value of collateral is not necessary for bubbles and overinvestment to arise. In fact, collateral can induce inefficient overinvestment even when all agents fully recognize that the collateralized asset lacks significant intrinsic value. This occurs when *higher-order uncertainty* plays a central role: even though both the lender and the borrower know that the asset has low intrinsic value to anyone in the economy, they may still believe it can be sold to a third party—potentially unaware of this fact (a "greater fool")—at a much higher price. We build a simple model of such *speculative bubbles* and examine the impact of macroprudential policies, as well as other regulatory measures such as interest rate hikes and the resolution of uncertainty.

More precisely, we consider an environment with three economic agents: a borrower, a lender, and a consumer. The borrower is an entrepreneur with an investment opportunity in a risky project but must borrow funds from the lender to pursue it. The lender, endowed with one unit of resource, allocates it between a safe project yielding a guaranteed but lower return, and the borrower's risky project, which offers a higher expected return. A key friction in this economy is contractual incompleteness: the parties cannot write a fully state-contingent contract ex ante, so repayment is determined through ex post renegotiation. This creates a classic hold-up problem—once the lender's funds are sunk, the borrower can bargain down the lender's claim—generating inefficiently low investment (Hart and Moore, 1994; Neher, 1999). Collateral matters for investment by providing downside protection to the lender. If the project fails, the lender can seize the pledged asset and liquidate it in the consumer market, limiting losses in bad states and expanding the set of feasible lending arrangements. To highlight the informational channel, we assume that the borrower and the lender commonly know whether the asset is valuable to the consumer, but they may not know whether the consumer knows this fact. Hence, even when the borrower and the lender commonly know that the asset is not valuable to the consumer, they may still assign it a high liquidation value, anticipating that the consumer could be unaware and therefore willing to buy the asset at a price that does not reflect its true value.

Within this framework, we compare two environments—one in which the asset can be used as collateral and one in which it cannot—and establish the following result:

Theorem. Collateral facilitates investment, even when *all* agents know it is worthless. Under certain conditions, it even induces overinvestment.

Our framework naturally fits the housing and stock markets, but it also extends to decentralized finance (DeFi) lending systems—particularly those relying heavily on crypto-collateral.[1] MakerDAO, for example, issues the Dai stablecoin, which maintains a stable value pegged to 1 USD. Users can collateralize cryptocurrencies such as Bitcoin or Ethereum to borrow the stablecoin for investment purposes.[2] These cryptocurrencies, arguably characterized by high volatility and speculative behavior, derive their value largely from market sentiment and higher-order uncertainty.[3] Nonetheless, DeFi lending has grown rapidly, with total value locked peaking at $50 billion in early 2022 from near zero in 2020 (Aramonte et al., 2022). Our analysis highlights the role of collateral in such environments shaped by higher-order uncertainty.

Policy Implications

The model's tractability allows us to analyze several important policies associated with bubbles and investments in environments with incomplete contracts: *macroprudential policy*, *increase in the interest rate*, and *resolving (higher-order) uncertainty*. As emphasized by Barlevy (2018), bridging the gap between policymakers and economic theories of bubbles is important; our analysis contributes to this connection.[4]

Macroprudential policy bans the trade of collateralized assets. This is a significant policy tool frequently used in practice. We ask: (i) can governments effectively control bubbles through such policies, and (ii) if so, to what extent should they intervene? We show that macroprudential policy influences both the size of bubbles and overall welfare. Further, the optimal strictness of such policy depends on the degree of overinvestment. If collateral only moderately facilitates investment, the optimal policy permits full asset trading. Conversely, when overinvestment is more severe, stricter policies that restrict certain trades are welfare-enhancing.

Interest rate policy serves as another crucial instrument for managing bubbles. In our model, an increase in the interest rate is reflected by a higher return on the safe project. However, it

[1] See Alamsyah et al. (2024) for an overview of the Defi ecosystem.

[2] According to Bloomberg, JP Morgan also plans to allow institutional clients to use Bitcoin and Ether as collateral for loans: "https://www.bloomberg.com/news/articles/2025-10-24/jpmorgan-to-allow-bitcoin-ether-as-collateral-in-crypto-push"

[3] Chiu et al. (2022) examine the role of cryptocurrency as collateral in the DeFi lending system when cryptocurrencies are used as a medium of exchange for consumption.

[4] See also Barlevy (2025), a textbook treatment of asset bubbles and macroeconomic policy.

also has an indirect effect: it reduces equilibrium investment in the risky project. If there is underinvestment, the overall welfare impact is ambiguous, as these two effects offset each other. Conversely, if there is overinvestment, raising the interest rate unambiguously improves welfare by both increasing the return from the safe project and curbing excessive investment.

Finally, we analyze the effect of resolving higher-order uncertainty by publicly announcing the value of collateralized assets. Such a policy introduces greater volatility into the economy—it boosts investment when assets are revealed to be valuable and reduces it when they are not. We provide an example in which this announcement policy is welfare-reducing.

Related Literature

Our paper is related to several strands of the literature. First of all, it is most closely connected to the literature on greater fool bubbles (see, e.g., Allen, Morris, and Postlewaite (1993), Conlon (2004), Awaya, Iwasaki, and Watanabe (2022); for an excellent summary, see Barlevy (2018)). Like ours, this literature examines rational bubbles arising due to information asymmetry and higher-order uncertainty among rational agents. Our main contribution is to show that such bubbles can emerge specifically in the context of *collateralized* assets—a topic not addressed in the existing literature. Further, to the best of our knowledge, many of the policy implications we derive are also novel.

The second relevant strand concerns bubbles under borrowing constraints (e.g., Martin and Ventura (2012) and Hirano and Yanagawa (2016)).[5] There are two main differences. First, the role of bubbles is different: in these papers, bubbles serve as a store of value, whereas in our paper, bubbles serve as collateral. Second, these papers consider symmetric information models, whereas we consider an asymmetric information model. One might conjecture that these differences are superficial if collateral is isomorphic to bubble exchanges. However, we demonstrate that the two are not equivalent.

More fundamentally, our private information framework reveals that bubbles can reduce the dispersion of asset prices relative to fundamentals. Without bubbles (i.e., when asset values are publicly known), prices reflect either zero or a high value depending on states, resulting in wide price dispersion. With bubbles, by contrast, asset prices remain strictly positive even when fundamentals are low, and become less high when fundamentals are high. This compression in asset prices reduces the variation in investment, suggesting that bubbles may play a

[5]See also Kocherlakota (1992), Santos and Woodford (1997), Caballero and Krishnamurthy (2006), Farhi and Tirole (2012), and Guerron-Quintana et al. (2023). Barlevy (2022) provides an excellent survey of this and other approaches.

socially useful role when reducing investment dispersion is welfare-enhancing—for example, in economies with both over- and under-investment relative to the efficient level. This mechanism, rooted in private information, differentiates our paper from the existing literature based on symmetric information models.

Finally, our paper also contributes to the literature on collateral and secured credit. In their seminal work, Kiyotaki and Moore (1997) show that collateral constraints are a central channel through which macroeconomic fluctuations are propagated and amplified. Awaya, Fukai, and Watanabe (2021) endogenize the Kiyotaki-Moore constraint by demonstrating that collateral can facilitate trade even when the underlying asset lacks sufficient intrinsic value, because it functions as a monitoring device to prevent reneging. In contrast, we propose a new mechanism to endogenize the collateral constraint: *bubbles* in collateralized assets, which arise due to information asymmetry—even when all agents are fully rational.

2 Model

Environment There are three risk-neutral agents: a borrower, a lender, and a consumer. The borrower is an entrepreneur with an investment opportunity in a risky project, whose outcome is either success $y = G$ with probability $p \in (0, 1)$ or failure $y = B$ with probability $1 - p$. If $x \geq 0$ is invested in this project, it generates a return $f(x)$ if $y = G$. If $y = B$, the return is 0 regardless of x. To pursue the project, the borrower must borrow funds from the lender, who has a total available resource of 1 and allocates it between the risky project and an alternative safe project yielding a guaranteed return $r > 0$ per unit. The return r can be interpreted as an interest rate, as it derives from a risk-free project.

Assumption 1. $f(x)$ is continuously differentiable with the following properties:

(i) $f(0) = 0$ and $f'(x) > 0 > f''(x)$ for all $x > 0$

(ii) $\lim_{x \to 0} f'(x) = \infty$ and $f'(1) < \frac{r}{p} < f(1)$.

The first item (i) means that the risky project's return is zero if no investment is made and increases in a strictly concave manner as more investments are made. The second item (ii) requires that the marginal return of the risky project approaches infinity when the investment level approaches zero; when all the resources are being invested, the average return (i.e., $f(1)/1 = f(1)$) remains high, but its marginal return falls below $\frac{r}{p}$. This assumption ensures that the efficient investment level x^* that maximizes the joint surplus of the borrower and the lender,

$$S(x) \equiv pf(x) + r(1-x),$$

is characterized by the first order condition, $pf'(x^*) - r = 0$ with $x^* \in (0,1)$.

Terms of Trade A key friction in this economy is a commitment problem in the relation: the borrower and the lender cannot write down a contract on the division of the outcome upon success ex-ante, and they have to bargain about the division ex-post. To capture this, we first consider the benchmark case where the agents can write down such a contract ex-ante in Section 3.1, and compare this to the case where the borrower and the lender split the project's return according to *exogenous* shares. More precisely, we assume that they engage in Nash bargaining for a realized return $f(x)$ with zero outside option payoffs, where the borrower's bargaining power $\sigma \in (0,1)$ is endogenous for the former, while exogenous for the latter.[6] In either case, the amount of the return given to the lender is $y = (1-\sigma)\, f(x)$, which solves

$$\max_{0 \le y \le f(x)} y^{1-\sigma} \left(f(x) - y\right)^{\sigma}.$$

Therefore, if x and $1-x$ are invested in the borrower's and the safe project, respectively, then the borrower's ex-ante payoff is

$$U_B(x) = p\sigma f(x),$$

and the lender's ex-ante payoff is

$$U_L(x) = p(1-\sigma)f(x) + r(1-x).$$

As $\sigma \to 1$, which is the focus of our paper, the hold-up problem becomes extreme: the borrower effectively holds control rights over the return from the investment on the risky project and makes a take-it-or-leave-it offer y to the lender. This can arise from the inalienability of the borrower's human capital or from the inability to commit not to renegotiate the lender's claim after investment (Hart and Moore, 1994; Neher, 1999; Gul, 2001; Pitchford and Snyder, 2004). In our model, this contractual incompleteness leads to underinvestment in the absence of collateral.

[6]The assumption of zero outside option payoffs can be relaxed as long as these values remain small, although it only complicates the analysis without offering additional insights.

Collateral The borrower has an asset that can be used as collateral, potentially serving as a safety device to the lender when the risky project does not work out. To make our results as stark as possible, this asset is assumed to be *worthless* for both the borrower and the lender, but it can be useful to the consumer.[7] More precisely, there are three states of the world with respect to the value of collateral, which are denoted by

$$\Omega \equiv \{\omega_v, \omega_0, \omega_\phi\}.$$

The asset has a value $V = v > 0$ for the consumer if the state is ω_v. Otherwise, in states $\{\omega_0, \omega_\phi\}$, it has no value to all agents, $V = 0$. The prior probability that the true state is ω_i is denoted by $q_i \in (0, 1)$ for each $i = v, 0, \phi$.

We will assume that the collateral cannot be partially transferred or stochastically shared between the parties due to the commitment problem and the contract incompleteness. That is, upon default (i.e., $y = B$), the lender seizes the entire collateral, whereas upon repayment (i.e., $y = G$), the borrower fully retains it. Our main qualitative result—collateral can boost investments even when every agent knows it is worthless—does not depend on this assumption, but it generates an additional friction that potentially leads to inefficient overinvestment with collateral. In Appendix B, we show that inefficiency disappears with more contingent and flexible contracts, for instance, when the randomized collateral transfer is allowed.

Information and Timing The information partition of the consumer is

$$\mathcal{P}^{\mathrm{c}} \equiv \{\{\omega_v, \omega_\phi\}, \{\omega_0\}\},$$

whereas the borrower and the lender's information partitions are equally

$$\mathcal{P} \equiv \{\{\omega_v\}, \{\omega_\phi, \omega_0\}\}.$$

Note that the second information set $\{\omega_0\}$ of the consumer corresponds to the case where the consumer knows that the collateral has no value, whereas the first information set $\{\omega_v, \omega_\phi\}$ indicates the case where the consumer does not know whether the collateral is worthless. Similarly, the first information set $\{\omega_v\}$ of the borrower and the lender corresponds to the case where they know that the collateral has a positive value to the consumer, whereas the second information set $\{\omega_\phi, \omega_0\}$ indicates that they know that the collateral is worthless but do not

[7]This assumption is only for clarifying our mechanism; the results still hold as long as the asset value is smaller for the borrower and the lender than for the consumer, and the consumer's valuation depends on the state of the world in a similar manner as above.

know whether the consumer knows this. In the following, we will often write as $\Omega_v^c = \{\omega_v, \omega_\phi\}$ and $\Omega_0^c = \{\omega_0\}$ for the information states of the consumer; $\Omega_v = \{\omega_v\}$ and $\Omega_0 = \{\omega_\phi, \omega_0\}$ for the information states of the borrower and the lender. Also, we will impose the following assumption later, requiring that the consumer's expected value of collateral is not too high.

Assumption 2. $E\left(V|\Omega_v^c\right) = \frac{q_v}{q_v+q_\phi}v < \frac{r}{1-p}$.

The sequence of events is as follows. First, the borrower offers x to the lender. Second, the lender either accepts or rejects the offer. If the offer is accepted, x and $1-x$ are invested in the risky and safe projects, respectively; if it is rejected, all resources are invested in the safe project. Finally, the collateral holder offers a price of the asset to the consumer, who then decides whether to purchase the collateral at that price or not.

Several remarks are in order. First, every agent in our model is fully rational. Second, the investment level is not observable to the consumer, so no additional information is available for the consumer to update his belief. Third, as explained in Section 3.4, the assumption that the borrower has the bargaining power over x is not crucial for our main results that asset bubbles can arise and facilitate investments in the presence of higher-order uncertainties. Fourth, each of the three states is a necessary ingredient of bubbles: state ω_v creates gains from the trade of the asset so that it can be used as collateral; state ω_0 establishes a situation where all agents know that the asset has no value; state ω_ϕ constructs a case where the asset value is zero, but the consumer does not know it—only the lender and the borrower know the fact that the value is zero.

Finally, while the setup is best illustrated by traditional markets such as housing and stocks, it can also be interpreted as the DeFi lending system: the borrower collateralizes cryptocurrency (e.g., Bitcoin or Ethereum) to borrow and invest an amount x of stablecoins (e.g., Dai stablecoin), which yields a return of $f(x)$ with probability p. In exchange, the lender is entitled to a repayment of $(1-\sigma)f(x)$ from the borrower, but if the borrower defaults, the lender retains the collateralized cryptocurrency. Importantly, they commonly know that the collateralized crypto asset has no intrinsic value, but there is uncertainty (asymmetric information) on consumers' knowledge about how much to appreciate it. Its potential value may come from capital gains (or losses) if resold further or liquidity premiums if used as a means of payment.

3 Analysis

3.1 Benchmark: No Hold-up Problem

First, we start with the benchmark case in which there is no hold-up problem. That is, the borrower optimally chooses his share σ and commits to it. We argue that the first-best outcome is achieved even without collateral, and hence, the collateral plays no role.

Note that if the accepted offer is a pair σ and x, the borrower's expected payoff is

$$U_B(\sigma, x) = p\sigma f(x),$$

while the lender's expected payoff is

$$U_L(\sigma, x) = p(1-\sigma)f(x) + r(1-x).$$

Since the lender would accept this offer if it yields at least as much as his outside option payoff r—the payoff when investing all the resources in the safe project—, the borrower will choose $\bar{\sigma}(x)$ such that the lender is indifferent between accepting the offer or not, that is,

$$p(1-\bar{\sigma}(x))f(x) + r(1-x) = r.$$

In particular, this means that the borrower's payoff from $\bar{\sigma}(x)$ and x is

$$U_B(\bar{\sigma}(x), x) = p\bar{\sigma}(x)f(x) = \underbrace{pf(x) + r(1-x)}_{=S(x)} - r.$$

Thus, the borrower will optimally offer the efficient investment level $x = x^*$ that maximizes the joint surplus $S(x)$, achieving the first-best outcome and payoff.

3.2 Without Collateral

We now move to a more natural case in which there is a commitment problem in the relation leading to the hold-up problem, treating σ as a model primitive. We maintain this assumption in the rest of the paper. We first consider the environment without collateral, in which case the information held by the borrower and the lender does not affect their investment incentives. The borrower's incentive compatibility condition is always satisfied since, for all $x \geq 0$,

$$U_B(x) = p\sigma f(x) \geq 0,$$

whereas the lender's incentive compatibility condition is

$$U_L(x) = p\,(1-\sigma)\,f(x) + r\,(1-x) \geq r \iff \mathcal{L}(x) \equiv p\,(1-\sigma)\,f(x) - rx \geq 0.$$

Note that $\mathcal{L}(0) = 0$; $\lim_{x \to 0} \mathcal{L}'(x) > 0$; and $\mathcal{L}(x)$ is strictly concave in x. Thus, there exists a unique cutoff $x^N \in (0, 1]$ such that the lender's incentive-compatibility condition is satisfied if and only if $x \leq x^N$. The following lemma summarizes this observation.

Lemma 1. Suppose Assumption 1 holds. Without collateral,

(i) The borrower is willing to borrow any $x \in [0, 1]$.

(ii) There exists $x^N \in (0, 1]$ such that the lender is willing to lend if and only if $x \leq x^N$.

There is a continuum of investment levels x satisfying both incentive-compatibility conditions. Among those, we focus on the maximum investment level in that the borrower has the bargaining power regarding the choice of x. As explained later, this is not crucial for our qualitative result that asset bubbles can arise and facilitate investments in the presence of higher-order uncertainties. We refer to this maximum as the *equilibrium investment level without collateral*, denoted by x^N.

The lender's opportunity cost of funding the risky project increases with σ, since $\mathcal{L}(x)$ monotonically decreases and converges to $-rx$ as σ increases to 1. Hence, the equilibrium investment level becomes lower than x^* when σ is sufficiently high, resulting in insufficient investment without collateral.

Lemma 2. Suppose Assumption 1 holds. Without collateral, $x^N < x^*$ for sufficiently large σ.

This is consistent with the standard underinvestment result in the hold-up literature, as the problem becomes more severe with higher σ.

3.3 With Collateral

This section analyzes the case where the borrower and the lender can trade the collateral. We consider a contract of the form: "the borrower keeps the collateral if the project is successful, but otherwise, it is held by the lender."

The value of holding the collateral comes from the fact that it could be sold to the consumer. To derive this value, recall that the consumer perfectly knows that the collateral is worthless at $\Omega_0^{\mathsf{c}} = \{\omega_0\}$, in which case she would not buy it at any strictly positive price. Otherwise, the expected consumption value of the collateral to the consumer is

$$\Pr\left(\omega_v \middle| \, \Omega_v^{\mathsf{c}}\right) \times v = \Pr\left(\omega_v \middle| \, \{\omega_v, \omega_\phi\}\right) \times v = \left(\frac{q_v}{q_v + q_\phi}\right) v > 0.$$

Suppose now that the borrower and the lender's information state is Ω_0. If the holder of the collateral sets price $t = \left(\frac{q_v}{q_v+q_\phi}\right)v$, the consumer will buy the good if and only if the true state is ω_ϕ, whose likelihood conditional on $\Omega_0 = \{\omega_\phi, \omega_0\}$ is $\frac{q_\phi}{q_\phi+q_0}$. Therefore, the expected values of holding the collateral to the borrower and the lender are given as: at $\Omega_0 = \{\omega_0, \omega_\phi\}$,

$$W_0 \equiv \underbrace{\frac{q_\phi}{q_\phi + q_0}}_{\text{Probability that the good is sold}} \times \underbrace{\left(\frac{q_v}{q_v + q_\phi}\right) v}_{\text{Price paid by the consumer}} > 0,$$

and at $\Omega_v = \{\omega_v\}$,

$$W_v \equiv \underbrace{1}_{\text{Probability the good is sold}} \times \underbrace{\left(\frac{q_v}{q_v + q_\phi}\right) v}_{\text{Price paid by the consumer}} > W_0.$$

The borrower's incentive-compatibility condition is thus

$$U_{B,k}(x) = p\left(W_k + \sigma f\left(x\right)\right) \geq W_k \iff f\left(x\right) \geq \frac{W_k\left(1-p\right)}{p\sigma},$$

which, by Assumption 1, is ensured to hold for some x when

$$f(1) > \frac{W_v(1-p)}{p\sigma}. \tag{1}$$

Compared to the case without collateral, the borrower would not trade with the lender unless the investment is large enough. Also, the minimum investment level satisfying the borrower's incentive condition, denoted by $\underline{x}_k$, depends on the state of the world.

Next, we consider the lender's incentive-compatibility condition. The lender would find it optimal to invest x in the borrower's project at Ω_k if and only if

$$\begin{aligned} & U_{L,k}(x) = p\left(1-\sigma\right) f\left(x\right) + \left(1-p\right) W_k + r\left(1-x\right) \geq r \\ \iff & \mathcal{L}_k\left(x\right) \equiv p\left(1-\sigma\right) f\left(x\right) + \left(1-p\right) W_k - rx \geq 0. \end{aligned}$$

Note that $\mathcal{L}_k\left(0\right) = \left(1-p\right) W_k > 0$, and, by Assumption 1,

$$\lim_{x\to 0} \mathcal{L}_k'\left(x\right) = p\left(1-\sigma\right) \lim_{x\to 0} f'(x) - r > 0.$$

Since $\mathcal{L}_k(x)$ is strictly concave in x, the lender's incentive condition imposes the upper bound on x. Also, compared to the case without collateral, the lender's incentive condition is relaxed due to the positive value of holding the collateral: for each x,

$$\mathcal{L}_k(x) = \mathcal{L}(x) + (1-p)W_k > \mathcal{L}(x).$$

Assuming condition (1), the necessary and sufficient condition for the existence of incentive-compatible investment is then, for each $k = v, 0$,

$$\mathcal{L}_k\left(\underline{x}_k\right) = p\left(1-\sigma\right) f\left(\underline{x}_k\right) + \left(1-p\right) W_k - r\underline{x}_k \geq 0.$$

If this holds, there is a nonempty interval $[\underline{x}_k, \bar{x}_k] \subset (0, 1]$ such that both the borrower's and the lender's incentive constraints are satisfied for $x \in [\underline{x}_k, \bar{x}_k]$. By the definition of $\underline{x}_k$ and strictly increasing f, it can be checked that $\mathcal{L}_k\left(\underline{x}_k\right) \geq 0$ is equivalent to $\underline{x}_k \leq \frac{(1-p)W_k}{r\sigma}$, or, by taking f on both sides,

$$\frac{(1-p)W_k}{p\sigma} \leq f\left(\frac{(1-p)W_k}{r\sigma}\right). \tag{2}$$

To summarize,

Lemma 3. Suppose Assumption 1 and conditions (1)–(2) hold. With collateral, there exists a nonempty interval $[\underline{x}_k, \bar{x}_k] \subset (0, 1]$ for state Ω_k such that

(i) The borrower is willing to borrow if and only if $x \geq \underline{x}_k$.

(ii) The lender is willing to lend if and only if $x \leq \bar{x}_k$.

It is immediate that $\bar{x}_v \geq \bar{x}_0$ and $\underline{x}_v \geq \underline{x}_0$, with each inequality being strict unless both bounds are equal to 1. Also, when the borrower's bargaining power is sufficiently large, Assumption 2, which imposes an upper bound on W_k's, turns out to ensure both conditions (1) and (2), leading to the following.

Corollary 1. Suppose Assumptions 1–2 hold. If σ is sufficiently large, conditions (1)–(2) hold, so both parties are willing to trade $x \in [\underline{x}_k, \bar{x}_k]$ at state Ω_k.

The maximum investment level satisfying both incentive constraints at Ω_k is called *equilibrium investment level with collateral at* Ω_k, and is denoted by x_k.

3.4 With vs. Without Collateral

3.4.1 Bubbles and Overinvestment

Now, we compare the equilibrium investment levels with and without collateral. The investment levels with collateral are strictly lower than 1 if

$$\mathcal{L}_v(1) < 0 \iff W_v < \frac{r - p(1-\sigma)}{1-p}.$$

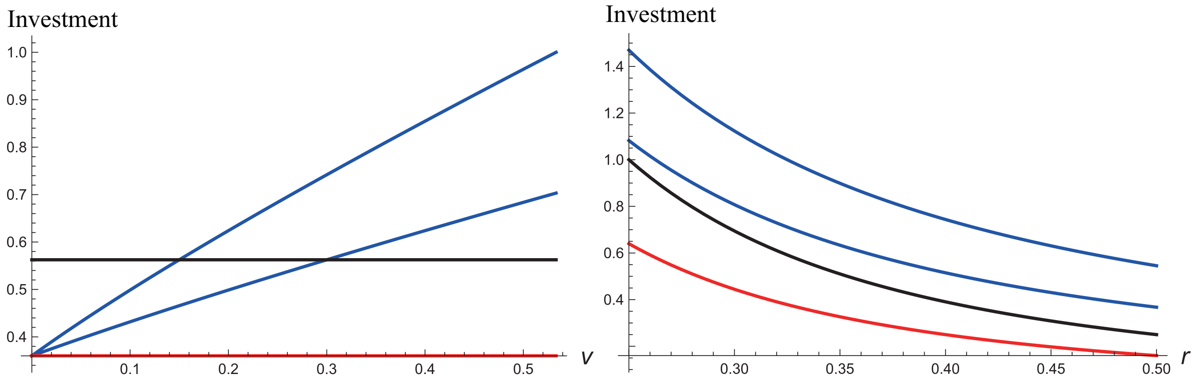

Figure 1: The efficient (black) and equilibrium investment levels without collateral (red) and with collateral (blue) as v (left) and r (right) vary when $f(x) = \sqrt{x}$, $p = \frac{1}{2}, \sigma = \frac{3}{5}$, $q = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$, and $r = \frac{1}{3}$ (left) and $v = \frac{1}{2}$ (right).

Under Assumption 2, we have $\frac{W_v(1-p)}{r} < 1$ since $\frac{r}{p} > f'(1)$, which implies that the above inequality holds for sufficiently large σ. Also, it can be checked that collateral induces overinvestment at Ω_k if and only if

$$\mathcal{L}_k\left(x^*\right) > 0 \iff p\left(1-\sigma\right) f\left(x^*\right) + \left(1-p\right) W_k - rx^* > 0.$$

For sufficiently large σ, this inequality holds when $W_k > \frac{r}{1-p}x^*$, while if $W_k < \frac{r}{1-p}x^*$, the reversed inequality holds, meaning that equilibrium investment level with collateral is still insufficient compared to the socially efficient one. This observation leads to the following main result of the paper.

Theorem 1. Suppose Assumptions 1–2 hold. For sufficiently large σ,

(i) If $\frac{rx^*}{1-p} < W_0$, then $x^N < x^* < x_0 < x_v$.

(ii) If $W_0 < \frac{rx^*}{1-p} < W_v$, then $x^N < x_0 < x^* < x_v$.

(iii) If $W_v < \frac{rx^*}{1-p}$, then $x^N < x_0 < x_v < x^*$.

Theorem 1 (i) pertains the main case of interest, where the asset bubble, driven by collateral, becomes so extreme that it leads to overinvestment in every state of the world. In particular, x_0 is the investment level when the state is ω_0, that is, when the collateralized asset has no value, and all agent knows this. The fact that $x_0 > x^N$ means that, despite that the collateralized asset has no value and all agent knows this, it still facilitates investment. Moreover, $x_0 > x^*$ means that it facilitates investment beyond the efficiency level.[8]

[8]Beyond the parameter space we focus on, overinvestment may occur even without collateral. Nonetheless, the presence of collateral still accelerates such overinvestments even further.

It is worth noting that, while the assumption that the borrower and the lender cannot commit to terms of trade before investment is crucial, our focus on the maximum incentive-compatible investment level is not. For instance, we may alternatively assume either that the lender makes an offer to the borrower or that they agree on x that maximizes their joint surplus conditional on the incentive constraints. In either case, one can find similar conditions ensuring the same ordering of equilibrium investment levels as in Theorem 1 (i). To see this, note that without collateral, the maximum incentive-compatible investment level decreases in σ whereas the minimum level remains at zero; with collateral, both the minimum and the maximum incentive-compatible investment levels increase in W_k. Thus, when σ is large enough, *any* incentive-compatible x without collateral must be smaller than x^*, and given such σ, *any* incentive-compatible x_k with collateral must exceed x^* when W_k is large enough.

Example. Suppose $f(x) = \sqrt{x}$. Since $f'(x) = \frac{1}{2\sqrt{x}}$, Assumption 1 holds so long as $\frac{1}{2} < \frac{r}{p} < 1$, and the efficient investment level is $x^* = \frac{p^2}{4r^2} \in (0, 1)$. Also, Assumption 2 and the condition in Theorem 1 (i) respectively impose the upper and lower bounds on the expected values of holding the collateral: the former requires $W_v < \frac{r}{1-p}$, while the latter requires $W_0 > \frac{rx^*}{1-p} = \frac{p^2}{4r(1-p)}$. If $p = \frac{1}{2}$, for instance, all the conditions hold when $\frac{1}{4} < r < \frac{1}{2}$ and $\frac{1}{8r} < W_0 < W_v < 2r$. Similarly, underinvestment occurs even with collateral (Theorem 1 (ii)) if $W_v < \frac{rx^*}{1-p} = \frac{p^2}{4r(1-p)}$, which becomes $W_v < \frac{1}{8r}$ when $p = \frac{1}{2}$. Figure 1 illustrates how the equilibrium investment levels depend on v and r for certain parameter values.[9] $\diamond$

3.4.2 Welfare

Based on these results, we analyze the impact of collateral on the joint surplus, assuming interior equilibrium investments. The ex-ante expected joint surplus with collateral is

$$S^C \equiv E\left(S\left(x_k\right)\right) = q_v S\left(x_v\right) + \left(1 - q_v\right) S\left(x_0\right),$$

whereas the expected joint surplus without collateral is

$$S^N \equiv S\left(x^N\right).$$

Clearly, as long as there is an underinvestment without collateral, the collateral improves social welfare when v is sufficiently small. Indeed, this condition can be rewritten as

$$S^C \geq S^N \iff E\left(x_k\right) - x^N \geq \left(\frac{(1-p)q_v}{\sigma r}\right) v, \tag{3}$$

[9] Note that we fix the value of σ at $\frac{3}{5}$ in this numerical exercise, so the figure does not exactly correspond to the conditions in Theorem 1.

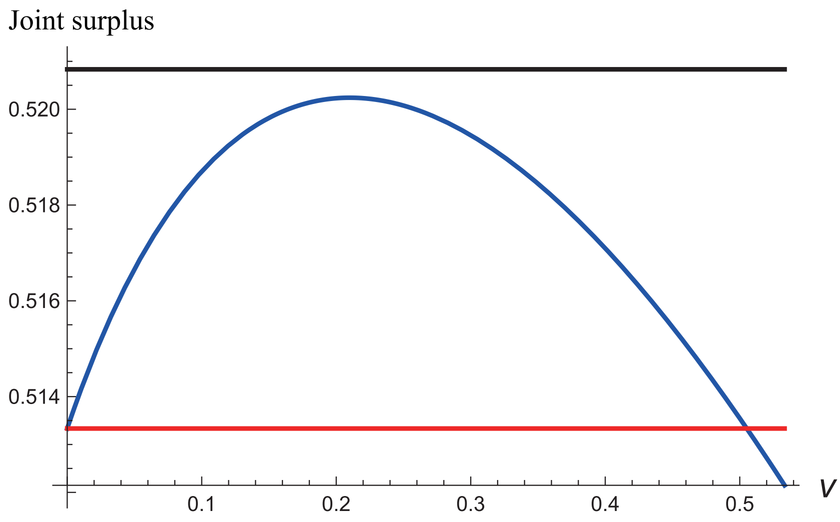

Figure 2: The ex-ante joint surplus under the efficient (black) and equilibrium investment levels without collateral (red) and with collateral (blue) as v varies when $f(x) = \sqrt{x}$, $p = \frac{1}{2}, \sigma = \frac{3}{5}, r = \frac{1}{3}$ and $q = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.

because

$$S^N = \left(\frac{r\sigma}{1-\sigma}\right) x^N + r, \quad S^C = \left(\frac{r\sigma}{1-\sigma}\right) E\left(x_k\right) - \frac{1-p}{1-\sigma} E\left(W_k\right) + r,$$

and

$$E(W_k) = q_v \left(\frac{q_v}{q_v + q_\phi}\right) v + (1 - q_v)\left(\frac{q_\phi}{1-q_v}\right)\left(\frac{q_v}{q_v+q_\phi}\right) v = q_v v.$$

Condition (3) shows that, in order for collateral to improve social welfare, the increase of investments facilitated by collateral must be sufficiently large relative to the expected value of holding the collateral. If v is too large, for instance, the collateralized trade may potentially lead to excessive overinvestments, resulting in lower social welfare. We confirm this intuition with the functional form example of $f(x) = \sqrt{x}$, as illustrated in Figure 2.

Note that although our analysis focuses on the joint surplus of the borrower and the lender, the comparison between the two environments is unchanged even when consumer surplus is taken into account. If trade between the borrower and the consumer is feasible in the absence of collateral, the borrower would optimally set the same price as under a collateralized contract, and the consumer's purchasing decision would likewise be unaffected. Consequently, consumer welfare is identical across the two settings.

4 Policy Implications

Given inherent contractual incompleteness, the tractability of our model allows us to study several key bubble-related policies. For simplicity, we assume interior equilibrium investment levels, i.e., $x^N, x_v, x_0 \in (0, 1)$.

4.1 Macroprudential Policy

Suppose the government bans the trade of the collateral with probability $\alpha \in [0, 1]$, which measures the strictness of macroprudential policy. Then, the expected value of holding the collateral for the borrower and the lender is reduced to $(1 - \alpha) W_k$.

Lemma 4. The equilibrium investment level x_k with collateral at Ω_k is strictly decreasing in α, σ, r, while strictly increasing in W_k.

Clearly, the lender's incentive constraint becomes tighter when α, σ, or r increases while being relaxed as W_k increases. The comparative statics results in the lemma then follows because the equilibrium investment levels are pinned down by the lender's incentive constraint.

Note that if the government slightly reduces α from 1, social welfare improves whenever the economy suffers underinvestment without collateral (i.e., $x^N < x^*$), as it boosts investment. By the same argument, macroprudential policy is only detrimental if v is initially small and $x^N < x^*$. If this is not the case (e.g., when condition (3) is violated), a positive degree of macroprudential policy can be welfare-improving.

Letting $\alpha^* \in [0, 1]$ denote the optimal strictness of macroprudential policy maximizing the joint surplus, the above observations can be summarized as follows (Figure 3).

Proposition 1. The ex-ante joint surplus with collateral S^C is strictly concave in α and coincides with the surplus without collateral when $\alpha = 1$. Therefore,

(i) If $x^N < x^*$, then $\alpha^* < 1$. That is, a complete ban is not socially optimal.

(ii) If $x^N < x^*$ and v is sufficiently small, then $\alpha^* = 0$. That is, full allowance is socially optimal.

Clearly, as v increases, overinvestment induced by the collateral becomes excessively inefficient, and the optimal macroprudential policy should be correspondingly stricter. Similarly, the change of the prior from $q = (q_v, q_\phi, q_0)$ to $q' = (q_v + \epsilon, q_\phi, q_0 - \epsilon)$ induces increases in W_v and W_0. With this change, the expected value of holding the collateral becomes higher as

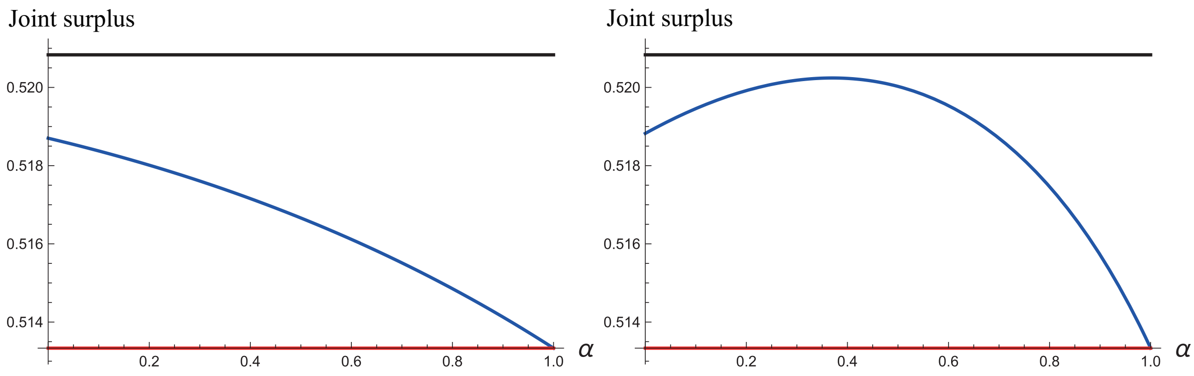

Figure 3: The ex-ante joint surplus under the efficient (black) and equilibrium investment levels without collateral (red) and with collateral (blue) as α varies when $f(x) = \sqrt{x}$, $p = \frac{1}{2}, \sigma = \frac{3}{5}, r = \frac{1}{3}, q = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$, and $v = \frac{1}{10}$ (left) and $v = \frac{1}{3}$ (right).

it is more likely that the asset has positive value v (i.e., $\omega = \omega_v$), while the event that every agent knows that the asset has no value (i.e., $\omega = \omega_0$) is less likely.[10] Finally, the optimal macroprudential policy induces $x_v > x^* > x_0$: if both x_v and x_0 exceed x^*, the government would rather increase α to lower these levels, while in the opposite case, the government would decrease α.

Proposition 2. Assume $\alpha^* \in (0, 1)$. Then, $x_v > x^* > x_0$ at $\alpha = \alpha^*$. Moreover

(i) α^* is strictly increasing in v.

(ii) α^* is strictly lower at $q = (q_v, q_\phi, q_0)$ than at $q' = (q_v + \epsilon, q_\phi, q_0 - \epsilon)$ for small $\epsilon > 0$.

Our analysis suggests that the impact of macroprudential policy crucially depends on the initial degree of the equilibrium investment with collateral. A stricter macroprudential policy may decrease social welfare if the collateral is facilitating investment moderately (the left panel of Figure 3), but may improve it if the collateral is inducing too much over-investment (the right panel of Figure 3).

Similar reasoning applies to other interventions that effectively cap equilibrium investment levels. For instance, a *haircut* can be defined as $h_k \equiv \frac{W_k - (x_k - x^N)}{W_k}$ for each state k, so a haircut floor is given by the lower bound $\underline{h}$ such that $\min_{k=0,v} h_k \geq \underline{h}$. That is, the haircut floor regulation imposes an upper bound on x_k, which brings about similar effects as increasing α.

[10] A similar conclusion cannot be made for the change to $q' = (q_v + \epsilon, q_\phi - \epsilon, q_0)$ because it reduces asset bubbles at Ω_0.

4.2 Interest Rate Regulation

Governments may also consider adjusting the interest rate, which is captured by r in our framework. A marginal increase in r impacts the joint surplus directly by improving the return of the safe project. However, there is also an indirect effect through the change of the equilibrium investment levels x_k's:

$$\frac{dS(x_k)}{dr} = S'(x_k)\frac{\partial x_k}{\partial r} + 1 - x_k = \underbrace{(pf'(x_k) - r)\frac{\partial x_k}{\partial r}}_{\text{Indirect effect}} + \underbrace{1 - x_k}_{\text{Direct effect}}.$$

One can apply the implicit function theorem to $\mathcal{L}_k(x_k) = 0$ to show that

$$\frac{\partial x_k}{\partial r} = \frac{x_k}{\mathcal{L}'_k(x_k)} < 0.$$

That is, as r becomes higher, the investments for the risky project are reduced in equilibrium because the lender's opportunity cost is increased.

The indirect effect is thus negative if and only if $S'(x_k) > 0 \iff x_k < x^*$. Hence, the direct and indirect effects are of opposite signs if there is underinvestment, $x_v, x_0 < x^*$, so it is unclear whether increasing the interest rate is welfare-improving. On the other hand, if there is overinvestment, $x_v, x_0 > x^*$, this policy unambiguously increases social welfare because not only is the return from the safe project improved, but also the equilibrium investment levels are reduced, mitigating overinvestment.

4.3 Resolving Uncertainty

Suppose that the central bank makes public announcements about the true state of the world. Since ω_i becomes common knowledge, the holder of the collateral knows that she can never sell the good to the consumer if $\omega \neq \omega_v$, meaning that the collateral does not affect the investment levels:

$$x_0^F = x^N.$$

On the other hand, if $\omega = \omega_v$, the value of holding the collateral to the borrower and the lender becomes $W_v = v$ since the probability of selling it is 1:

$$x_v^F = \min\{1, \bar{x}\},$$

where $\bar{x}$ is the unique solution to

$$p(1-\sigma)f(x) + (1-p)v - rx = 0.$$

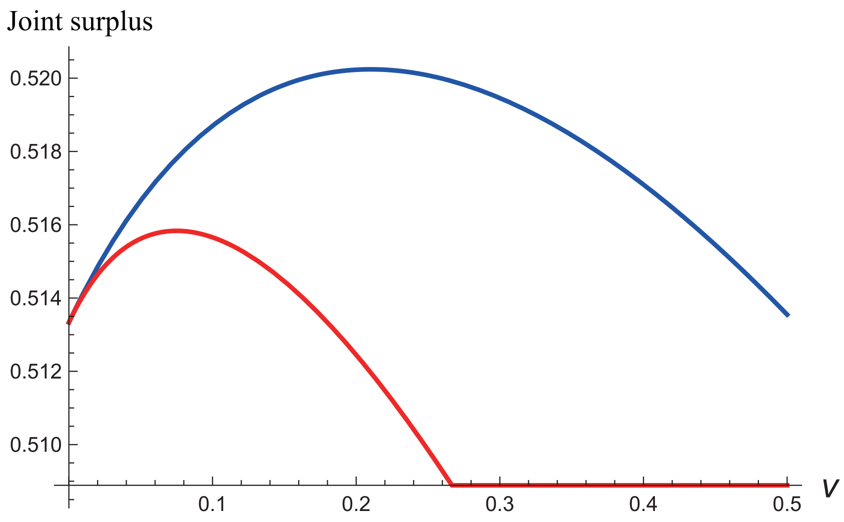

Figure 4: The ex-ante joint surplus under the equilibrium investment levels with uncertainty (blue) and without uncertainty (red) as v varies when $f(x) = \sqrt{x}$, $p = \frac{1}{2}, \sigma = \frac{3}{5}, r = \frac{1}{3}$ and $q = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.

Thus, resolving uncertainty improves social welfare if and only if

$$q_v S\left(x_v^F\right) + (1 - q_v) S\left(x^N\right) \geq q_v S(x_v) + (1 - q_v) S(x_0).$$

Notice that this policy increases the equilibrium investment level at ω_v (i.e., $x_v^F > x_v$) while lowering it at $\omega \neq \omega_v$ (i.e., $x_0 > x^N$). In other words, the dispersion in the asset's price with respect to fundamentals is *increased* by resolving uncertainty, which is unlike the models with symmetric information (e.g., Martin and Ventura (2012) and Hirano and Yanagawa (2016)). Therefore, the policy *strictly lowers* social welfare whenever overinvestment occurs only at Ω_v initially:

$$x_0 < x^* < x_v.$$

Our numerical exercise suggests that, even in other cases, resolving uncertainty does not improve social welfare, and that bubbles can play a socially beneficial role. (Figure 4). We leave further characterizations to future research.[11]

5 Payment Puzzle

Lagos (2011) raised the payment puzzle: "Why even to these days are assets used as collateral

[11]The effect of information policies aiming to burst bubbles is well-examined in the literature (Asako and Ueda, 2014; Conlon, 2015; Holt, 2019). Holt (2019) considers risk-averse agents and, like ours, identifies environments where such policies are detrimental.

instead of simply as a means of payment?" His point is also suggested by Martin and Ventura (2016).[12] In many settings, these two are equivalent, as these papers have shown. But are there any positive reasons why collateral, rather than payment, is used in reality?

In our framework, payment and collateral are not equivalent. To see this, suppose the borrower *sells* the asset to the lender to get x, and the return from the project is solely consumed by the borrower. Then, the lender would be willing to buy the asset at Ω_k if

$$r(1-x) + W_k \geq r \iff x \leq \frac{W_k}{r},$$

and the borrower would be willing to sell the asset if

$$pf(x) \geq W_k \iff x \geq f^{-1}\left(\frac{W_k}{p}\right).$$

Thus, there exists x at which both the borrower and the lender would trade if and only if

$$\begin{aligned} f^{-1}\left(\frac{W_k}{p}\right) \leq \frac{W_k}{r} \iff \frac{W_k}{p} \leq f\left(\frac{W_k}{r}\right) &\iff f\left(\frac{W_k}{r}\right) - \frac{W_k}{p} \geq 0 \\ &\iff \frac{f(W_k/r)}{W_k/r} \geq \frac{r}{p}. \end{aligned} \tag{4}$$

Compared to the equilibrium condition (2) for the collateralized contract,

$$\frac{(1-p)W_k}{p\sigma} \leq f\left(\frac{(1-p)W_k}{r\sigma}\right) \iff \frac{f((1-p)W_k/(r\sigma))}{(1-p)W_k/(r\sigma)} \geq \frac{r}{p},$$

we can check that the two conditions are equivalent only in the knife-edge case, $1-p=\sigma$, whereas if $1-p<\sigma$, condition (2) is strictly implied by condition (4). That is, an equilibrium may fail to exist with the selling contract while the collateralized trading mechanism supports an equilibrium. Further, even when trade is possible under both mechanisms, they predict different investment levels and social welfare in general. For instance, if $\sigma \approx 1$, then the collateralized contract induces $x_k \approx \frac{(1-p)W_k}{r}$, which is strictly smaller than the maximum investment level $x = \frac{W_k}{r}$ that the lender would accept under the selling contract. In this case, the collateralized contract induces a higher joint surplus than the selling contract when the size of bubbles is too high.

The crucial difference between the two is the following: Under the payment contract, the borrower can compensate the lender only through the asset, and its value is unconditional on the project's success or its return (i.e., lump-sum transfer). In contrast, under the collateralized contract, the compensation made to the lender consists of a share of the investment return and holding the asset upon the project's failure, which is contingent but varies in size with the investment level. This distinction plays a key role in shaping both parties' incentives to trade and, consequently, the equilibrium conditions and outcomes.

[12] See also Awaya et al. (2021) and Madison (2024).

Appendix A: Omitted Proofs

Proof of Corollary 1. Since $f(1) > \frac{r}{p}$, condition (1) is satisfied for sufficiently large σ if $\frac{r}{p} > \frac{W_v(1-p)}{p}$, which is equivalent to Assumption 2. Further, condition (2) is equivalent to

$$\frac{f\left((1-p)W_k/(r\sigma)\right)}{(1-p)W_k/(r\sigma)} \geq \frac{r}{p}.$$

Under Assumption 2, $\frac{(1-p)W_k}{r\sigma} < 1$ for sufficiently large σ. The strict concavity of $f(x)$ with $f(0) = 0$ then implies

$$\frac{f\left((1-p)W_k/(r\sigma)\right)}{(1-p)W_k/(r\sigma)} > f(1) > \frac{r}{p},$$

and therefore the condition (2) holds for large enough σ.

Proof of Lemma 4. Recall that x_k is the unique solution to

$$\mathcal{L}_k(x) = p(1-\sigma)f(x) + (1-p)(1-\alpha)W_k - rx = 0, \tag{5}$$

and that $\mathcal{L}'(x_k) = p(1-\sigma)f'(x_k) - r < 0$. By the implicit function theorem, we have

$$\frac{\partial x_k}{\partial \alpha} = \frac{(1-p)W_k}{p(1-\sigma)f'(x_k) - r} < 0.$$

Similarly,

$$\frac{\partial x_k}{\partial \sigma} = \frac{pf(x_k)}{p(1-\sigma)f'(x_k) - r} < 0,$$

$$\frac{\partial x_k}{\partial r} = \frac{x_k}{p(1-\sigma)f'(x_k) - r} < 0,$$

and

$$\frac{\partial x_k}{\partial W_k} = \frac{-(1-p)(1-\alpha)}{p(1-\sigma)f'(x_k) - r} > 0.$$

Proof of Proposition 1. It is clear that $S^C = S^N$ when $\alpha = 1$. To see strict concavity, note that, by the definition of x_k,

$$f(x_k) = \frac{rx_k - (1-p)(1-\alpha)W_k}{p(1-\sigma)}.$$

Therefore,

$$S^C = E\left(pf\left(x_k\right) + r\left(1 - x_k\right)\right) = \frac{r\sigma}{1-\sigma}E\left(x_k\right) - \frac{(1-p)(1-\alpha)}{1-\sigma}E\left(W_k\right) + r,$$

and so

$$\frac{\partial S^C}{\partial \alpha} = \frac{1-p}{1-\sigma}\left(r\sigma E\left(\frac{W_k}{p(1-\sigma)f'(x_k)-r}\right) + E\left(W_k\right)\right).$$

By Lemma 4, x_k is decreasing in α. Also, the strict concavity of f implies that $f'(x_k)$ is increasing in α, and thus $\frac{\partial S^C}{\partial \alpha}$ decreases in α. This shows that S^C is strictly concave in α, which, in turn, implies that if v is sufficiently small so that $\frac{\partial S^C}{\partial \alpha} \leq 0$ at $\alpha = 0$, we have $\frac{\partial S^C}{\partial \alpha} \leq 0$ for all α. The optimal macroprudential policy in this case is $\alpha^* = 0$.

Finally, observe that the derivative of S^C with respect to α can be written as

$$\frac{\partial S^C}{\partial \alpha} = E\left(\left(pf'\left(x_k\right) - r\right)\frac{\partial x_k}{\partial \alpha}\right). \tag{6}$$

Since $x^N < x^*$ implies $f'\left(x^N\right) > \frac{r}{p}$, we have

$$\frac{\partial S^C}{\partial \alpha}\Big|_{\alpha=1} = E\left(\left(pf'\left(x^N\right) - r\right)\frac{\partial x_k}{\partial \alpha}\right)\Big|_{\alpha=1} < 0$$

when $x^N < x^*$.

Proof of Proposition 2. From equation (6), the following first-order condition should be satisfied:

$$E\left(\left(pf'\left(x_k\right) - r\right)\frac{\partial x_k}{\partial \alpha}\right) = 0 \text{ at } \alpha = \alpha^*.$$

Since $\frac{\partial x_k}{\partial \alpha} < 0$, it follows that

$$\left(pf'\left(x_v\right) - r\right)\left(pf'\left(x_0\right) - r\right) < 0.$$

Since $x_v > x_0$ and f is strictly concave, this implies

$$pf'\left(x_v\right) - r < 0 < pf'\left(x_0\right) - r,$$

which is equivalent to $x_v > x^* > x_0$.

To check (i), note that, at $\alpha = \alpha^*$,

$$\begin{aligned}
& E\left(\left(pf'\left(x_k\right) - r\right)\frac{\partial x_k}{\partial \alpha}\right) = 0 \\
\iff & q_v\left(1-p\right)W_v\left(\frac{pf'\left(x_v\right)-r}{p\left(1-\sigma\right)f'\left(x_v\right)-r}\right) + \left(1-q_v\right)\left(1-p\right)W_0\left(\frac{pf'\left(x_0\right)-r}{p\left(1-\sigma\right)f'\left(x_0\right)-r}\right) = 0 \\
\iff & q_v\left(\frac{1}{1-p\sigma\left(p-\frac{r}{f'(x_v)}\right)^{-1}}\right) + \left(1-q_v\right)\left(\frac{q_\phi}{q_\phi+q_0}\right)\left(\frac{1}{1-p\sigma\left(p-\frac{r}{f'(x_0)}\right)^{-1}}\right) = 0 \\
\iff & q_v\left(\frac{1}{1-p\sigma\left(p-\frac{r}{f'(x_v)}\right)^{-1}}\right) + q_\phi\left(\frac{1}{1-p\sigma\left(p-\frac{r}{f'(x_0)}\right)^{-1}}\right) = 0.
\end{aligned} \tag{7}$$

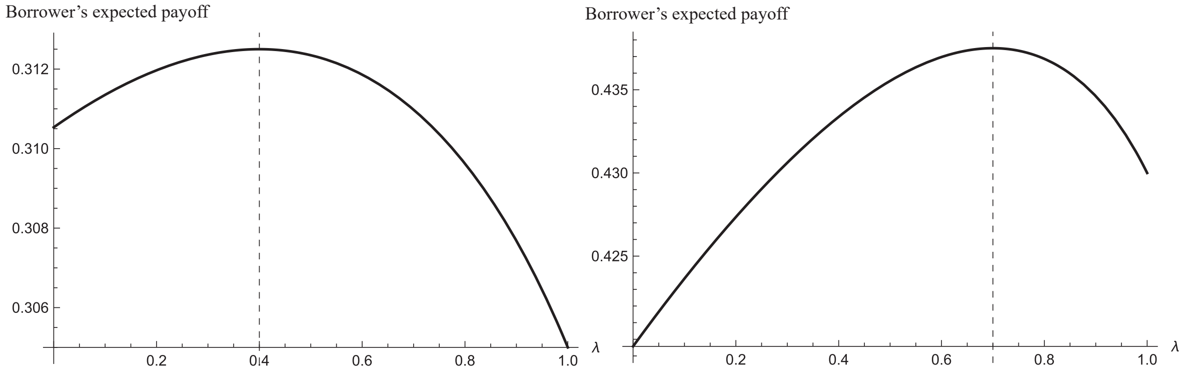

Figure 5: The borrower's expected payoff as λ varies at the information state $\Omega_0 = \{\omega_\phi, \omega_0\}$ (left) and $\Omega_v = \{\omega_v\}$ (right) when $f(x) = \sqrt{x}, p = \frac{1}{2}, \sigma = \frac{3}{5}, r = \frac{1}{3}, v = \frac{1}{2}$, and $q = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.

As v increases, x_k also increases, so $f'(x_k)$ decreases. Thus, the left-hand side in equation (7) is increased with v when α remains unchanged. Therefore, α^* should increase correspondingly because x_k is decreasing in α.

Finally, consider a change of prior from $q = (q_v, q_\phi, q_0)$ to $q' = (q_v + \epsilon, q_\phi, q_0 - \epsilon)$. Since the expected values of holding collateral,

$$W_v = \left(\frac{q_v}{q_v + q_\phi}\right) v = \left(\frac{1}{1 + \frac{q_\phi}{q_v}}\right) v,$$

and

$$W_0 = \left(\frac{q_\phi}{q_\phi + q_0}\right)\left(\frac{q_v}{q_v + q_\phi}\right) v = \left(\frac{1}{1 + \frac{q_0}{q_\phi}}\right)\left(\frac{1}{1 + \frac{q_\phi}{q_v}}\right) v,$$

become higher, $f'(x_k)$ decreases for each $k = v, 0$. As a result, each term in the bracket of the left-hand side in equation (7) is increased. Furthermore, the first bracket is strictly positive, implying that the change in the prior induces a strictly positive overall effect, causing the left-hand sideto increase strictly. Therefore, α^* becomes higher because x_v and x_0 are strictly decreasing in α.

Appendix B: Partial Collateralization

In this appendix, we consider the following generalized contracts: "the borrower keeps the collateral if the project is successful, but otherwise, it is held by the lender with probability $1-\lambda$". The two environments analyzed in the main text are the special cases of this generalized contract, where the case without collateral corresponds to $\lambda = 1$, while the case with collateral corresponds to $\lambda = 0$. The lender's incentive-compatibility condition becomes

$$p\left(1-\sigma\right)f\left(x\right)+\left(1-p\right)\left(1-\lambda\right)W_k+r\left(1-x\right)\geq r$$
$$\Longleftrightarrow \mathcal{L}_k\left(x;\lambda\right)\equiv p\left(1-\sigma\right)f\left(x\right)+\left(1-p\right)\left(1-\lambda\right)W_k-rx\geq 0,$$

whereas the borrower's incentive-compatibility condition becomes

$$\left(p+\left(1-p\right)\lambda\right)W_k+\sigma pf\left(x\right)\geq W_k \iff f\left(x\right)\geq\frac{W_k\left(1-\left(p+\left(1-p\right)\lambda\right)\right)}{p\sigma}.$$

Clearly, an increase in λ reduces the maximum level (resp. minimum level) of investments that the lender (resp. borrower) would find it profitable to trade. Therefore, partial collateralization (i.e., $\lambda > 0$) reduces the investment levels, allowing for improvement of joint surplus when investments are inefficiently excessive.

Then, does the borrower have an incentive to offer a contract with partial collaterlization? This involves a clear trade-off: as a contract with higher λ is signed, the borrower is more likely to retain the asset, but this also reduces the amount that can be borrowed for investment. That is,

$$\text{Borrower's payoff}=\underbrace{\left(p+\left(1-p\right)\lambda\right)W_k}_{\text{Increasing in }\lambda}+\underbrace{\sigma pf\left(\bar{x}_k(\lambda)\right)}_{\text{Decreasing in }\lambda\text{ due to reduced }\bar{x}_k(\lambda)}$$

where $\bar{x}_k(\lambda)$ is the maximum investment level satisfying $\mathcal{L}_k\left(x;\lambda\right)\geq 0$. Due to this tradeoff, it may be optimal to offer partial collateralization (See Figure 5).[13] Importantly, the borrower's optimal contract will induce the efficient investment level because

$$\begin{aligned}\text{Borrower's payoff}&=\left(p+\left(1-p\right)\lambda\right)W_k+\sigma pf\left(\bar{x}_k(\lambda)\right)\\&=pf\left(\bar{x}_k\left(\lambda\right)\right)-r\bar{x}_k\left(\lambda\right)+W_k\\&=S(\bar{x}_k(\lambda))+W_k-r,\end{aligned}$$

where the first equality follows from $\mathcal{L}_k(\bar{x}_k(\lambda);\lambda) = 0$. Clearly, the borrower's payoff is maximized at $\bar{x}_k(\lambda) = x^*$, suggesting that inefficiency disappears once the borrower and the lender can commit to more contingent contracts.

[13]Note that we ignore a subtle potential issue: if different λ is chosen at different information states, Ω_0 and Ω_v, then the consumer would (imperfectly) infer about the state of world by observing "who sells the asset", which in turn affects her willingness-to-pay. We assume that this type of learning does not arise—this would be the case, for instance, when the asset is sold anonymously or indirectly.

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