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Search in Credence Goods Markets

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Abstract

We study consumer search in markets for horizontally differentiated expert services. A consumer faces a problem of unknown severity and consults experts for diagnosis and treatment. Because experts differ in match value, consumers search not only for honest recommendations but also for a better match. We show that an honest equilibrium can arise when search costs are sufficiently low, whereas in markets with homogeneous experts, any positive search cost eliminates honesty. Thus, horizontal differentiation provides a novel mechanism through which consumer search disciplines expert conduct. We also show that improving the search pool can unintentionally increase expert cheating.

Keywords: Credence goods, expert services, search, horizontal differentiation, honesty, platform design

JEL: D00, D42, D80, D82, D83, L00

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1 Introduction

Consumers often find themselves in situations where they must rely on experts to diagnose a problem and recommend a solution, simply because they lack the technical knowledge to evaluate what they truly need. This reliance is ubiquitous—whether a patient is deciding between medical procedures, a homeowner is responding to a contractor's assessment, or an investor is receiving financial advice. A persistent concern in these settings is that experts may exploit their informational advantage by recommending unnecessary services for financial gain. Not surprisingly, consumers often try to protect themselves by seeking second opinions.

While consumers seek honest opinions, expert services are rarely homogeneous. Doctors differ in bedside manner and communication style; handymen vary in expertise and availability; financial advisors offer distinct investment philosophies. Consumers, therefore, care not only about receiving an honest recommendation but also about finding an expert with whom they have a good match. Despite the prevalence of horizontal differentiation in expert markets, the credence goods literature has largely abstracted away from this dimension.

This paper studies consumer search when consumers value both honest recommendations and horizontal match quality. We ask the following questions: What types of equilibria arise when consumers search for honesty and fit? How does the degree of product differentiation shape expert incentives? Do improved matching technologies such as digital platforms that pre-screen experts promote or undermine honest conduct? And how does price competition interact with search to discipline or distort expert behavior?

Our paper bridges two literature. The first is the large literature on credence goods and expert markets, which highlights how asymmetric information about treatment needs creates the potential for fraud. The second is the literature on consumer search in horizontally differentiated product markets. We show that combining these two forces generates a novel disciplinary mechanism: search for fit can indirectly discipline dishonest expert behavior. Due to the intricate interplay between recommendation incentives and product differentiation, technologies designed to improve matching can unintentionally weaken this mechanism and backfire.

Our paper builds on Wolinsky (1993) who studied consumer search in a market with homogeneous expert services. A puzzling insight from Wolinsky is that honest equilibrium cannot exist, even when search costs are arbitrarily small. The intuition goes as follows. If consumers expect recommendations to be honest, they do not search; but if they never search, experts always have an incentive to recommend the expensive treatment, making honesty unsustainable.

We depart from Wolinsky (1993) by introducing horizontal product differentiation and two-sided asymmetric information. We consider a market with $N \geq 2$ ex ante identical experts. A consumer knows that she has a problem but does not know whether its severity is high or low. Treating a high-severity problem is more costly than treating a low-severity one. When a consumer accepts an expert's recommendation, she obtains (i) the benefit of resolving the problem and (ii) a match value with the expert. This match value is an independent and identically distributed random draw across experts and captures horizontal product differentiation.

The consumer begins by randomly visiting an expert and privately learns her match value with that expert. The expert, who does not observe the consumer's search history or match value, perfectly diagnoses the problem and privately observes its severity. He then recommends either the inexpensive or the expensive treatment. The consumer chooses whether to accept the recommendation or reject it and incur a search cost to seek a second opinion. If she rejects, the problem remains unresolved, and she receives no match value. If she accepts, the expert is liable for resolving the problem.

Our results are threefold. First, we characterize the symmetric pure strategy equilibria for any fixed prices. Depending on the price profile, the equilibrium may feature honest recommendations, where all experts recommend the appropriate treatments, or fraudulent recommendations, where all experts recommend the expensive treatment regardless of the actual problems.

When search costs are sufficiently low, an honest equilibrium exists provided that the profit margin from cheating is not too large. This stands in sharp contrast to Wolinsky (1993), where expert services are homogeneous. The intuition is as follows. In an honest equilibrium, consumers search only to obtain a better match, so their optimal stopping rule is characterized by a reservation match value. When a consumer receives a recommendation for expensive treatment, she accepts it if her current match value exceeds this reservation value; otherwise, she continues to search. Consequently, if an expert recommends the expensive treatment when it is unnecessary, he risks losing the consumer when she shops around for a better match and inadvertently discovers the fraud. When the gain from cheating is small relative to the profit margin from honest recommendations, this risk deters experts from upselling.

In contrast, when expert services are homogeneous, consumers have no reason to search in an honest equilibrium, so cheating goes undetected and honesty cannot be sustained. However, with product differentiation, the search for a better fit by the consumer generates a powerful disciplinary mechanism, highlighting the important role of horizontal differentiation in curbing dishonest expert behavior.

A fraudulent equilibrium exists when the profit margin from cheating is sufficiently high. In such an equilibrium, all experts always recommend the expensive treatment, so consumers' stopping rule is again characterized by a reservation match value: an expert retains consumers with high match values and loses those with low match values. If an expert deviates and recommends the inexpensive treatment, that deviation acts as a credible signal of honesty and induces the consumer to accept the recommendation with probability one. Thus, by deviating, the expert secures a higher acceptance rate but earns a lower profit margin. A fraudulent equilibrium is sustainable when the gain from the higher margin on the expensive treatment outweighs the loss in demand from consumers with low match values.

The price range that supports an honest equilibrium expands as product differentiation increases or search costs fall. When expert services are more differentiated or when searching is cheaper, consumers are more willing to incur the search cost to find a better match. A higher search propensity increases the likelihood that a consumer who receives an upsell recommendation will continue shopping and inadvertently uncover the fraud. This increased risk strengthens the incentives of experts to behave honestly. In contrast, the price range that supports a fraudulent equilibrium expands as differentiation decreases or search costs increase.

Fraudulent equilibria also become harder to maintain as the number of experts increases. With more experts in the market, each expert has a smaller equilibrium market share. If an expert deviates by recommending the inexpensive treatment, the boost in demand he receives from signaling honesty can more than offset the loss in profit margin. As a result, deviation becomes more attractive, making fraudulent behavior more difficult to sustain when competition is strong.

For any finite number of experts, the honest equilibrium is the unique symmetric pure-strategy equilibrium when the profit margin from cheating is sufficiently low, while the fraudulent equilibrium is unique when the margin is sufficiently high. When the profit margin lies in an intermediate range, both equilibria can coexist. As the number of experts increases, this intermediate region shrinks and converges to a single threshold. In the limit, honesty is sustainable if and only if the profit margin from cheating is below this threshold, while fraud is sustainable if and only if the margin is above it.

Second, we examine how an improved search pool affects experts' incentives to cheat. This question is particularly relevant given the rise of digital platforms that aim to refine consumer–expert matching. Platforms such as HomeAdvisor, ZocDoc, and ZOE Finance help consumers connect with contractors, physicians, and financial advisors.[1] While such platforms can increase the consumers'

[1]These platforms generate a list of experts based on self-reported preferences such as type of service, personal taste,

expected match value, it is not obvious whether better matching promotes or undermines honest expert behavior. In particular, we ask whether improved matching might inadvertently lead to more fraudulent recommendations.

We model an improved search pool by truncating the match value distribution from below, capturing the idea that platforms pre-screen experts and exclude those below a minimum standard. For example, a platform can remove experts who are unavailable for urgent repairs or who are located outside the preferred radius of the consumer. We show that, although an improved search pool increases the consumer's expected match value, it may unintentionally strengthen experts' incentives to upsell. The intuition is that better matching makes consumers more likely to be satisfied with the first expert they meet, reducing their propensity to search. The lower search propensity diminishes the chance that cheating is detected, thus weakening the disciplinary effect of consumer search. As a result, improving the search pool can make it harder for honest equilibria to sustain and increase the scope of fraudulent recommendations. These findings highlight that platform design can have unintended consequences: by decreasing the frequency of consumer search, well-intentioned matching improvements may inadvertently encourage dishonest expert behavior.

Third, we investigate how price competition affects expert conduct. Price competition undermines experts' incentives to provide honest recommendations by eroding the profit margin associated with inexpensive treatment. Nevertheless, the honest equilibrium can still be sustained under certain conditions. Specifically, when the market contains a large number of experts and search costs are sufficiently low, an honest equilibrium exists if the distribution of match values has a thin upper tail. This finding suggests that unfettered price competition, while potentially beneficial for consumers in other markets, can deteriorate expert conduct in credence goods settings.

The credence goods literature studies experts' incentive to cheat in various market environments (Pitchik and Schotter (1987), Wolinsky (1993), Fong (2005), Dulleck and Kerschbamer (2006), Liu (2011), Chiu and Karni (2021), Fong et al. (2022), Karni (2024), and Cao et al. (2025)). The existing literature assumes expert services are homogeneous, and consumers search for honest recommendations. Our paper departs from the literature by considering a market with differentiated expert services where consumers search for honest recommendations and a better fit. Cao et al. (2025) studied a market where consumers search for honest recommendations and cheaper prices. They assume that the market consists of informed consumers who observe prices but search for honest recommendations and uninformed consumers who incur a search cost to learn recommendations

and urgency.

along with prices. In contrast, we do not model informed consumers; instead, we introduce product differentiation.

Another main difference between our paper and the existing literature is that our model features two-sided asymmetric information; the consumer privately learns her match value with an expert, whereas the expert privately learns the nature of the consumer's problem. In contrast, the existing literature focuses on one-sided asymmetric information. That is, the expert has private information.

Our paper is also related to the consumer search literature. Weitzman (1979), Wolinsky (1986), and Anderson and Renault (1999) studied consumer search with differentiated products. In these papers, consumers do not learn any underlying state as they sample products. Hence, the information about sampled products is independent of the attributes of unsampled products. In our paper, experts' recommendations are based on the consumer's problem and therefore are correlated.

The paper is organized as follows. Section 2 introduces the model. Section 3 characterizes equilibria with fixed prices. Section 4 analyzes the impact of improved matching technologies on expert conduct. Section 5 endogenizes the prices. Section 6 gives additional discussion on the setting used in Section 3. Section 7 concludes. In the Online Appendix, we provide an additional analysis with mixed strategies.

2 Model

A consumer becomes aware of a problem θ, which is either H(high) or L(low). It is common knowledge that $Pr(\theta = H) = \alpha$, with $\alpha \in (0, 1)$. The consumer does not know the nature of the problem and consults experts for diagnoses and services. There are N, $N \geq 2$, experts who are ex ante identical for consumers. Each expert can provide two types of treatment, $t = L, H$, at prices p_L and p_H, with $p_L < p_H$. The expert incurs a cost c_θ to fix a problem θ, $\theta = H, L$, with $c_L < c_H$.

We normalize a consumer's utility from not receiving any treatment to zero. If a consumer has a θ-problem and receives treatment t, her benefit from the treatment is $b_{\theta t}$, where

$b_{\theta t}$	$t = L$	$t = H$
$\theta = L$	b	b
$\theta = H$	0	b

and $b > 0$. The payoff table says that the consumer receives a benefit b if treatment t matches the problem θ. Moreover, treatment H can fix both problems, whereas treatment L can fix only problem L. If Expert i fixes the consumer's problem, the consumer receives the benefit b and a match value

v_i. The consumer's match values are independently and identically distributed across experts. They are drawn from a distribution $F(v)$ on an interval $[\underline{v}, \bar{v}]$, where $F(v)$ is differentiable everywhere. If Expert i fixes the consumer's problem at a price $p \in \{p_L, p_H\}$, the consumer receives a payoff

$$u = \lambda v_i + (1 - \lambda)b - p,$$

where $\lambda \in (0, 1)$ is the weight of the match value. The parameter λ captures the degree of product differentiation: experts' services are more differentiated with a larger λ.[2] We assume the match value is generated only by a successful treatment. The consumer does not enjoy the match value with Expert i if the treatment does not address her problem or if she rejects the expert recommendation. The consumer incurs a search cost $s > 0$ for each expert she samples.

Hence, if the consumer visited n experts in total and ended up being treated by Expert i at a price p, her payoff is $\lambda v_i + (1 - \lambda)b - p - ns$. The consumer's payoff is $-ns$ if she rejects all treatments offered by the n experts.

Experts are risk-neutral and wish to maximize profits. Following the literature, we assume that the consumer does not observe the severity of her problem or the type of treatment the expert provides.[3] However, the consumer can verify the outcome of a treatment, i.e., whether the treatment benefit is b or 0.[4] Because the treatment outcome is verifiable, an expert must provide the necessary treatment to resolve the consumer's problem upon payment. This is termed *Liability* in the literature. The non-observability of the implemented treatments and Liability together imply that if an expert recommends treatment θ' to a θ-problem, the expert will provide treatment θ and incur a cost c_θ. If the consumer accepts the recommendation θ', the expert's payoff is

$$\pi(r = \theta' | \theta) = p_{\theta'} - c_\theta,$$

where r denotes a recommendation.

Because $p_L < p_H$, experts have incentives to recommend treatment H for an L-problem and overcharge consumers. We make the following assumptions and discuss their role at the end of this section:

A1. Prices are fixed at (p_L, p_H), with $c_L \leq p_L$, $c_H \leq p_H$, and $p_L < p_H$.

[2]Expert services are homogeneous credence goods if $\lambda = 0$ and pure search goods if $\lambda = 1$.

[3]This means that a treatment cost is not verifiable.

[4]When consumers cannot verify the treatment provided by experts, trade will not occur if experts are not liable for the treatment outcome.

A2. Price difference, $\Delta p \equiv p_H - p_L$, is sufficiently large such that $\Delta p \geq \Delta v$, where $\Delta v = \bar{v} - \underline{v}$. This assumption is termed "price dominance."

A3. Consumers' benefit b is sufficiently large.

A4. Experts cannot observe the consumer's search history.

A5. Consumers search with costless recall.

Timing of the game:

Stage 1. Nature draws consumers' problems. Consumers are unaware of the severity of their problems and randomly visit an expert.

Stage 2. Upon visiting an expert, the consumer privately observes the match value with the expert.

Stage 3. The expert perfectly diagnoses the consumer's problem and recommends a treatment.

Stage 4. The consumer decides whether to take the treatment or sample another expert.

Strategies: A consumer's history after visiting k experts is $H^k = [\underline{v}, \bar{v}]^k \times [L, H]^k$. The consumer's strategy upon visiting expert $k+1$ is $A : H^k \times [\underline{v}, \bar{v}] \times [L, H] \to \{0, 1\}$. Under the liability assumption, experts must provide treatment H for problem H. Because an expert does not observe the consumer's search history, his strategy is a probability $\beta_\theta = Pr(r = H|\theta) \in [0, 1]$, $\theta \in \{H, L\}$, which prescribes the probability that the expert recommends treatment H to a θ-problem.

We characterize symmetric perfect Bayesian equilibria. A symmetric PBE consists of a pair $\{A^*(.), \beta^*\}$ and a belief system $(\mu^e(.), \mu^c(.))$, where $\mu^e(.)$ is the expert's beliefs about the consumer's search history and match value, and $\mu^c(.)$ is the consumer's belief about the nature of her problem and experts' recommendation strategies. Players' strategies are mutually best responses given their beliefs. Beliefs are updated according to Bayes' rule whenever possible.

It is useful to discuss the assumptions before proceeding with the analysis. Assuming fixed prices (A1) allows us to focus on consumer search for honest recommendations and better fits, and it also has real-world applications. For example, prices for healthcare services are often regulated. In Section 5, we analyze the case where experts also compete in prices. There, consumers search for cheaper prices in addition to honest recommendations and a better fit.

The price dominance condition (A2) ensures that an expert's honesty is more important to the consumer than her match value with the expert. Under this assumption, match values determine the consumer's choice of experts only if experts make the same recommendations. If the consumer receives conflicting recommendations, she strictly prefers the experts recommending p_L to those recommending p_H, irrespective of her match values. This assumption highlights the credence nature of experts' services and reflects the observation that honesty is a primary concern for consumers in credence goods markets.

Assumptions A3-A5 are adopted in the literature (Wolinsky (1993)). A sufficiently high b ensures that the consumer will not exit the market with an unresolved problem. The unobservability of search history rules out discriminatory recommendations based on the consumer's search history. Search with costless recall means the consumer can return to an expert she has visited before without incurring an additional search cost. We maintain these assumptions to facilitate comparison with Wolinsky (1993).

3 Equilibria under fixed prices

In this section, we study equilibria with fixed prices. We restrict our attention to the price range $(p_L, p_H) \in [c_L, c_H) \times [c_H, \bar{p}]$, where $\bar{p} = \lambda \underline{v} + (1-\lambda)b$. The upper bound $\bar{p}$ guarantees that consumers will accept a treatment from an expert. Because $p_L < c_H \leq p_H$, an expert will recommend treatment H to an H-problem. In this section, we focus on the expert's recommendation strategy when the consumer has an L-problem. We restrict our attention to this price range because, under endogenous pricing, low search costs intensify competition and will drive equilibrium p_L below c_H (see Section 5). We discuss the equilibria under prices $(p_L, p_H) \in (c_H, \bar{p}]^2$ in Section 6.

3.1 Honest equilibrium

Honest equilibrium refers to a separating equilibrium in which experts recommend treatment θ to a θ-problem, $\theta \in \{L, H\}$. Suppose there exists an honest equilibrium. In such an equilibrium, all experts will recommend the same treatment, which matches a consumer's problem. Therefore, the consumer will only search for a better match value. Following Wolinsky (1983), if the consumer has a θ-problem, her optimal stopping policy is characterized by a reservation match value denoted by v_θ^*. The consumer will sample another expert if and only if her match value is below a cutoff v_θ^*, which is determined by

$$s = \lambda \int_{v_\theta^*}^{\bar{v}} (x - v_\theta^*) dF(x) = \lambda(\bar{v} - v_\theta^* - \int_{v_\theta^*}^{\bar{v}} F(x)dx) = \lambda \int_{v_\theta^*}^{\bar{v}} (1 - F(x))dx. \tag{1}$$

The right-hand side strictly decreases in v_θ^*. Assume $0 < s < \bar{s} \equiv \lambda \int_{\underline{v}}^{\bar{v}} (1 - F(x))dx$. So, there exists a unique reservation match value $v_\theta^* = v^* \in (\underline{v}, \bar{v}), \theta = L, H$. Note that the reservation match value is the same for both types of problems.

In an honest equilibrium, given a recommendation r, a consumer's optimal strategy is to stop searching if and only if the highest sampled match value exceeds v^*. If the consumer's match values with all the experts are below v^*, she accepts the treatment from the expert who gives her the highest match value.

Suppose that an expert, say Expert i, diagnoses a consumer with problem L. We derive the expert's payoff from recommending treatment L, given that all other experts also recommend treatment L. We begin by deriving the conditional probability that the expert is the n-th expert visited by the consumer. The consumer visits Expert i with probability

$$\frac{1}{N} + \frac{F(v^*)}{N} + \frac{F(v^*)^2}{N} + \frac{F(v^*)^{N-1}}{N} = \frac{1}{N} \frac{1 - F(v^*)^N}{1 - F(v^*)},$$

where $\frac{F(v^*)^{n-1}}{N}, n = 1, ..., N$ represents the probability that Expert i is the n-th expert the consumer sampled in the sequence. The conditional probability that Expert i is the n-th expert the consumer sampled is

$$\frac{F(v^*)^{n-1}(1 - F(v^*))}{1 - F(v^*)^N}. \tag{2}$$

The consumer will accept treatment L from Expert i if her match value with Expert i exceeds the reservation value v^*, or if her match value with Expert i falls below v^* but is highest among all experts. The probability that the consumer accepts treatment L from Expert i is

$$\begin{aligned}
&\sum_{n=1}^{N} \frac{F(v^*)^{n-1}(1 - F(v^*))}{1 - F(v^*)^N} \left(\Pr(v_i \geq v^* | i = n) + \Pr(\max_{j \neq i}\{v_j\} < v_i < v^* | i = n) \right) \\
&= \sum_{n=1}^{N} \frac{F(v^*)^{n-1}(1 - F(v^*))}{1 - F(v^*)^N} \left(1 - F(v^*) + \frac{\int_{\underline{v}}^{v^*} F(v_i)^{N-1} dF(v_i)}{F(v^*)^{n-1}} \right) \\
&= 1 - F(v^*) + \frac{(1 - F(v^*))F(v^*)^N}{1 - F(v^*)^N} \\
&= \frac{1 - F(v^*)}{1 - F(v^*)^N}.
\end{aligned} \tag{3}$$

In the above equations, $\Pr(v_i \geq v^* | i = n)$ is the probability that the consumer stops searching after sampling Expert i, conditional on Expert i being the n-th expert in the sequence. Similarly,

$\Pr(\max_{j\neq i}\{v_j\} < v_i < v^*|i = n)$ is the probability that the consumer returns to Expert i after sampling all experts, conditional on Expert i being the n-th in the sequence. The second equality follows after substituting $\int_{\underline{v}}^{v^*} F(v_i)^{N-1} dF(v_i) = \frac{F(v^*)^N}{N}$. By (3), the expert's equilibrium payoff from recommending treatment L is

$$\pi(r = L|\theta = L) = (p_L - c_L)\frac{1 - F(v^*)}{1 - F(v^*)^N}.$$

Now, we analyze an expert's cheating incentives. We assume that, if a consumer receives conflicting recommendations, she believes that her problem is L. Suppose that Expert i deviates to recommending treatment H. The consumer is one of two types: (i) an inexperienced consumer who has not visited any other expert, and (ii) an experienced consumer who has visited at least one other expert. Conditional on arrival, the consumer is inexperienced with probability $\frac{1-F(v^*)}{1-F(v^*)^N}$ (see (2)) and experienced with complementary probability. This deviation is on the equilibrium path for an inexperienced consumer who believes that her problem is H, and that all other experts also recommend H. The inexperienced consumer accepts the recommendation if her match value with the deviating expert exceeds v^*, and continues searching otherwise.

Alternatively, if the consumer is experienced or inexperienced with a match value below v^*, she will sample multiple experts. Because all the sampled experts other than i will recommend treatment L, hence, the consumer will detect the cheating by Expert i. Under the price dominance condition (A2), the consumer will not accept Expert i's treatment. It follows that Expert $i's$ payoff from cheating is

$$\pi(r = H|\theta = L) = (p_H - c_L)\Pr(i = 1)\Pr(v_i > v^*) = (p_H - c_L)\frac{(1 - F(v^*))^2}{1 - F(v^*)^N},$$

where $Pr(i = 1) = \frac{1-F(v^*)}{1-F(v^*)^N}$ is the probability that expert i is the first expert consulted by the consumer. The no deviation constraint requires $\pi(r = L|\theta = L) \geq \pi(r = H|\theta = L)$, which gives

$$\frac{p_H - c_L}{p_L - c_L} \leq \frac{1}{1 - F(v^*)}. \tag{4}$$

Recall that the price dominance condition (A2) requires $p_H - p_L \geq \Delta v$. Hence, both the price dominance condition and the no cheating constraint (4) are satisfied if

$$\Delta v + p_L - c_L \leq p_H - c_L \leq \frac{p_L - c_L}{1 - F(v^*)}. \tag{5}$$

The set $[\Delta v + p_L - c_L, \frac{p_L - c_L}{1-F(v^*)}]$ is non-empty if and only if

$$p_L - c_L \geq \frac{\Delta v(1 - F(v^*))}{F(v^*)}. \tag{6}$$

Now, suppose that Expert i diagnoses a consumer with an $H-$problem. Because $p_L < c_H \le p_H$, recommending treatment L will at best lead to zero profit and is, therefore, weakly dominated by recommending treatment H. It follows that the expert will recommend treatment H for problem H.

We present the existence conditions for an honest equilibrium in the following proposition (proof omitted to avoid repetition).

Proposition 1. *For any $s \in (0, \bar{s})$, there exists an honest equilibrium if $p_L - c_L \ge \frac{\Delta v(1-F(v^*))}{F(v^*)}$ and $\Delta v + p_L - c_L \le p_H - c_L \le \frac{p_L - c_L}{1-F(v^*)}$.*

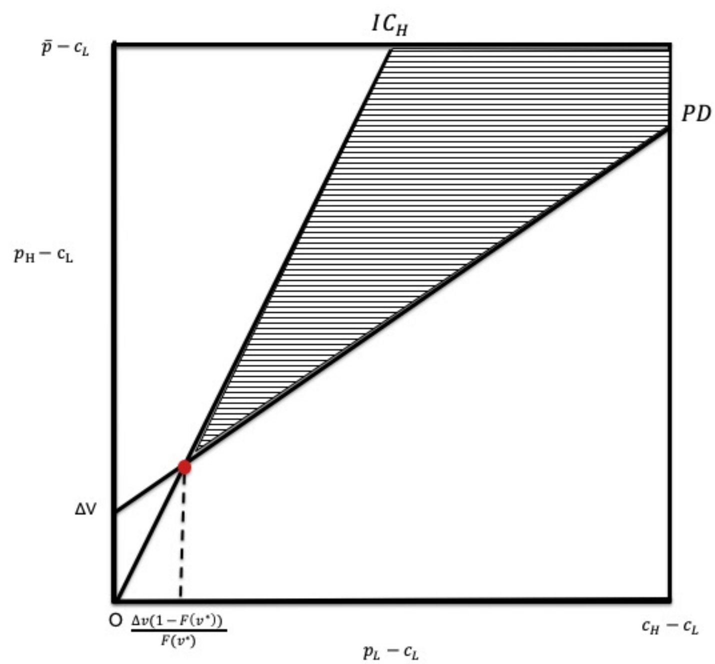

Figure 1: Honest equilibrium

PD corresponds to a binding price dominance condition (A2), and IC_H corresponds to a binding (4).

Figure 1 illustrates Proposition 1. The line PD is the binding price dominance constraint, and the line IC_H is the binding no cheating constraint (4). An honest equilibrium exists when the prices are in the shaded area. Note that, for an honest equilibrium to exist, the profit margin of an honest recommendation must exceed a threshold value $\frac{\Delta v(1-F(v^*))}{F(v^*)}$. This is because when $p_L - c_L$ falls below the threshold, the price dominance condition requires a price p_H too high to satisfy the no cheating constraint (4).

Proposition 1 draws a sharp contrast with the case of homogeneous expert services (Wolinsky, 1993). Note that the case of homogeneous expert services corresponds to the case $\lambda = 0$. In this case, an honest equilibrium does not exist for any arbitrarily small search costs. In our model, as $\lambda \to 0$, $v^* \to \underline{v}$.[5] Then, the no cheating condition (4) is violated for any $p_L < p_H$, rendering the honest equilibrium unsustainable.

The intuition is as follows. If the consumer has the same match values with all experts, she will only search for honest recommendations. The consumer will not incur a costly search for a second opinion if she expects all experts to be honest. However, if the consumer always accepts the first opinion, experts will always recommend the expensive treatment because cheating will never be detected. In our model, even if the consumer believes that experts are honest, she may still search for better match values. The consumer may inadvertently detect cheating in the quest for better match values. Hence, when an expert makes a fraudulent recommendation, he trades off a higher profit margin against a reduced demand. The expert will make honest recommendations if $\frac{p_H - c_L}{p_L - c_L}$, the profit margin from cheating relative to making the honest recommendation, is lower than $\frac{1}{1-F(v^*)}$. The existence of an honest equilibrium highlights the importance of horizontal product differentiation in disciplining expert conduct.

Lemma 1. *The price range in which an honest equilibrium exists expands as the search cost decreases or the degree of product differentiation increases. If $s \to 0$, an honest equilibrium exists for any $(p_L, p_H) \in [c_L, c_H) \times [c_H, \bar{p}]$, with $\Delta p > \Delta v$.*

As illustrated in Figure 1, there is an honest equilibrium in the price region between the line PD and IC_H. The price dominance condition (PD) does not depend on the search cost or the degree of product differentiation. However, the no cheating constraint (IC_H) becomes more relaxed as the ratio $\frac{s}{\lambda}$ decreases. To see it, we can rewrite (1) as

$$\frac{s}{\lambda} = \int_{v^*}^{\bar{v}} (1 - F(x))dx.$$

It is straightforward to see that v^* decreases in $\frac{s}{\lambda}$. Hence, as $\frac{s}{\lambda}$ decreases, $\frac{1}{1-F(v^*)}$ increases, and the line IC_H rotates counterclockwise, resulting in an expansion of the shaded area. As $s \to 0$, the line IC_H coincides with the vertical axis, and there is an honest equilibrium for prices that are higher than the treatment costs and satisfy the price dominance condition. This is a sharp contrast to the homogeneous expert services case (Wolinsky(1993)), where the honest equilibrium exists at $s = 0$, but it does not exist for any arbitrarily small s.

[5] Fix a small s, if $\lambda \to 0$, equation (1) fails to hold, and $s > \lambda \int_{\underline{v}}^{\bar{v}} (1 - F(x))dx$. The consumer will stop searching for any match value, i.e. $v^* = \underline{v}$.

3.2 Fraudulent equilibrium

We call an equilibrium a fraudulent equilibrium if experts recommend p_H to both types of problems. This section characterizes the conditions under which a fraudulent equilibrium exists. Suppose there exists an equilibrium in which all experts always recommend treatment H. In this case, a consumer will only search for a better match value. The consumer's optimal search policy is the same as in the honest equilibrium: she stops search if the highest sampled match value exceeds v^* (defined in (1)); if the consumer's match values with all the experts are below v^*, she buys from the expert who gives her the highest match value.

Suppose that Expert i diagnoses a consumer with problem L. If the expert recommends treatment H, the probability that the consumer accepts the recommendation is (3), the same as the probability that a consumer accepts treatment L in the honest equilibrium case. This is because (i) in both the honest and the fraudulent equilibria, a consumer only searches for a better match value, and (ii) the consumer adopts the same search policy. It follows that Expert i's expected payoff from cheating is

$$\pi(r = H|\theta = L) = (p_H - c_L)\frac{1 - F(v^*)}{1 - F(v^*)^N}. \tag{7}$$

If the consumer is recommended treatment L, we specify the following off-equilibrium belief: (i) her problem is L, and (ii) all other experts adhere to the equilibrium recommendation H. Off-equilibrium belief (i) is reasonable because recommending L with $p_L < c_H$ to fix problem H is weakly dominated by recommending H. Belief (ii) is common in the search literature, which assumes that one seller's deviation does not change a buyer's belief about other sellers' equilibrium strategies.[6] Under the price dominance condition and given the off-equilibrium belief, the consumer will accept recommendation L with probability one. Therefore, Expert i's payoff from recommending treatment L is

$$\pi(r = L|\theta = L) = (p_L - c_L). \tag{8}$$

Expert i will not deviate if and only if

$$\begin{gathered}\pi(r = H|\theta = L) \geq \pi(r = L|\theta = L)\\ \frac{p_H - c_L}{p_L - c_L} \geq \frac{1 - F(v^*)^N}{1 - F(v^*)}.\end{gathered} \tag{9}$$

The next proposition presents the existence condition for a fraudulent equilibrium (proof omitted).

Proposition 2. *A fraudulent equilibrium exists if* $p_H - c_H \geq \max\{p_L - c_L + \Delta v, \frac{(1-F(v^*)^N)(p_L-c_L)}{1-F(v^*)}\}$.

[6]For example, Wolinsky (1986, 1993) and Anderson and Renault (1999).

Proposition 2 is illustrated in Figure 2. Prices that support a fraudulent equilibrium lie in the shaded area, which is above the line PD (the price dominance constraint (A2)) and the line IC_F (the cheating condition (9)). Intuitively, experts need a high cheating margin to maintain incentives to misreport an $L-$problem as an $H-$problem.

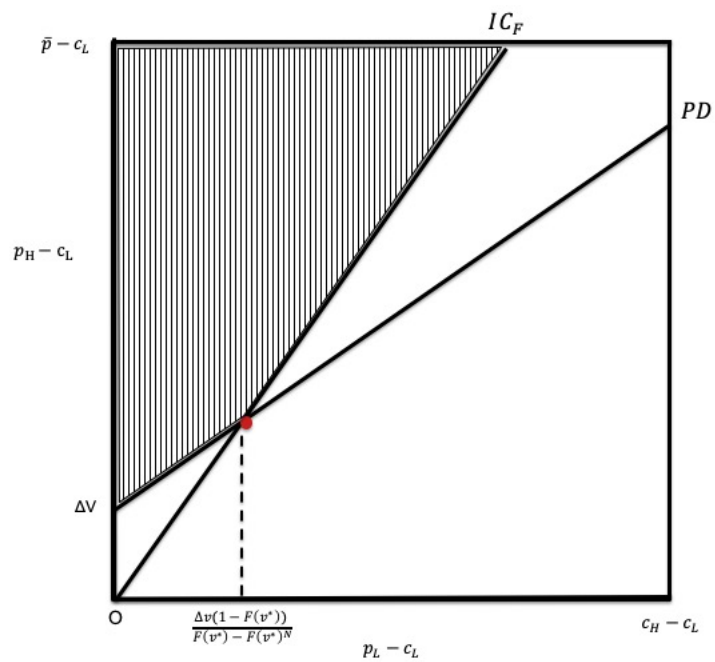

Figure 2: Fraudulent equilibrium

PD corresponds to a binding price dominance condition (A2), and IC_F corresponds to a binding (9).

Lemma 2. *The price range where there is a fraudulent equilibrium shrinks if the search cost decreases or the degree of product differentiation increases. When $s \to 0$, there is a fraudulent equilibrium if $p_H - c_L \geq \max\{p_L - c_L + \Delta v, N(p_L - c_L)\}$.*

We can write the threshold in equation (9)

$$\frac{1 - F(v^*)^N}{1 - F(v^*)} = \sum_{n=1}^{N} F(v^*)^{n-1},$$

which increases in v^*. Because v^* decreases in $\frac{s}{\lambda}$, $\frac{1-F(v^*)^N}{1-F(v^*)}$ decreases in $\frac{s}{\lambda}$. Hence, as s decreases or λ increases, the line IC_F rotates counterclockwise, which reduces the shaded area.

Note that, when expert services are homogeneous, $\lambda = 0$, $v^* = \underline{v}$ and $\frac{1-F(v^*)^N}{1-F(v^*)} = 1$. In this case, the line IC_F lies below the line PD, and there is a fraudulent equilibrium for prices that satisfy the price dominance condition. Interestingly, when $s \to 0$, a fraudulent equilibrium can still exist if the profit ratio $\frac{p_H - c_L}{p_L - c_L}$ exceeds the number of experts. Hence, extensive consumer search alone is insufficient to eliminate cheating when the market has only a small number of experts.

We summarize the pure strategy equilibria for any given (p_L, p_H) in the following corollary and illustrate it in Figure 3:

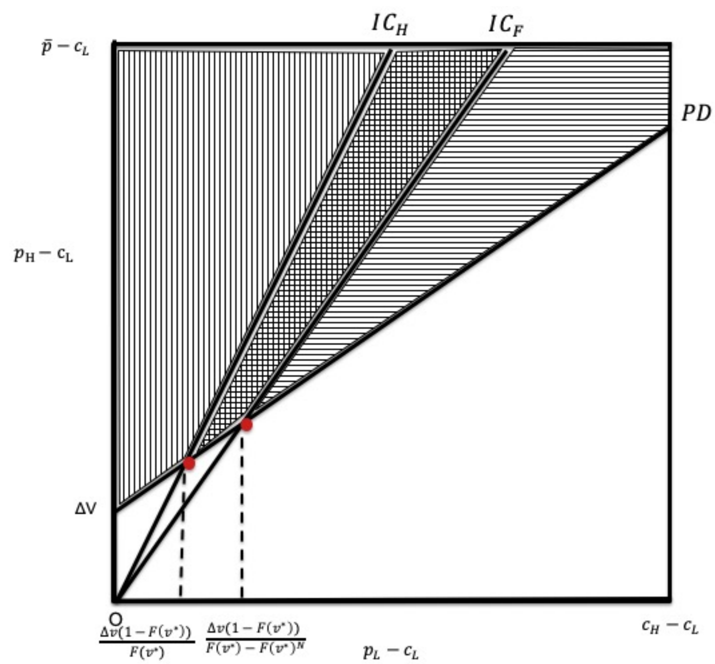

Figure 3

PD corresponds to a binding price dominance condition, IC_H corresponds to a binding (4), and IC_F corresponds to a binding (9).

Corollary 1. *We characterize the pure strategy equilibria for any price configuration:*

- *if* $p_L - c_L + \Delta v \leq p_H - c_L < \frac{(1-F(v^*)^N)(p_L - c_L)}{1-F(v^*)}$ *(the area between PD and IC_F), Honest equilibrium is the unique symmetric pure strategy equilibrium ;*

- *if* $p_H - c_L > \max\{\frac{p_L - c_L}{1-F(v^*)}, p_L - c_L + \Delta v\}$ *(the area above IC_H and PD), fraudulent equilibrium is the unique symmetric pure strategy equilibrium;*

- *if* $\max\{p_L - c_L + \Delta v, \frac{(1-F(v^*)^N)(p_L - c_L)}{1-F(v^*)}\} \leq p_H - c_L \leq \frac{p_L - c_L}{1-F(v^*)}$ *(The area above PD and IC_F but below IC_H), both the honest and the fraudulent equilibria exist.*

If $N \to \infty$, $\frac{(1-F(v^*)^N)(p_L-c_L)}{1-F(v^*)} \to \frac{(p_L-c_L)}{1-F(v^*)}$.

- *Honest equilibrium is the unique symmetric pure strategy equilibrium if* $p_L - c_L + \Delta v < p_H - c_L < \frac{p_L-c_L}{1-F(v^*)}$, *and fraudulent equilibrium is the unique symmetric pure strategy equilibrium if* $p_H - c_L > \max\{\frac{p_L-c_L}{1-F(v^*)}, p_L - c_L + \Delta v\}$*;*

- *Both equilibria exist if* $p_H - c_L = \frac{p_L-c_L}{1-F(v^*)}$.

If $s \to 0$,

- *Honest equilibrium is the unique symmetric pure strategy equilibrium if* $p_L - c_L + \Delta v < p_H - c_L < N(p_L - c_L)$*;*

- *Both equilibria exist if* $p_H - c_L \geq \max\{N(p_L - c_L), p_L - c_L + \Delta v\}$.

4 Improving search pool

In this section, we examine how an enhanced search pool impacts experts' cheating incentives. We model an improved search pool by replacing the match value distribution $F(v)$ with a distribution $G(v)$ that truncates $F(v)$ below a value $\hat{v} \in (\underline{v}, \bar{v})$.[7] It follows that $G(v) = \frac{F(v)-F(\hat{v})}{1-F(\hat{v})}$ on the support $[\hat{v}, \bar{v}]$. Truncating the distribution $F(v)$ from below captures the situation where a platform selects experts whose characteristics meet a certain minimum standard. For example, when looking for a contractor on HomeAdvisor, a consumer can indicate that she wants a service within the next two days. HomeAdvisor can exclude contractors who are unavailable in the next two days.

Denote $F(v) = \eta$ and $F^{-1}(\eta) = v$. Hence, v is the ηth percentile. Let v_F^* denote the reservation match value under distribution $F(v)$ and η_F^* denote the corresponding search propensity. Similarly, we define $G(v) = \eta$, $G^{-1}(\eta) = v$, and $G^{-1}(\eta_G^*) = v_G^*$ under the distribution function $G(v)$.

Proposition 3. *If* $f(v)$ *is a weakly increasing function, then*

(i) $G(v_G^*) < F(v_F^*)$,

(ii) *the honest equilibrium can be supported in a wider price range under the distribution* $F(v)$ *than under the distribution* $G(v)$*, and*

[7]See, for example, Zhou (2022) and Zhong (2023).

(iii) *the fraudulent equilibrium can be supported in a wider price range under the distribution $G(v)$ than under the distribution $F(v)$.*

Proof. Proof for proposition 3: We begin by proving (i). By changing variable $F(v) = \eta$, the optimal stopping rule under $F(v)$ becomes

$$\int_{v_F^*}^{\bar{v}} (1 - F(x))dx = \int_{\eta_F^*}^{1} \frac{1-\eta}{f(F^{-1}(\eta))} d\eta = s/\lambda. \tag{10}$$

The optimal stopping rule under $G(v)$ is

$$\int_{\eta_G^*}^{1} \frac{1-\eta}{g(G^{-1}(\eta))} d\eta = s/\lambda. \tag{11}$$

Using (10) and (11), substituting $g(v) = \frac{f(v)}{1-F(\hat{v})}$, we have

$$\int_{\eta_F^*}^{1} \frac{1-\eta}{f(F^{-1}(\eta))} d\eta = \int_{\eta_G^*}^{1} \frac{1-\eta}{g(G^{-1}(\eta))} d\eta = \int_{\eta_G^*}^{1} \frac{(1-\eta)(1-F(\hat{v}))}{f(G^{-1}(\eta))} d\eta.$$

It follows that $\eta_G^* < \eta_F^*$ if

$$1 - F(\hat{v}) < \frac{f(G^{-1}(\eta))}{f(F^{-1}(\eta))}, \forall \eta \in [0, 1]. \tag{12}$$

Let v_G be a realization from $G(\cdot)$ and v_F be a realization from $F(\cdot)$. Recall $G(v_G) = \frac{F(v_G)-F(\hat{v})}{1-F(\hat{v})}$. If $G(v_G) = F(v_F) = \eta$, then

$$F(v_G) = F(v_F) + (1 - \eta)F(\hat{v}).$$

It follows that $v_G > v_F$ and, therefore, $G^{-1}(\eta) > F^{-1}(\eta)$. If $f(v)$ is weakly increasing, Condition (12) is satisfied, and hence $G(v_G^*) = \eta_G^* < \eta_F^* = F(v_F^*)$.

Next, we prove (ii) and (iii). Suppose the price dominance condition (A2) holds $\Delta p \geq \Delta v$. Recall by (4), the honest equilibrium exists under the distribution $F(v)$ if $\frac{p_H - c_L}{p_L - c_L} \leq \frac{1}{1-F(v_F^*)} = \frac{1}{1-\eta_F^*}$. Under the truncated distribution $G(v)$, the honest equilibrium exists if $\frac{p_H - c_L}{p_L - c_L} \leq \frac{1}{1-G(v_G^*)} = \frac{1}{1-\eta_G^*}$. Because $\eta_G^* < \eta_F^*$, $\frac{1}{1-\eta_G^*} < \frac{1}{1-\eta_F^*}$.

The fraudulent equilibrium exists under the distribution $F(v)$ if $\frac{p_H - c_L}{p_L - c_L} \geq \frac{1-F(v_F^*)^N}{1-F(v_F^*)} = \frac{1-(\eta_F^*)^N}{1-\eta_F^*}$. (condition (9)) Under the truncated distribution $G(v)$, the fraudulent equilibrium exists if $\frac{p_H - c_L}{p_L - c_L} \geq \frac{1-(\eta_G^*)^N}{1-\eta_G^*}$. Because $\frac{1-x^N}{1-x}$ increases in x and $\eta_G^* < \eta_F^*$, we have $\frac{1-(\eta_G^*)^N}{1-\eta_G^*} < \frac{1-(\eta_F^*)^N}{1-\eta_F^*}$. $\square$

A truncation of the function $F(v)$ from below will affect consumers' search propensity by changing the distribution of match values and consumers' reservation match values. Holding constant the reservation match values, a consumer is less likely to search if bad apples are removed from the search pool. However, as the search pool improves, consumers may also increase their reservation match value, rendering an ambiguous impact on consumers' search propensity. Proposition 3 states that if the density function $f(v)$ is weakly increasing, removing experts with low match values will reduce consumers' search propensity. This is because if $f(v)$ is weakly increasing, a truncation of the lower value has a relatively small impact on the reservation match value and reduces search propensity. This reduces experts' risk of cheating and makes it harder to support the honest equilibrium and easier to support the fraudulent equilibrium. Examples include uniform distribution, truncated exponential distribution, power-law distribution on a bounded interval, and triangular distribution. Therefore, when we assume a uniform distribution, which is frequently used in the literature, improvements in matching technologies may inadvertently shift the equilibrium in an unintended direction.

In what follows, we provide an example when $F(\cdot)$ follows a uniform distribution on the interval $[\underline{v}, \bar{v}]$.

Honest equilibrium threshold: The honest equilibrium exists if $\frac{p_H - c_L}{p_L - c_L} \leq \frac{1}{1-F(v^*)}$ ((4)). Suppose $F(v)$ is a uniform distribution on the interval $[\underline{v}, \bar{v}]$. Then,

$$\frac{1}{1-F(v^*)} = \frac{\bar{v} - \underline{v}}{\bar{v} - v^*},$$

where

$$v^* = \bar{v} - \sqrt{2s(\bar{v} - \underline{v})/\lambda}.$$

It follows that

$$\frac{1}{1-F(v^*)} = \frac{\bar{v} - \underline{v}}{\bar{v} - (\bar{v} - \sqrt{2s(\bar{v} - \underline{v})/\lambda})} = \frac{\sqrt{(\bar{v} - \underline{v})/\lambda}}{\sqrt{2s}}.$$

An increase in $\underline{v}$ decreases the threshold $\frac{1}{1-F(v^*)}$, making it harder to support the honest equilibrium.

Fraudulent equilibrium threshold The fraudulent equilibrium exists if $\frac{p_H - c_L}{p_L - c_L} \geq \frac{1-F(v^*)^N}{1-F(v^*)}$((9)). Under the uniform distribution,

$$F(v^*) = \frac{v^* - \underline{v}}{\bar{v} - \underline{v}} = 1 - \frac{\sqrt{2s}}{\sqrt{(\bar{v} - \underline{v})/\lambda}}.$$

As $\underline{v}$ increases, $F(v^*)$ decreases, and the threshold $\frac{1-F(v^*)^N}{1-F(v^*)}$ also decreases, implying that it is easier to sustain the fraudulent equilibrium.

5 Endogenous price

In this section, we endogenize experts' pricing strategies. Section 3 shows that an honest equilibrium exists when prices are fixed and satisfy some mild conditions. In this section, we investigate whether the honest equilibrium is robust if experts choose prices endogenously. This analysis will also shed light on how price competition affects expert conduct. Specifically, we modify the third stage of the game as follows. After an expert perfectly diagnoses a consumer's problem, he recommends fixing it for a price $p \in [c_L, \bar{p}]$.[8]

We characterize a symmetric honest equilibrium in which all experts recommend the same prices, p_H^* in state H and p_L^* in state L. We specify a consumer's off-equilibrium belief upon receiving a price recommendation $p' \notin \{p_H^*, p_L^*\}$ as follows: (i) all other experts adhere to the equilibrium strategies, (ii) her problem is H if (a) p' is the first recommendation she sampled and $p' \geq c_H$, or (b) she has sampled multiple recommendations before the price recommendation $p' \geq c_H$ and all other price recommendations are p_H^*, (iii) her problem is L if (a) $p' < c_H$, or (b) she has sampled multiple price recommendations before the price recommendation p' and at least one recommendation is p_L^*.

Before proceeding with analysis, we provide justifications for the above off-equilibrium beliefs. It is common in the literature of consumer search to assume that a price deviation by one seller does not change consumers' beliefs about other sellers' price strategies.[9] Moreover, we assume that a consumer holds a favorable belief toward an expert following a price deviation $p' \geq c_H$ unless she has already drawn an honest recommendation at p_L^*. Under the liability assumption, it is a weakly dominated strategy for sellers to recommend fixing an H-problem at a price below c_H. Hence, the consumer believes that she has an $L-$problem upon a recommendation at $p' < c_H$.

It is difficult to establish the uniqueness of the equilibrium under a general function $F(v)$ with a finite number of experts. Hence, we focus on the limiting equilibrium when the number of experts is sufficiently large, i.e. $N \to \infty$.

We assume the match value distribution $F(v)$ has a monotone hazard rate:

Assumption 6. $\frac{1-F(v)}{f(v)}$ *is decreasing in* v.

The following proposition characterizes the existence conditions for an honest equilibrium. Define $\Delta c \equiv c_H - c_L$.

[8] It is a dominated strategy to set a price below c_L, and the upper bound of the price ensures that consumers will not exit the market without having the problem solved.

[9] See, for example, Wolinsky (1986) and Anderson and Renault (1999).

Proposition 4. *Suppose the number of experts is infinite. An honest equilibrium exists with* $p_H^* = c_H + \lambda \frac{1-F(v^*)}{f(v^*)}$ *and* $p_L^* = c_L + \lambda \frac{1-F(v^*)}{f(v^*)}$ *if*

$$\Delta c \geq \lambda \left(\frac{1 - F(v^*)}{f(v^*)} \right) + \Delta v, \tag{13}$$

and

$$(\hat{p} - c_L) \left(1 - F \left(v^* - \frac{p_H^* - \hat{p}}{\lambda} \right) \right) \leq \lambda \left(\frac{1 - F(v^*)}{f(v^*)} \right), \tag{14}$$

where

$$\hat{p} = \begin{cases} \tilde{p} & \text{if } \tilde{p} > c_H \\ c_H & \text{otherwise} \end{cases}$$

and $\tilde{p}$ *is the solution for*

$$p - \lambda \left(\frac{1 - F \left(v^* - \frac{p_H^* - p}{\lambda} \right)}{f(v^* - \frac{p_H^* - p}{\lambda})} \right) = c_L. \tag{15}$$

Note that the prices p_H^* and p_L^* are the equilibrium prices under complete information, where the severity of a consumer's problem is public information. To see that these prices constitute an honest equilibrium in our model, suppose that a consumer has an $H-$problem. An expert's price deviation $p' \geq c_H$ does not change the consumer's belief that her problem is H. Hence, the optimal price remains the same as that under complete information.

Now, consider a consumer with an $L-$problem. Given the off-equilibrium belief that the consumer's problem is L for any $p' < c_H$, p_L^* is more profitable than any price deviation $p' < c_H$. The challenge is to show that any price deviation $p' \geq c_H$ is not profitable.

Condition (13) ensures that the price dominance condition is satisfied for any $p' \geq c_H$. That is, the consumer will not buy from an expert who recommends $p' \geq c_H$ if she has already sampled the price recommendation p_L^* from another expert. Under the price dominance condition, if an expert deviates to $p' \geq c_H$, he can only sell to consumers who have not visited any other expert and whose valuation is sufficiently high for them to stop searching. If the expert deviates to $p' \geq c_H$, the most profitable price deviation is $\hat{p}$, which is between c_H and p_H^*. Condition (14) ensures that any price deviation above c_H is not profitable.

Proof. Proof for Proposition 4. We first show $p_H^* = c_H + \lambda \frac{1-F(v^*)}{f(v^*)}$ constitutes an honest equilibrium. Consider an honest equilibrium in which experts recommend p_θ^* for problem θ, $\theta = L, H$. Suppose that a consumer has an $H-$problem. Upon receiving $p' \geq c_H$, the consumer believes that she has an H-problem and hence will stop searching if and only if $\lambda v - p' \geq \lambda v^* - p_H^*$. If an expert charges a price $p' \geq c_H$, he can only sell to consumers who visit him for the first time because there is no returning consumers when $N \to \infty$. Therefore, the expert's profit-maximizing price solves

$$\max_{p'}(p' - c_H)\left(1 - F\left(v^* - \frac{p_H^* - p'}{\lambda}\right)\right).$$

The first order condition gives

$$p' = c_H + \lambda \frac{1 - F\left(v^* - \frac{p_H^* - p'}{\lambda}\right)}{f\left(v^* - \frac{p_H^* - p'}{\lambda}\right)}. \tag{16}$$

Given the Monotone Hazard Rate (MHR) property by Assumption 6, the unique symmetric equilibrium price is $p_H^* = c_H + \lambda \frac{1-F(v^*)}{f(v^*)}$.

Next, we show $p_L^* = c_L + \lambda \frac{1-F(v^*)}{f(v^*)}$. Condition (13) implies $p_L^* < c_H$. Suppose a consumer has an L-problem. If an expert deviates to $p' < c_H$, the consumer believes that her problem is L, and she will stop searching if $\lambda v - p' \geq \lambda v^* - p_L^*$. If the expert deviates to $p' \geq c_H$, the consumer believes that her problem is H if she has not visited any other experts. In this case, the consumer expects all other experts to recommend p_H^*, and she will stop searching if $\lambda v - p' > \lambda v^* - p_H^*$. If, however, the consumer has visited at least one other expert who recommended p_L^*, she infers that her problem is L and will refuse to accept p' because of the price dominance. We divide the analysis into $p' < c_H$ and $p' \geq c_H$.

We first show that given all other experts recommending $p_L^* = c_L + \lambda \frac{1-F(v^*)}{f(v^*)}$ for an L-problem, an expert makes a higher profit from recommending p_L^* for a L-problem than recommending any price $p' < c_H$. An expert's profit from recommending $p' < c_H$ for fixing a L-problem is

$$(p' - c_L)\left(1 - F\left(v^* - \frac{p_L^* - p'}{\lambda}\right)\right).$$

The first order condition gives

$$p' = c_L + \lambda \frac{1 - F\left(v^* - \frac{p_L^* - p'}{\lambda}\right)}{f\left(v^* - \frac{p_L^* - p'}{\lambda}\right)}. \tag{17}$$

It is easy to verify that the expert's profit is maximized at $p' = p_L^* < c_H$. Hence, the expert does not have a profitable deviation to any price below c_H.

Next, we show that the expert does not have a profitable deviation to $p' \geq c_H$. First, we show that under the condition (13), price dominance holds for $p' \geq c_H$. Substituting $p_L^* = c_L + \lambda \frac{1-F(v^*)}{f(v^*)}$, we obtain

$$p' - p_L^* = p' - c_L - \lambda \frac{1 - F(v^*)}{f(v^*)} \geq \Delta c - \lambda \frac{1 - F(v^*)}{f(v^*)} \geq \Delta v,$$

where the first inequality follows from $p' \geq c_H$ and the last inequality follows from (13). Under the price dominance condition, if the expert deviates to recommending $p' \geq c_H$, he can only sell to consumers who have not visited any other expert and decide to stop searching. By (2), as $N \to \infty$, the fraction of consumers who have not visited any other experts is $1 - F(v^*)$. The profit-maximizing price deviation is therefore the solution for

$$\max_{p' \geq c_H} (p' - c_L)(1 - F(v^*)) \left(1 - F\left(v^* - \frac{p_H^* - p'}{\lambda}\right)\right).$$

Define the solution for the unconstrained maximization problem as $\tilde{p}$, and $\tilde{p}$ is determined by (15). If $\tilde{p} > c_H$, it is the interior solution for the constrained optimization problem. If $\tilde{p} \leq c_H$, the expert's deviation profit is maximized at the corner solution $p' = c_H$. Thus, if the expert deviates to recommending $p' \geq c_H$ for fixing a L-problem, his most profitable deviation is

$$\hat{p} = \begin{cases} \tilde{p} & \text{if } \tilde{p} > c_H \\ c_H & \text{otherwise} \end{cases}$$

The no deviation condition requires that the expert makes a higher profit from recommending p_L^* than $\hat{p}$ for fixing an L-problem, which gives

$$\begin{aligned} (\hat{p} - c_L)(1 - F(v^*)) \left(1 - F\left(v^* - \frac{p_H^* - \hat{p}}{\lambda}\right)\right) &\leq (p_L^* - c_L)(1 - F(v^*)) \\ (\hat{p} - c_L) \left(1 - F\left(v^* - \frac{p_H^* - \hat{p}}{\lambda}\right)\right) &\leq \lambda \left(\frac{1 - F(v^*)}{f(v^*)}\right), \end{aligned}$$

where the second inequality follows from (17) by substituting $p_L^* - c_L = \lambda \frac{1-F(v^*)}{f(v^*)}$. □

Lemma 3. *As $s \to 0$, $p_H^* \to c_H$, $p_L^* \to c_L$, $\hat{p} \to c_H$; the honest equilibrium exists if*

$$\Delta v \leq \Delta c \leq \frac{\lambda}{2f(\bar{v})}. \tag{18}$$

Proof. Proof for Lemma 3. We begin by showing $\tilde{p} \leq p_H^*$. Consider the following equation

$$p - \lambda \frac{1 - F\left(v^* - \frac{p_H^* - p}{\lambda}\right)}{f\left(v^* - \frac{p_H^* - p}{\lambda}\right)} = c_\theta. \tag{19}$$

Here, p_H^* is the solution for (19) when $c_\theta = c_H$, and $\tilde{p}$ solves (19) when $c_\theta = c_L$. Under Assumption 6, the LHS of (19) is strictly increasing in p. Since $c_L < c_H$, $\tilde{p} < p_H^*$.

Next, we show that as $s \to 0$, condition (14) is satisfied if $\Delta c \leq \frac{\lambda}{2f(\bar{v})}$. Note that $v^* \to \bar{v}$ as $s \to 0$. Therefore, $p_H^* = c_H + \lambda \frac{1-F(v^*)}{f(v^*)} \to c_H$ as $s \to 0$. Because $\tilde{p} < p_H^*$, there exists a cutoff $\hat{s}$ such that $\hat{p} = c_H$ if $s < \hat{s}$. If $s < \hat{s}$, we can rewrite the condition (14) as

$$c_H - c_L \leq \frac{\lambda(1 - F(v^*))}{f(v^*)\left(1 - F\left(v^* - \frac{p_H^* - c_H}{\lambda}\right)\right)}.. \tag{20}$$

As $s \to 0$, $v^* \to \bar{v}$, both the numerator and the denominator of the right-hand-side fraction of (20) are zero. By L'Hôpital's rule, as $v^* \to \bar{v}$, the fraction becomes $\frac{\lambda}{2f(\bar{v})}$, Hence, the condition (14) is satisfied in the limiting case if $\Delta c \leq \frac{\lambda}{2f(\bar{v})}$.

Lastly, as $s \to 0$, (13) becomes $\Delta c \geq \Delta v$, As a result, the honest equilibrium exists in the limiting case if

$$\Delta v \leq \Delta c \leq \frac{\lambda}{2f(\bar{v})}.$$

□

Lemma 3 states that as the search cost approaches zero, the honest equilibrium exists if the difference in the treatment costs is in some intermediate range. Because, at equilibrium, $p_H^* - p_L^* = \Delta c$ holds, the price dominance condition (13) requires $\Delta v \leq \Delta c$. The no-cheating condition (14) is satisfied if $\Delta c \leq \frac{\lambda}{2f(\bar{v})}$. To see the intuitions, note that, when the search cost approaches zero, consumers' reservation match values approach to $\bar{v}$. A dense upper tail at $\bar{v}$ will entail more intense price competition and faster erosion of markups from honest recommendations, making it harder to satisfy the no-cheating constraint.

For $\lambda > 0$, the interval $[\Delta v, \frac{\lambda}{2f(\bar{v})}]$ is nonempty if $f(\bar{v})$ is sufficiently small. Note that the condition (18) is violated for the uniform distribution but can be satisfied for the triangle distribution, the truncated normal distribution, the truncated exponential distribution, and the truncated Pareto distribution. Lemma 3 shows that, although lower search costs can discipline experts by encouraging active consumer search, price competition will undermine this effect. More frequent consumer search will result in a decrease in the price for treatment L. However, a lower p_L will erode experts' profits from making honest recommendations and make it harder to sustain an honest equilibrium. Policies aimed at promoting honest recommendations through consumer search will be more effective when combined with price regulation.

6 Discussion

In this section, we extend the analysis in Section 3 to the price range $(p_L, p_H) \in [c_H, \bar{p}]^2$, while fixing prices. We begin by characterizing the conditions under which an honest equilibrium still exists. Suppose an expert diagnoses a consumer with an $L-$problem, the analysis of the expert's cheating incentives is the same as in Section 3.1. Hence, the conditions for the expert to recommend treatment L for problem L remain (5).

Because $p_L \geq c_H$, we need to consider the possibility that the expert recommends treatment L for problem H. Suppose the expert diagnoses a consumer with problem H. If the expert recommends treatment H, the consumer will accept the recommendation with a probability $\frac{1-F(v^*)}{1-F(v^*)^N}$ (see (3)), which is the equilibrium probability that a consumer accepts treatment L from an expert when her problem is L. This is because, in an honest equilibrium, the consumer is only searching for a better match value. Hence, her optimal stopping rule is independent of her state. It follows that the expert's equilibrium profit from recommending treatment H is

$$\pi(r = H|\theta = H) = (p_H - c_H)\frac{1 - F(v^*)}{1 - F(v^*)^N}.$$

If the expert deviates and recommends treatment L, under the price dominance condition, the expert will sell to the consumer with probability 1, as all other experts will recommend treatment H. Because problem H can only be resolved by treatment H, under the Liability assumption, the expert must incur the cost c_H to fix problem H. Hence, the expert's payoff from recommending treatment L is

$$\pi(r = L|\theta = H) = (p_L - c_H).$$

The no deviation condition becomes

$$p_H - c_H \geq \frac{(p_L - c_H)(1 - F(v^*)^N)}{1 - F(v^*)}.$$

We can rewrite the above condition as

$$p_H - c_L \geq \frac{(p_L - c_L)(1 - F(v^*)^N)}{1 - F(v^*)} - \frac{\Delta c(F(v^*) - F(v^*)^N)}{1 - F(v^*)}. \tag{21}$$

Proposition 5. *For a given price list $(p_L, p_H) \in [c_H, \bar{p}]^2$, an honest equilibrium exists if*

$$p_L - c_L \geq \frac{\Delta v(1 - F(v^*))}{F(v^*)}$$

and

$$\max\{\Delta v + p_L - c_L, \frac{(p_L - c_L)(1 - F(v^*)^N)}{1 - F(v^*)} - \frac{\Delta c(F(v^*) - F(v^*)^N)}{1 - F(v^*)}\} \leq p_H - c_L \leq \frac{p_L - c_L}{1 - F(v^*)}.$$

Compared with the case $p_L < c_H$, if $p_L \geq c_H$, an honest equilibrium exists with an added constraint (21) to prevent a deviation in state H. Since

$$\frac{(p_L - c_L)(1 - F(v^*)^N)}{1 - F(v^*)} - \frac{\Delta c(F(v^*) - F(v^*)^N)}{1 - F(v^*)} < \frac{p_L - c_L}{1 - F(v^*)},$$

the set of prices which support an honest equilibrium is non-empty if $\Delta v + p_L - c_L \leq \frac{p_L - c_L}{1 - F(v^*)}$, which is satisfied when $p_L - c_L \geq \frac{\Delta v(1 - F(v^*))}{F(v^*)}$ from equation (6). The price range that supports the honest equilibrium is illustrated in Figure 4.

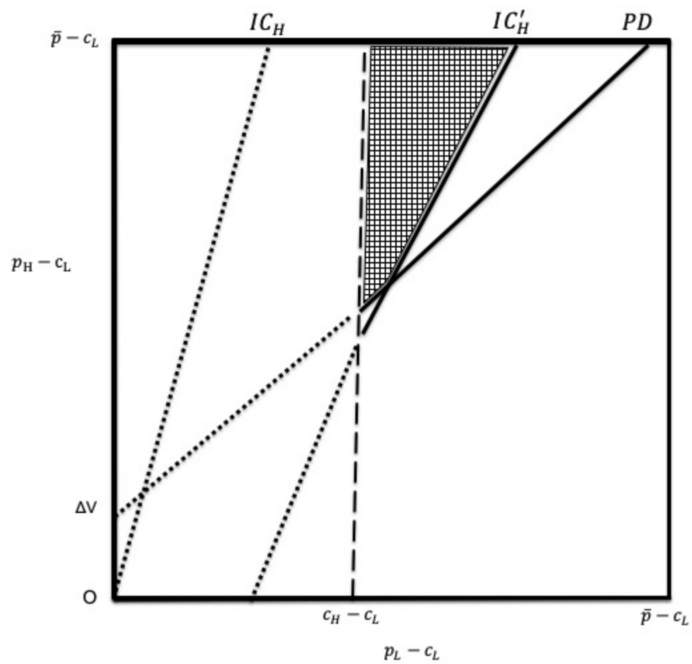

Figure 4

PD corresponds to a binding price dominance condition (A2), IC_H corresponds to a binding (4), and $IC_H^{'}$ corresponds to a binding (21).

Next, we argue that a fraudulent equilibrium in which experts always recommend p_H exists under the same condition in Proposition 2. The condition in Proposition 2 ensures that an expert does not want to deviate to recommending p_L for an $L-$problem. When $p_L \geq c_H$, we also need to verify that, under the same condition, the expert will not deviate to recommending p_L for an $H-$problem. In a fraudulent equilibrium, the expert's payoff from recommending p_H for an $H-$problem is $\pi(r = H|\theta = H) = (p_H - c_H)\frac{1 - F(v^*)}{1 - F(v^*)^N}$. If the expert deviates to recommending p_L, he will sell to the consumer with probability one and make $\pi(r = L|\theta = H) = p_L - c_H$. It is easy to verify that the condition in Proposition 2 guarantees that the expert will not deviate in state H. This implies that, if the expert does not want to deviate in state L, he will not deviate in state H. The intuition is simple. When deviating to p_L, the expert trades off more demand against a lower profit margin. The expert's incentive to deviate to p_L is weaker in state H than in state L due to a higher cost.

Finally, a price profile $(p_L, p_H) \in [c_H, \bar{p}]^2$ will admit a trivial equilibrium in which experts always recommend p_L. In such an equilibrium, a recommendation p_H is off-the-equilibrium path. We assume that, upon receiving an off-the-equilibrium price recommendation, the consumer believes that all other experts will adhere to their equilibrium strategies, i.e., recommending p_L for both types of problems. Under this assumption and the price dominance assumption, the consumer will reject p_H, making a deviation to p_H unprofitable.

7 Conclusion

We analyze consumer search in markets for horizontally differentiated expert services. Because experts vary in both treatment recommendations and match value, consumers search for honest recommendations as well as better matches. We show that horizontal product differentiation plays a critical role in disciplining expert conduct. When expert services are homogeneous, no honest equilibrium exists under positive search costs, i.e., experts cheat with positive probability in any symmetric equilibrium. By contrast, with horizontal differentiation, a symmetric honest equilibrium emerges if the search cost is below a threshold, even though consumers search only for better matches.

Our analysis also uncovers unintended consequences of improved matching technologies. Excluding experts with low match values can reduce consumers' incentives to continue searching, thereby increasing experts' incentives to cheat. Thus, enhancements in matching efficiency do not necessarily improve consumer welfare.

The results remain robust under endogenous price competition. Although price competition reduces the profitability of honest recommendations and increases the temptation to cheat, we show that honesty can still be sustained—particularly with many competing experts and sufficiently low search costs.

Overall, these findings highlight important policy implications. Technological improvements and stronger price competition, which are typically beneficial in other markets, may worsen expert conduct in credence-goods markets. Our results, therefore, offer guidance for platform design and competition policy in markets where consumers rely on expert recommendations.

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Supplementary Appendix (Online Appendix)

Search in Credence Goods Markets

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A Mixed strategy equilibrium

In this Appendix, we characterize a mixed strategy equilibrium with two experts, $N = 2$. We study equilibria with fixed prices, and restrict our attention to the price range $(p_L, p_H) \in [c_L, c_H) \times [c_H, \bar{p}]$ as in Section 2. We consider a symmetric equilibrium in which experts recommend treatment H with probability $\beta \in (0, 1)$. After the first recommendation, the consumer updates her belief of having problem H to α_1^r, where $r = H, L$ denotes the recommendation by the first expert. Denote by v_1 and v_2 the match value with Expert 1 and 2 if the consumer searches.

A.1 Consumer's optimal search policy

We begin with a consumer's optimal search policy. It is without loss of generality to assume that the consumer first samples Expert 1.

Suppose Expert 1 recommends treatment H. The consumer's belief of having problem H after recommendation H is updated upward to $\alpha_1^H = \frac{\alpha}{\alpha+(1-\alpha)\beta} > \alpha$. If the consumer searches for a second opinion and Expert 2 also recommends p_H, the consumer will buy from Expert 2 if $v_2 \geq v_1$ and from Expert 1, otherwise. However, if Expert 2 recommends p_L, the consumer will buy from Expert 2. This is due to the price dominance condition (A2) in the main text. Hence, the consumer will accept an honest second opinion, irrespective of her match values with the two experts.

Given a match value v_1, the consumer's payoff from accepting Expert 1's service is

$$u = \lambda v_1 + (1 - \lambda) b - p_H.$$

Conditional on a match value v_1 and a recommendation H, the consumer's expected gain from search without including a search cost, is

$$\begin{aligned} E^S(u|H, v_1) &= (1 - \alpha_1^H)(1 - \beta)(p_H - p_L) \\ &\quad + \lambda \left[(\alpha_1^H + (1 - \alpha_1^H)\beta) \int_{v_1}^{\bar{v}} (v_2 - v_1) dF(v_2) + (1 - \alpha_1^H)(1 - \beta) \int_{\underline{v}}^{\bar{v}} (v_2 - v_1) dF(v_2) \right]. \end{aligned} \tag{22}$$

If $\beta < 1$, at equilibrium,

$$E^S(u|H, v_1 = \bar{v}) = (1 - \alpha_1^H)(1 - \beta) \left[(p_H - p_L) + \lambda \int_{\underline{v}}^{\bar{v}} (v_2 - \bar{v}) dF(v_2) \right] > 0,$$

where the inequality holds under the price dominance condition (A2). Unlike a standard search good, the consumer still derives a positive search benefit when her match value with an expert is at the highest level. This search benefit is realized when the consumer samples a fraudulent first opinion and an honest second opinion.

Suppose Expert 1 recommends treatment L. The consumer's belief about having problem H is updated downward to $\alpha_1^L = 0$ because of the liability constraint with the price dominance assumption (A2). If the consumer accepts Expert 1's recommendation, her payoff is $u = \lambda v_1 + (1-\lambda)b - p_L$. Because experts are liable for the outcome, the consumer infers that she must have a problem L. The consumer searches for a second opinion only if her match value with Expert 1 is low.

Suppose Expert 2 also recommends p_L. In this case, the consumer will accept Expert 2's recommendation if $v_2 \geq v_1$ and will return to Expert 1 otherwise.

Under the price dominance condition (A2), the consumer will always return to Expert 1 if Expert 2 cheats. So, we can write the consumer's gain from search as

$$E^S(u|L, v_1) = \lambda(1-\beta)\int_{v_1}^{\bar{v}}(v_2 - v_1)dF(v_2). \tag{23}$$

When the price effect dominates the match value effect, match values affect the consumer's purchase decision only when two experts make the same recommendations. Upon receiving conflicted recommendations, the consumer always buys from the expert who recommends treatment L. Therefore, if Expert recommends L, the consumer will buy from Expert 2 only if Expert 2 also recommends L and gives the consumer a higher match value. The consumer's expected gain from search decreases in experts' cheating probabilities. Moreover, the consumer draws the highest possible match value with Expert 1, her gain from the search is zero.

Lemma 4. *(i) $E^S(u|H, v_1)$ and $E^S(u|L, v_1)$ strictly decrease in v_1. (ii) $E^S(u|H, v_1) > E^S(u|L, v_1)$, $\forall\beta \in (0,1)$ and $v_1 \in [\underline{v}, \bar{v}]$.*

Proof. Proof of Lemma 4 We prove that Lemma 4 holds for any prices.

We first prove (i). Applying integration by part, we can rewrite

$$\int_{v_1}^{\bar{v}}(v_2 - v_1)dF(v_2) = 1 - v_1 - \int_{v_1}^{\bar{v}} F(v_2)dv_2 = \int_{v_1}^{\bar{v}}(1 - F(v_2))dv_2. \tag{24}$$

Then we have

$$\frac{\partial E^S(u|H, v_1)}{\partial v_1} = -\lambda\left[(\alpha_1^H + (1-\alpha_1^H\beta)(1-F(v_1)) + (1-\alpha_1^H)(1-\beta)\right] < 0.$$

Next, we show $\frac{\partial E^S(u|L,v_1)}{\partial v_1} < 0$. It is written as

$$\frac{\partial E^S(u|L,v_1)}{\partial v_1} = -\lambda(1-\beta)(1-F(v_1)) < 0.$$

Now, we show (ii). This comes simply from the fact that (22)−(23)> 0. □

Fixed the recommendation of the first opinion, the consumer's gain from search decreases in her match value with the first expert. Note that the consumer's match value with the first expert does not affect the recommendation she expects to receive from the second expert. Hence, an increased match value with the first expert reduces the consumer's expected gain from search, as her match value with the first expert is more likely to dominate that with the second expert. A decreasing gain from search implies a unique reservation match value that makes the consumer indifferent between whether or not to search for a second opinion. The consumer strictly prefers searching to purchasing from the first expert if her match value is lower than the reservation match value, and the opposite holds if her match value with the first expert is greater than the reservation match value.

(ii) says the consumer's expected gain from search is greater if the first opinion recommends treatment H than treatment L. If the first expert recommends treatment H instead of treatment L, in addition to a better match value, the consumer has an additional benefit from receiving an honest second opinion and may enjoy a saving in treatment expenditure.

Let v_i denote the consumer's reservation match value for search if the first recommendation is r, $r = L, H$, where v_H and v_L solve

$$E^S(u|H, v_H) = s; \tag{25}$$

$$E^S(u|L, v_L) = s \tag{26}$$

and $E^S(u|H, v_H)$ and $E^S(u|L, v_L)$ are defined in (22) and (23) respectively.

Lemma 4 leads to the following corollary:

Corollary 2. *(i) $\underline{v} \le v_L < v_H \le \bar{v}$. (ii) If s is sufficiently small, $\underline{v} < v_L < v_H = \bar{v}$.*

We illustrate the corollary in figures A.1 and A.2. The consumer is more likely to search for a second opinion if the first opinion recommends treatment H instead of treatment L. For algebra simplicity, for the rest of the paper, we assume a small search cost such that the consumer always searches for a second opinion upon a treatment recommendation H, i.e., $v_H = \bar{v}$.

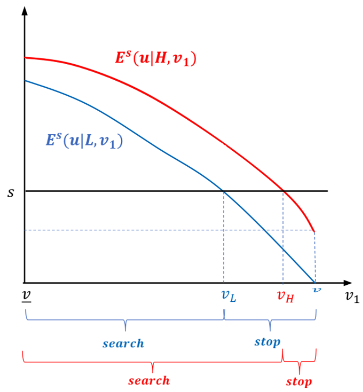

Figure A.1: large s

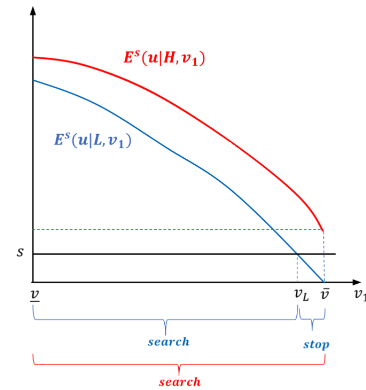

Figure A.2: small s

Using (23), v_L is solution for

$$\lambda(1-\beta)\int_{v_L}^{\bar{v}}(v_2 - v_L)dF(v_2) = \lambda(1-\beta)\int_{v_L}^{\bar{v}}(1-F(v_2))dv_2 = s. \tag{27}$$

It is easy to verify that $\frac{\partial v_L}{\partial \beta} = -\frac{\lambda\int_{v_L}^{\bar{v}}(1-F(v_2))dv_2}{(1-\beta)(1-F(v_L))} < 0$, and $\frac{\partial v_L}{\partial \lambda} = \frac{\lambda\int_{v_L}^{\bar{v}}(1-F(v_2))dv_2}{(1-\beta)(1-F(v_L))} > 0$. Upon recommended treatment L on the first visit, a consumer accepts the recommendation if $v_1 > v_L$ and searches for a second opinion, otherwise. Conditional on treatment recommendation L, the consumer's reservation value for search decreases in experts' cheating probability (β) and the importance of the credence goods attribute $(1-\lambda)$. If the first expert recommends treatment L, the consumer infers that she must have problem L and, therefore, will only search for a better match value. If the consumer expects the second expert to cheat with a high probability, she is more willing to accept the treatment from the first expert despite a low match value. Similarly, if the credence attribute is more important for the consumer (λ becomes smaller), the consumer is more likely to accept the first opinion at a low match value to avoid sampling fraudulent recommendations.

Below we explicitly solve for v_L when the consumer's match value is uniformly distributed.

Closed-form solutions of v_L. We focus on v_1 and v_2 following the uniform distribution to solve for a closed-formed solution of v_L. Using the uniform distribution, (23) can be extended to

$$\begin{aligned} E^S(u|L, v_1) &= \left[\frac{\lambda(1-\beta)}{2(\bar{v}-\underline{v})}\right]\left[v_1^2 - 2\bar{v}v_1 + \bar{v}^2\right] \\ &= \left[\frac{\lambda(1-\beta)}{2(\bar{v}-\underline{v})}\right](v_1-\bar{v})^2. \end{aligned}$$

The cutoff v_L is determined by $E^S(u|L, v_L) = s$. If $v_1 = \bar{v}$, then $E^S(u|L,\bar{v}) = 0 < s$. If $v_1 = \underline{v}$, then $E^S(u|L,\underline{v}) = \frac{\lambda(1-\beta)(\bar{v}-\underline{v})}{2} > 0$. Suppose $0 < s < \frac{\lambda(1-\beta)(\bar{v}-\underline{v})}{2}$. Then, at the reservation utility v_L, we have $E(u|L, v_L) = s$.

$$v_L = \bar{v} - \sqrt{\frac{2(\bar{v} - \underline{v})s}{\lambda(1-\beta)}}. \tag{28}$$

Given $0 < s < \frac{\lambda(1-\beta)(\bar{v}-\underline{v})}{2}$, $\underline{v} < v_L < \bar{v}$.

Let us comment on the role of search attributes in shaping the consumer's search policy. Suppose $\lambda = 0$. The product only has the credence attributes. Then, by (22)

$$E^S(u|H, v_H) = (1 - \alpha_1^H)(1-\beta)(p_H - p_L) = \frac{(1-\alpha)\beta(1-\beta)}{\alpha + (1-\alpha)\beta}(p_H - p_L),$$

and by (23), $E^S(u|L, v_L) = 0$.

Then, (25) gives a quadratic function of β, which has two roots, and (26) gives $v_L = 0$. As λ converges to 0, the consumer's optimal stopping rule converges to Wolinksy (1993), which says that the consumer stops searching upon recommendation L, and she is indifferent between accepting the treatment H or searching for a second opinion when the expert cheats with probability β solving (25). Hence, the search attribute incentivizes consumers to search more often than when the product only has credence attributes.

A.2 Expert's recommendation strategy

We now turn to experts' recommendation strategies. In the following analysis, we restrict attention to a small search cost given which a consumer always seeks a second opinion upon recommended treatment H. If the first opinion recommends treatment L, then the consumer samples another expert if and only if her match value with the first expert is lower than a cutoff v_L. This corresponds to Figure A.2.

Suppose Expert 2 diagnoses that the consumer has a minor problem ($\theta = L$). There are three possibilities: (i) the consumer has not visited Expert 1, (ii) the consumer has visited Expert 1 and Expert 1 recommended treatment H, or (iii) Expert 1 recommended treatment L, and the consumer's match value with Expert 1 is lower than v_L.

The conditional probabilities of the events (i), (ii), and (iii) are, respectively,

$$Pr(\emptyset|L) = \frac{1/2}{1/2 + 1/2[\beta + (1-\beta)F(v_L)]} = \frac{1}{1 + \beta + (1-\beta)F(v_L)}, \tag{29}$$

$$Pr(H|L) = \frac{(1/2)\beta}{1/2 + 1/2[\beta + (1-\beta)F(v_L)]} = \frac{\beta}{1 + \beta + (1-\beta)F(v_L)}, \tag{30}$$

$$Pr(L|L) = \frac{(1/2)(1-\beta)F(v_L)}{1/2 + 1/2[\beta + (1-\beta)F(v_L)]} = \frac{(1-\beta)F(v_L)}{1 + \beta + (1-\beta)F(v_L)}. \tag{31}$$

From equation (29), given v_L, $Pr(\emptyset|L)$ is decreasing in β, $Pr(L|L)$ is decreasing in β and $Pr(H|L)$ is increasing in β because $Pr(\emptyset|L) + Pr(H|L) + Pr(L|L) = 1$

Suppose Expert 2 recommends treatment H. Given the consumer's optimal search policy, the consumer will accept the offer only if she has consulted both experts and her match value with Expert 2 exceeds that of Expert 1. Hence, Expert 2's expected payoff from recommending p_H to a type L consumer is

$$\begin{aligned}\pi(r = H|\theta = L) &= (p_H - c_L)\left[Pr(H|L)\int_{\underline{v}}^{\bar{v}}\int_{v_1}^{\bar{v}} dF(v_2)dF(v_1) + Pr(\emptyset|L)\beta\int_{\underline{v}}^{\bar{v}}\int_{v_1}^{\bar{v}} dF(v_2)dF(v_1)\right]\\ &= (p_H - c_L)\left[Pr(H|L)1/2 + Pr(\emptyset|L)\beta 1/2\right] \qquad (32)\end{aligned}$$

Suppose Expert 2 recommends treatment L. The consumer will accept the offer if (i) she has not visited Expert 1 and her match value with Expert 2 exceeds v_L, (ii) she has visited Expert 1, who recommends treatment H, and (iii) Expert 1 recommends treatment L and her match value with Expert 2 exceeds her match value with Expert 1. Expert 2's expected payoff from recommending treatment L is

$$\begin{aligned}&\pi(r = L|\theta = L)\\ &= (p_L - c_L)Pr(\emptyset|L)\left(\int_{v_L}^{\bar{v}} dF(v_2) + \int_{\underline{v}}^{v_L}\left(\beta + (1-\beta)\int_{\underline{v}}^{v_2} dF(v_1)\right)dF(v_2)\right)\\ &+ (p_L - c_L)\left(Pr(H|L) + Pr(L|L)\int_{\underline{v}}^{v_L}\int_{v_1}^{\bar{v}} dF(v_2)dF(v_1)\right)\\ &= (p_L - c_L)Pr(\emptyset|L)\left(1 - (1-\beta)\int_{\underline{v}}^{v_L}(1 - F(v_2))dF(v_2)\right) + (p_L - v_L)Pr(H|L)\\ &+ (p_L - v_L)Pr(L|L)\int_{\underline{v}}^{v_L}(1 - F(v_1))dF(v_1). \qquad (33)\end{aligned}$$

At a mixed strategy equilibrium, Expert 2 is indifferent between recommending treatment L and H if,

$$\pi(r = H|\theta = L) = \pi(r = L|\theta = L). \qquad (34)$$

A.3 Mixed-strategy equilibrium

In this subsection, we first characterize the conditions for the existence of a symmetric mixed strategy equilibrium. Multiple symmetric equilibria may exist. We then provide sufficient conditions for a unique symmetric mixed strategy equilibrium.

A symmetric equilibrium is a pair (β, v_L) that solves equations (27) and (34), where (27) characterizes the consumer's optimal stopping rule upon recommendation L and (34) is the expert's indifference condition for randomizing between making honest recommendation and cheating in state L. Under the price dominance assumption (A2), the consumer will seek a second opinion upon recommendation H, irrespective of her match value with the first sampled expert.

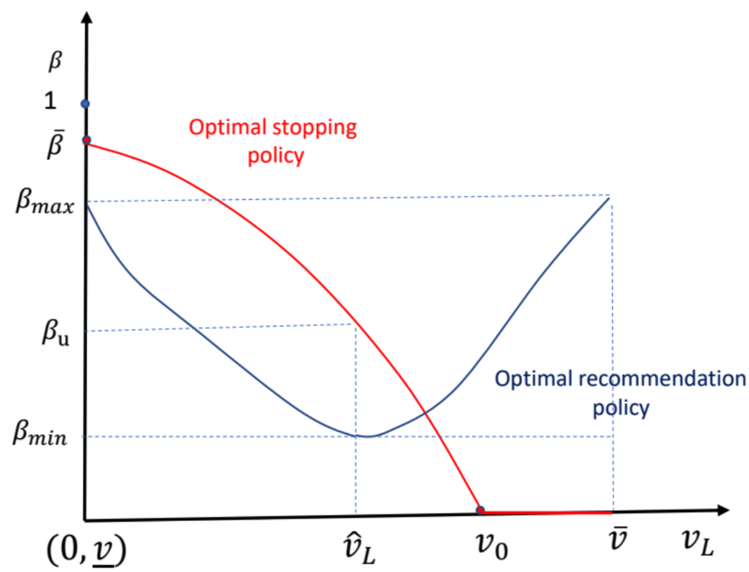

Figure A.3

Proposition 6. *Suppose* $\frac{p_H - c_L}{p_L - c_L} > 2$. *There exists* $\hat{s}(p_H, p_L)$ *such that an symmetric mixed strategy equilibrium exists if* $s < \hat{s}(p_H, p_L)$. *In the equilibrium, experts recommend treatment H with probability* $\beta^* \in (0, 1)$ *when the consumer has an L-problem. The consumer always searches for a second opinion if the first opinion recommends treatment H. If the first opinion recommends treatment L, the consumer will elicit a second opinion if and only if her match value with the first expert is below* v_L^*, *where* v_L^* *satisfies* $\lambda(1-\beta^*) \int_{v_L^*}^{\bar{v}} (1 - F(v_2)) dv_2 = s$. *If the consumer samples both experts and receives conflicting recommendations, she accepts treatment L. If experts recommend the same treatment, the consumer accepts the recommendation from the expert with whom she derives a higher match value.*

Using (32) and (33), we can write the expert's indifference condition (payoff equivalence) as

$$\begin{aligned}
\pi(r=L|\theta=L)-\pi(r=H|\theta=L) &= Pr(L|L)\underbrace{(p_L-c_L)\int_{\underline{v}}^{v_L}(1-F(v_1))dF(v_1)}_{\Delta(L|L)} + Pr(H|L)\underbrace{\left((p_L-c_L)-\frac{(p_H-c_L)}{2}\right)}_{\Delta(H|L)} \\
&+ Pr(\emptyset|L)\underbrace{\left[(p_L-c_L)\left(1-(1-\beta)\int_{\underline{v}}^{v_L}(1-F(v_2))dF(v_2)\right)-\frac{(p_H-c_L)\beta}{2}\right]}_{\Delta(\emptyset|L)} \\
&= 0. \qquad (35)
\end{aligned}$$

We can decompose the profit difference between making an honest recommendation and cheating into profit difference from selling to (i) a consumer who has already sampled a recommendation L ($\Delta(L|L)$), (ii) a consumer who has already sampled a recommendation H ($\Delta(H|L)$), and (iii) a fresh consumer ($\Delta(\emptyset|L)$).

If the consumer has already sampled an honest recommendation, she will buy from Expert 2 only if Expert 2 also recommends treatment L and gives her a higher match value than Expert 1. Therefore, the expert's gain from recommending treatment L relative to treatment H is $\Delta(L|L)$. Suppose the consumer has sampled a fraudulent recommendation H. Expert 2 will sell to the consumer with probability 1 if he makes an honest recommendation L. If the expert recommends treatment H, he will sell to the consumer only if he gives the consumer a higher match value than Expert 1, which occurs with probability $1/2$. So, Expert 2's profit difference from selling to this group of consumers is $\Delta(H|L)$, which is positive iff $\frac{p_H-c_L}{p_L-c_L} < 2$. Lastly, suppose the consumer is fresh and Expert 2 recommends treatment L. The consumer will buy from Expert 2 unless her match value with Expert 2 is below the stopping cutoff v_L, which triggers a search, and she obtains a higher match value with Expert 1 who also happens to be honest. If Expert 2 recommends treatment H, the consumer will return only if Expert 1 also recommends H, and she derives a higher match value with Expert 2. Therefore, Expert 2's profit difference from selling to fresh consumers is $\Delta(\emptyset|L)$, which can be positive or negative.

We can rewrite $\Delta(\emptyset|L)$ as

$$\left(1-(1-\beta)\int_{\underline{v}}^{v_L}(1-F(v_2))dF(v_2)\right)\left[(p_L-c_L)-\frac{(p_H-c_L)}{2}\cdot k\right],$$

where $k \equiv \frac{\beta}{\left(1-(1-\beta)\int_{\underline{v}}^{v_L}(1-F(v_2))dF(v_2)\right)} < 1$ because $v_L < \bar{v}$. It follows that $\Delta(H|L) \geq 0$ implies $\Delta(\emptyset|L) > 0$.

If $\frac{p_H-c_L}{p_L-c_L} \leq 2$, then $\Delta(H|L) \geq 0$ and $\Delta(\emptyset|L) > 0$. In this case, $\pi(L|L) > \pi(H|L)$. Hence, for the expert to randomize between making an honest and fraudulent recommendation, it is necessary that $\frac{p_H-c_L}{p_L-c_L} > 2$.

The existence condition is illustrated in Figure A.3. The consumer's optimal stopping rule implies that β is strictly decreasing in v_L, reaching 0 at a cutoff $v_0 < \bar{v}$. According to the expert's indifference condition, β first decreases in v_L and then increases in v_L; it reaches the maximum at $v_L = \underline{v}$ and $v_L = \bar{v}$ and the minimum at $v_L = \hat{v}_L$. A sufficient condition for the two best response functions to cross is when $\bar{\beta} > \beta_{max}$. This condition is satisfied if $\frac{p_H-c_L}{p_L-c_L} > 2$ and s is sufficiently small.

Note that the existence condition does not imply uniqueness. Generally, the two best response functions can cross multiple times in the region when both curves decrease.

Proof. From equations (32) and (33), the indifference condition (34) can be rewritten as

$$\frac{2(p_H-c_L)\beta}{1+\beta+(1-\beta)F(v_L)}\int_{\underline{v}}^{\bar{v}}\int_{v_1}^{\bar{v}} dF(v_2)dF(v_1) = (p_L-c_L)\frac{1}{1+\beta+(1-\beta)F(v_L)}\left(\int_{v_L}^{\bar{v}} dF(v_2)+\beta\int_{\underline{v}}^{v_L} dF(v_2)+(1-\beta)\int_{\underline{v}}^{v_L}\int_{\underline{v}}^{v_2} dF(v_1)dF(v_2)\right)$$
$$+(p_L-c_L)\left(\frac{\beta}{1+\beta+(1-\beta)F(v_L)}+\frac{(1-\beta)F(v_L)}{1+\beta+(1-\beta)F(v_L)}\int_{\underline{v}}^{v_L}\int_{v_1}^{\bar{v}} dF(v_2)dF(v_1)\right).$$

Multiplying $1+\beta+(1-\beta)F(v_L)$ to both sides and rearrange terms, we get

$$\beta = B/A,$$

where

$$A = \frac{2(p_H-c_L)}{(p_L-c_L)}\int_{\underline{v}}^{\bar{v}}\int_{v_1}^{\bar{v}} dF(v_2)dF(v_1) - \int_{\underline{v}}^{v_L} dF(v_2) + \int_{\underline{v}}^{v_L}\int_{\underline{v}}^{v_2} dF(v_1)dF(v_2) - 1 + F(v_L)\int_{\underline{v}}^{v_L}\int_{v_1}^{\bar{v}} dF(v_2)dF(v_1),$$

and

$$B = \int_{v_L}^{\bar{v}} dF(v_2) + \int_{\underline{v}}^{v_L}\int_{\underline{v}}^{v_2} dF(v_1)dF(v_2) + F(v_L)\int_{\underline{v}}^{v_L}\int_{v_1}^{\bar{v}} dF(v_2)dF(v_1).$$

We can rewrite A as

$$\begin{aligned} A &= B + 2\left[\frac{p_H-c_L}{p_L-c_L}\int_{\underline{v}}^{\bar{v}}\int_{v_1}^{\bar{v}} dF(v_2)dF(v_1) - 1\right] \\ &= B + \left[\frac{p_H-c_L}{p_L-c_L} - 2\right], \end{aligned}$$

where the second equality follows from $\int_{\underline{v}}^{\bar{v}}\int_{v_1}^{\bar{v}} dF(v_2)dF(v_1) = \int_{\underline{v}}^{\bar{v}}(1-F(v_1))dF(v_1) = 1/2$. Substituting A,

$$\beta = \frac{B}{B+\left(\frac{p_H-c_L}{p_L-c_L}-2\right)}.$$

Because $B > 0$, $\beta \in (0,1)$ iff $\frac{p_H - c_L}{p_L - c_L} > 2$. We can rewrite

$$B = 1 - F(v_L)(1 - F(v_L))(1 - \frac{F(v_L)}{2}).$$

Take the derivative

$$\frac{\partial B}{\partial F(v_L)} = -3/2(F(v_L))^2 + 3F(v_L) - 1.$$

$\frac{\partial B}{\partial F(v_L)} < 0$ for $0 \le F(v_L) < 1 - \frac{1}{\sqrt{3}}$ and $\frac{\partial B}{\partial F(v_L)} > 0$ for $1 - \frac{1}{\sqrt{3}} < F(v_L) \le 1$. Let $\hat{v}_L$ denote the solution for $F(v_L) = 1 - \frac{1}{\sqrt{3}}$. So, B is decreasing in v_L for $v_L \in [\underline{v}, \hat{v}_L]$ and B is increasing in v_L for $v_L \in [\hat{v}_L, \bar{v}]$.

Suppose $\frac{p_H - c_L}{p_L - c_L} > 2$. Then, β is increasing in B. It follows that β is decreasing in v_L for $v_L \in [\underline{v}, \hat{v}_L]$ and increasing in v_L for $v_L \in [\hat{v}_L, \bar{v}]$. β reaches the minimum value

$$\beta_{min} \equiv \frac{1}{1 + \left(\frac{3\sqrt{3}}{3\sqrt{3}-1}\right)\left(\frac{p_H - c_L}{p_L - c_L} - 2\right)},$$

at $v_L = \hat{v}_L$ and the maximum value

$$\beta_{max} = \frac{1}{1 + \left(\frac{p_H - c_L}{p_L - c_L} - 2\right)}, \tag{36}$$

at $v_L = \underline{v}$ or $v_L = \bar{v}$.

Using (27),

$$\beta = 1 - \frac{s}{\lambda \int_{v_L}^{\bar{v}} (1 - F(v_2)) dv_2}.$$

β is decreasing in v_L, reaches the maximum value

$$\bar{\beta} = 1 - \frac{s}{\lambda \int_{\underline{v}}^{\bar{v}} (1 - F(v_2)) dv_2},$$

at $v_L = \underline{v}$, and equals 0 for $v_0 \le v_L \le \bar{v}$, where v_0 solves $\lambda \int_{v_0}^{\bar{v}} (1 - F(v_2)) dv_2 = s$.

The existence conditions are

$$s < \lambda [E(v) - \underline{v}] \tag{37}$$

$$\begin{aligned} s &< E\left(u | H, v_1 = \bar{v}\right) \qquad (38) \\ &= \frac{(1-\alpha)\beta(1-\beta)}{\alpha + (1-\alpha)\beta} \left[(p_H - p_L) + \lambda \int_{\underline{v}}^{\bar{v}} (v_2 - \bar{v}) dF(v_2) \right] \\ &= \frac{(1-\alpha)\beta(1-\beta)}{\alpha + (1-\alpha)\beta} \left[(p_H - p_L) - \lambda(\bar{v} - E(v)) \right], \end{aligned}$$

$$\frac{p_H - c_L}{p_L - c_L} > 2, \tag{39}$$

and

$$\begin{aligned}
\bar{\beta} &> \beta_{max} \rightarrow \\
\frac{p_H - c_L}{p_L - c_L} &> 1 + \frac{\lambda(E(v) - \underline{v})}{\lambda(E(v) - \underline{v}) - s} \\
s &< \frac{\lambda(E(v) - \underline{v})(p_H - 2p_L + c_L)}{p_H - p_L}.
\end{aligned} \tag{40}$$

Condition (37) ensures $\underline{v} < v_L < \bar{v}$. Condition (38) ensures the consumer always samples a second opinion upon recommendation H, i.e., $v_H = \bar{v}$. Since the equilibrium cheating probability is $\beta^* \in [\beta_{min}, \beta_{max}]$ and $E^S(u|H, v_1 = \bar{v})$ is concave in β, condition (38) is satisfied at β^* if it is satisfied at both β_{min} and β_{max}. Conditions (39) and (40) ensure the consumer and experts' best response functions will intersect at least once. Note that conditions (39) and (40) imply (37). Let

$$\begin{aligned}
&\hat{s}(p_L, p_H) \equiv \\
&\min\left\{\frac{\lambda(E(v) - \underline{v})(p_H - 2p_L + c_L)}{p_H - p_L},\right. \\
&\left.\min\{\frac{(1-\alpha)\beta_{min}(1-\beta_{min})}{\alpha + (1-\alpha)\beta_{min}}, \frac{(1-\alpha)\beta_{max}(1-\beta_{max})}{\alpha + (1-\alpha)\beta_{max}}\}\left[(p_H - p_L) - \lambda(\bar{v} - E(v))\right]\right\}.
\end{aligned} \tag{41}$$

.

In summary, a symmetric mixed-strategy exists if (39) and $s < \hat{s}(p_L, p_H)$. □

Next, we characterize the condition for uniqueness. Let β_u be the value of β such that β_u is the solution of the consumer's indifference condition (27) when $v_L = \hat{v}_L$. Then, β_u is characterized as

$$\begin{aligned}
\lambda(1 - \beta_u) \int_{\hat{v}_L}^{\bar{v}} (1 - F(v_2)) dv_2 &= s \\
\beta_u &= 1 - \frac{s}{\lambda \int_{\hat{v}_L}^{\bar{v}} (1 - F(v_2)) dv_2}.
\end{aligned} \tag{42}$$

Figure A.3 shows that a sufficient condition for uniqueness is $\beta_u \geq \beta_{max}$.

Proposition 7. *If $\frac{p_H - c_L}{p_L - c_L} > 2$ and $s < \hat{\hat{s}}(p_H, p_L)$, the equilibrium characterized in Proposition 6 is the unique symmetric mixed-strategy equilibrium.*

Proof. Proof for Proposition 7 First, we show $\beta_u \geq \beta_{max}$ is sufficient to ensure the uniqueness of the mixed strategy equilibrium. Let $\beta^E(v_L)$ denote the solution for the expert's indifference condition (35).

Let $\beta^C(v_L)$ denote the solution for the consumer's indifference condition (27). Proof for Proposition 6 shows $\beta^E(v_L)$ is decreasing for $v_L < \hat{v}_L$ and increasing for $v_L > \hat{v}_L$. It is easy to verify that $\beta^C(v_L)$ is strictly decreasing.

Suppose $\beta_u \geq \beta_{max}$. Then $\beta^C(v_L) > \beta^E(v_L), \forall v_L < \hat{v}_L$, because

$$\beta^C(v_L) > \beta^C(\hat{v}_L) \equiv \beta_u \geq \beta_{max} \geq \beta^E(v_L), \forall v_L < \hat{v}_L.$$

Hence, the solution for $\beta^E(v_L) = \beta^C(v_L)$ must be at some $v_L > \hat{v}_L$. Moreover, because $\beta^C(v_L)$ is decreasing and $\beta^E(v_L)$ is increasing for $v_L > \hat{v}_L$, there is at most one solution for $\beta^E(v_L) = \beta^C(v_L)$.

Using (36) and (42),

$$\begin{aligned} &\beta_u \geq \beta_{max} \\ &s \leq \frac{P-2}{P-1}\left[\lambda \int_{\hat{v}_L}^{\bar{v}} (1 - F(v_2)) dv_2\right], \end{aligned}$$

where $P \equiv \frac{p_H - c_L}{p_L - c_L}$.

Recall the existence condition requires $P > 2$. Let

$$\hat{\hat{s}}(p_H, p_L) \equiv \min\left\{\hat{s}(p_H, p_L), \frac{P-2}{P-1}\left[\lambda \int_{\hat{v}_L}^{\bar{v}} (1 - F(v_2)) dv_2\right]\right\},$$

where $\hat{s}(p_H, p_L)$ is the cutoff search cost for the existence of the mixed strategy equilibrium.

□

A.4 Comparative Statics

The equilibrium analysis we have shown so far is under a general distribution function $F(\cdot)$. In this section, we want to shed some light on how an improvement in matching value affects experts' cheating incentives. To facilitate this analysis, we focus on uniform distribution and provide numerical examples of the comparative statistics when we change the support of the $F(\cdot)$.

We conduct three experiments: (i) increasing $\bar{v}$, (ii) increasing $\underline{v}$, and (iii) increasing both $\bar{v}$ and $\underline{v}$ while keeping Δv constant. (i) corresponds to the scenario in which a platform seeks to find the expert who best fits the consumer's idiosyncratic needs. Case (ii) can be interpreted as a strategy to weed out the experts who are poor matches for the consumer. (iii) combines the two cases.

We report three comparative statics in each experiment: changes in experts' cheating probability, changes in the consumer's reservation match value, and changes in the probability that the consumer will sample another expert after drawing an honest opinion.

We set $p_H = 10$, $p_L = 2$, $s = 1$ and $\lambda = 0.5$ in the following examples. Figures A.4 - A.6 provide comparative statics when $\bar{v}$ increases. Figures A.7 - A.9 provide comparative statics when $\underline{v}$ increases. Figures A.10 - A.12 provide comparative statics when both $\bar{v}$ and $\underline{v}$ increase while Δv remains constant.

Experiments (i) and (ii) show that increasing $\bar{v}$ or $\underline{v}$ has the opposite effect on honesty. When $\bar{v}$ increases, the consumer will search more often, hoping to find an expert who can provide an extremely high match value. Consumer search will discipline experts and reduce cheating. Increasing $\underline{v}$ has the opposite effect because it reduces consumer search frequency, leading to more cheating.

In Experiment (iii), we increase both $\bar{v}$ and $\underline{v}$ while keeping Δv constant. Figures A.10-A.12 show that consumer search frequency does not change. As a result, the expert's cheating probability remains the same.

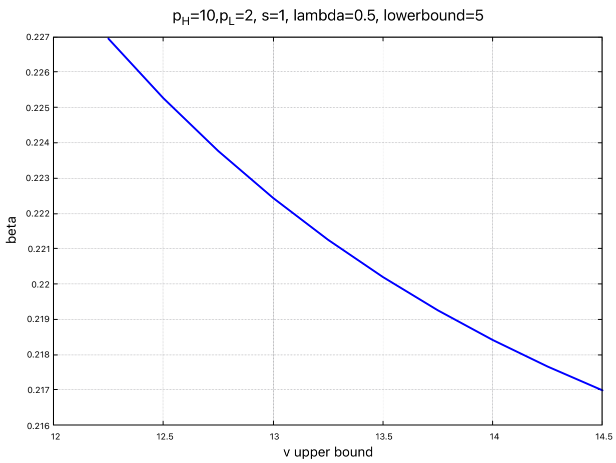

Figure A.4: Change in $\bar{v}$ on β^*

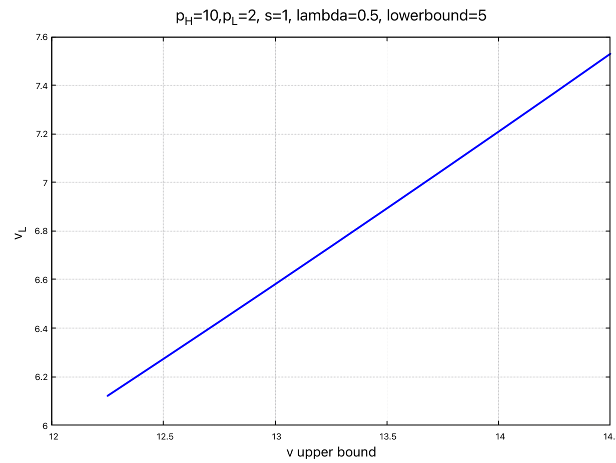

Figure A.5: Change in $\bar{v}$ on v_L^*

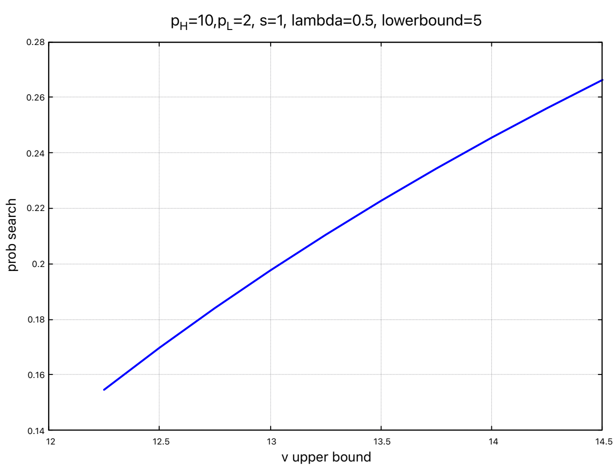

Figure A.6: Change in $\bar{v}$ on search probability

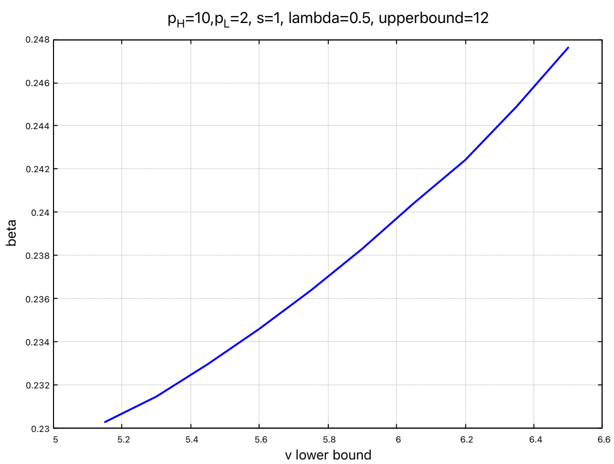

Figure A.7: Change in $\underline{v}$ on β^*

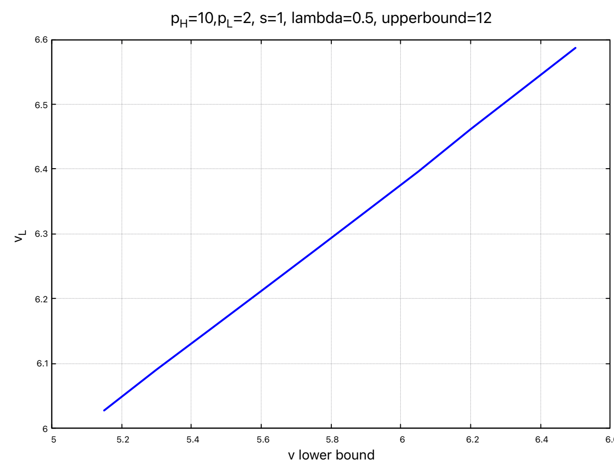

Figure A.8: Change in $\underline{v}$ on v_L^*

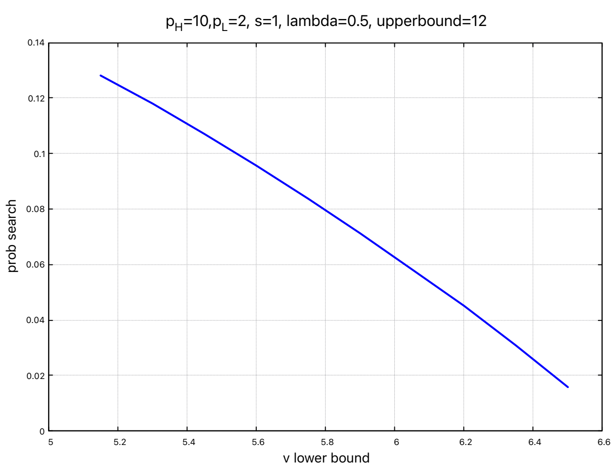

Figure A.9: Change in $\underline{v}$ on search probability

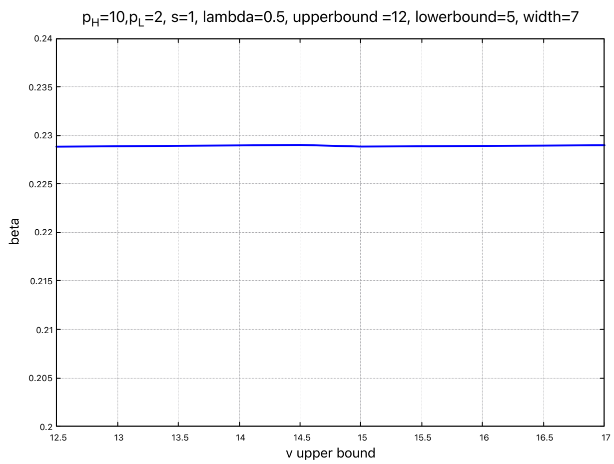

Figure A.10: Move of both $\bar{v}$ and $\underline{v}$ to right on β^*

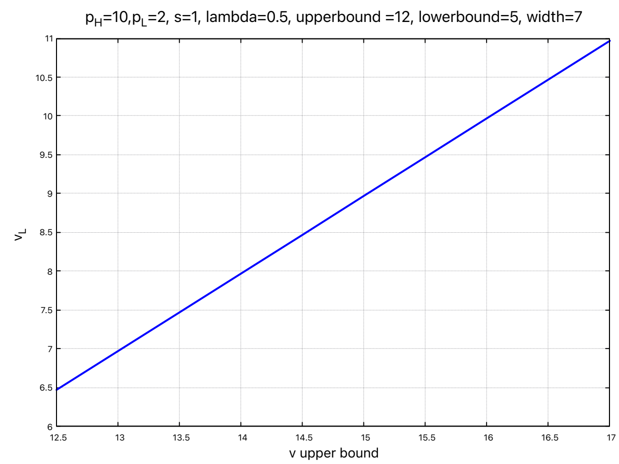

Figure A.11: Move of both $\bar{v}$ and $\underline{v}$ to right on v_L^*

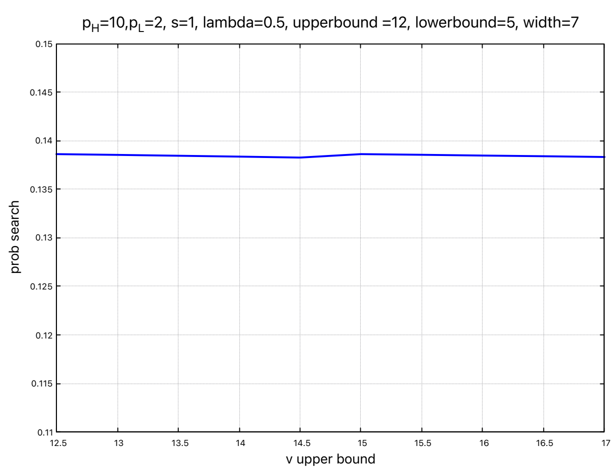

Figure A.12: Move of both $\bar{v}$ and $\underline{v}$ to right on search probability

References

Wolinsky, Asher. 1993. “Competition in a Market for Informed Experts’ Services.” *The RAND Journal of Economics* 24 (3): 380-398.