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**Communication Technology Advances and
Consequences:
Using Two-sided Search Model**

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Abstract

Do advances in communication technology, such as online dating sites and social networking services, change the value of being in a relationship? This paper constructs a non-stationary, two-sided search market equilibrium model, including cohabitation, to analyze the quantitative effects of advances in communication technology on individuals' marital behavior and welfare. This paper provides a new proof of the existence of a non-stationary market equilibrium and establishes its local uniqueness. Using the model's equilibrium condition, I develop a new identification argument to separately identify parameters previously considered difficult to identify. This paper estimates the structural model with indirect inference, using the NLS 72 and the NLSY 97. I show that changes in mating preference contribute more to changes in marital behaviors than do advances in communication technology.

Keywords: Marriage, divorce, cohabitation, two-sided search model, market equilibrium, structural estimation.

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1 Introduction

Communication technology has significantly changed our lives over the past decades. People today use texting services for daily communication, and many couples now meet online. It is important to understand how people react and change their marital behavior at equilibrium. This paper presents an empirical, non-stationary, two-sided search market equilibrium model of marital formation that includes cohabitation. Using the model, this paper quantifies the effects of the technological change on relationship formation patterns and welfare.

These advances expand the choice set. At first glance, these changes appear beneficial. More choices can lead to higher overall welfare, which might be unattainable with fewer options (see, for example, Mas-Colell et al. (1995)). However, while theory suggests a potential rise in overall welfare, the resulting equilibrium effects can leave certain individuals at a disadvantage, thereby fundamentally altering the value of forming a partnership in society.

Beyond evaluating the overall impact of technological changes, it is important to understand the specific channels through which society is affected; for example, individual preferences and/or matching technology. Depending on the channels, the implications would differ. This research identifies which parts of the model change with the advent of communication technology advances.

This paper makes two main contributions: First, this paper quantifies the complicated impacts of the advances in communication technology on marital behaviors and on welfare. Second, this paper provides a new identification argument: In the search-matching literature, identifying parameters of the model is challenging (see, for example, Flinn and Heckman (1982)). Suppose we have observed more matches between men and women with particular characteristics (for example, a highly educated man and a highly educated woman). This may result from (i) they like each other more (a deterministic part of a match value, or preference); (ii) there are more opportunities for them to meet (matching technology); (iii) there are just more people with the particular characteristics (stocks of singles issue), and/or (iv) a probability of having good draws differs (variance of a stochastic parts in the model). Typically, however, we can observe only data about “duration of being single,” “who matches with whom,” and “how long they remain matched.” This data limitation is common in structural estimation for marriage matching

studies, which this paper addresses.

In the literature, identification often relies heavily on specific functional forms for a meeting probability or, in some cases, abstracts from possible aspects of a model (as an extreme example, a frictionless marriage market assumption (Choo and Siow (2006))). Including all of the above four possibilities, this paper presents a method for identifying parameters with a new type of moments, using a market equilibrium concept. This identification method is widely applicable to other search studies without relying on a unique dataset. This method can apply to not only two-sided labor markets but also to any model in any field with an endogenous aggregate market object involved.¹ Also, related to the second contribution, this paper proves existence of equilibrium under non-stationarity and limited commitment assumptions, and its local uniqueness of equilibrium.

This study uses the National Longitudinal Study of High School Class of 1972 (NLS 72) and the National Longitudinal Survey of Youth 1997 (NLSY 97) as two different cohorts. The NLS 72 is assumed to represent the cohort before advances in the communication technology, and the NLSY 97 is assumed to represent the cohort after advances in the communication technology.

This paper estimates a dynamic discrete choice model with three alternatives: single, cohabitation, and marriage. In this equilibrium model, stocks of individuals are also equilibrium objects. Individuals optimally decide whether to be single, cohabiting, or married, while considering dynamics of the endogenously determined aggregate stocks of individuals. The structural parameters are estimated through indirect inference. In matching moments, this paper uses a *rational expectations* condition associated with the dynamics of equilibrium stocks, providing new sources of identification. These appear as new moment conditions within the market equilibrium framework.

This paper performs a series of counterfactual experiments. In these experiments, this paper assesses welfare changes caused by technological advancements and decompose through which channels the marital behaviors change, incorporating *equilibrium effects*. The estimation results indicate that individuals in the NLSY 97 cohort enjoy more efficient matching technology. However, welfare — defined as an ex-ante expected lifetime payoff in the marital game while

¹We can use the same identification strategy with a stationary environment because a stationary economy is a special case of a non-stationary environment.

normalizing a flow value of being single to 0 — is lower in the NLSY 97 cohort when equilibrium effects are considered. Second, the counterfactual experiments reveal the primary driver of changes in marital behavior between the NLS 72 and NLSY 97 cohorts is not the change in the matching technology itself, or the changes in the distribution of the stochastic component of a match that is also assumed technology-dependent. This paper finds that most of the reduction in the ex-ante expected lifetime payoff between the NLS 72 and NLSY 97 cohorts comes from changes in preferences. In addition, this paper finds that changes in the distribution of lifetime relationship experiences are more strongly influenced by changes in separation costs than by the technological innovation.

To isolate the impact of these technological advances, this paper controls for other changes. Previous literature mentions changes along several dimensions.² However, this paper emphasizes that most of the changes pointed out by previous literature can be reinterpreted roughly as a change in preferences, separation costs, non-stationary distributions of individual types (stocks), and attitudes toward cohabitation.³ By extending a model used in previous studies and developing the rigorous identification argument, this paper can control for other changes. In other words, this paper provides an approach to extract the effects of the technological changes through structural estimation under data limitations without explicit technology variations.⁴

Broadly, this study can be characterized from the following three perspectives: the use of a non-stationary market equilibrium model; the incorporation of search frictions; and the inclusion of both cohabitation and marriage. In these aspects, this study differs from previous studies. I introduce some of the previous works related to my research.

²Various changes occurred between the two cohorts, as documented in previous literature. These include alterations in divorce costs (Friedberg (1998) and Voena (2015)), a decline in the gender wage gap (Blau and Kahn (2000)), advancements in household technology (Greenwood and Vandenbroucke (2005)), shifts in the gender ratio in colleges (Goldin et al. (2006)), changing perceptions toward cohabitation (Stevenson and Wolfers (2007)), adjustments in the minimum wage (Flinn (2006)), and evolving social norms (Fernández (2013)).

³For example, changes in divorce laws between the two cohorts are captured as changes in separation costs. Changes in wage differences between men and women are captured through changes in a match value. Changes in the gender ratio are captured through changes in the distribution of individual types (stocks). Changes in social norms are captured through changes in preferences. This paper includes cohabitation as a choice to control for changes in trends toward cohabitation.

⁴Strictly speaking, we can still interpret it as a cohort effect. However, as discussed above, the model indirectly controls for other major changes between the two cohorts. It might be difficult to conceive of factors other than communication technologies that could have affected the matching technology (Hitsch et al. (2010)). It allows me to think that one of most appropriate interpretations is a communication technology shock. However, regardless of the interpretation, the value of this paper's main contribution — a new identification method to separate matching technology from mating preference — is not diminished.

First, the recent marriage search models developed by, for example, Akin and Platt (2016), Beauchamp et al. (2018), and Shephard (2019) are most closely related. A main difference is that they do not focus on technological changes. Nor do they consider cohabitation: In the US, cohabitation has become increasingly common and is emerging as an alternative form of marital behavior (Stevenson and Wolfers (2007)). Excluding it could lead to misleading results. One significant change resulting from technological advances is the advent of online dating sites. However, marriage may not be directly influenced by the new style of dating using online dating sites. Modeling cohabitation allows indirect capture of online datings effects, which may be overlooked by focusing solely on marriage.

Second, this model relates to matching studies assuming a large market with perfect information, such as Chiappori (1988) and Choo and Siow (2006). Although a large-market, perfect-information approach simplifies identification and estimation, its assumption of no search friction remains controversial. Incorporating meeting probability, conceptually representing search frictions, is central to this research. Thus, this study is closer to works that include search friction, such as Goussé et al. (2017) and Shephard (2019).

Third, this model contributes to the literature dealing with both cohabitation and marriage, including works by Brien et al. (2006), Matouschek and Rasul (2008), and Blasutto (2024). Brien et al. (2006) and Blasutto (2024) incorporate cohabitation with marriage through a learning structure but focus only on the women's side. Matouschek and Rasul (2008) employ a basic non-cooperative game framework, incorporating cohabitation as a choice.⁵ Unlike Brien et al. (2006), Matouschek and Rasul (2008), and Blasutto (2024), this study explicitly employs a non-stationary market equilibrium framework to capture strategic interactions and marriage market dynamics. Even though many previous studies assume stationarity of an economy, individuals' dynamic behaviors naturally should cause non-stationarity of an economy which should be captured (see Manea (2017) and Ke et al. (2025)). This study accommodates non-stationarity of the economy to explicitly control for the variations of the stocks of singles.

Section 2 documents the trend of relationship formulation in the US. Section 3 describes the basic environment of the model. Section 4 describes how a player decides their optimal behavior

⁵Matouschek and Rasul (2008) use a two-player simultaneous non-cooperative game setting. However, they assume a symmetric game where a husband and a wife have the same payoff. Therefore, essentially, the game can be considered as a single-agent model.

under the game rule. Section 5 provides the properties of reservations values. Section 6 discusses the equilibrium concept employed in this paper. In Section 7, I provide the description of data sets. Section 8 explains the identification strategy. Section 9 provides the estimation method to estimate the model primitives. Section 10 provides the estimation results. In Section 11, I introduce several counterfactual experiments. Section 12 concludes. Proofs, algorithms, and details are in the Online Appendix.

2 Empirical trend about relationship formation

One of the most striking changes US society has experienced over the last 40 years is the change in ways of communication and dating styles. People did not even have a cell phone 40 years ago. So, if a person wanted to communicate with or ask for a date from someone, he/she needed to do it in person or through home phones. However, gradually, our lifestyles have changed alongside the advent of new technology. People started using messaging services that made communication among people much easier. Today, with the advent of the internet, almost everyone uses their own smartphone with several messaging services, social networking services, and even online dating apps. This phenomenon has definitely induced changes in relationship formation.

With the advent of new technology, one surprising fact we have seen associated with relationship formation is that, today, most couples meet their partners online in the US. In the past, the typical ways of meeting a partner were within their network, for example, through friends of friends, their colleagues, their religious group, or at a bar near where they lived. In this sense, their choice set was restricted. However, the rise of the internet has allowed individuals to use it even in choosing their partners.

Figure 1 illustrates how couples that formed in a given year first met each other. For example, Rosenfeld et al. (2019) document that the percentage of couples who met online is 0 percent before 1995. About 22 percent of couples in 2009 meet online. It increased even further to 39 percent by 2017.^{6, 7} Figure 1 provides evidence of how much our society has accepted online

⁶Figure 1 comes from the How Couples Meet and Stay Together (HCMST) survey. The HCMST is a nationally representative longitudinal survey of adults in the US with a spouse or partner, conducted in 2009 and 2017. See Rosenfeld et al. (2019) for more detailed information.

⁷It is important to note that these percentages represent the specific channel through which couples successfully led to the formation of a partnership, not necessarily that individuals used only that single platform; they may have

dating. This phenomenon has the potential to alter cohabitation and marriage patterns. Consistent with Figure 1, aggregate relationship choices evolve markedly across cohorts; detailed series are reported in Online Appendix A3.

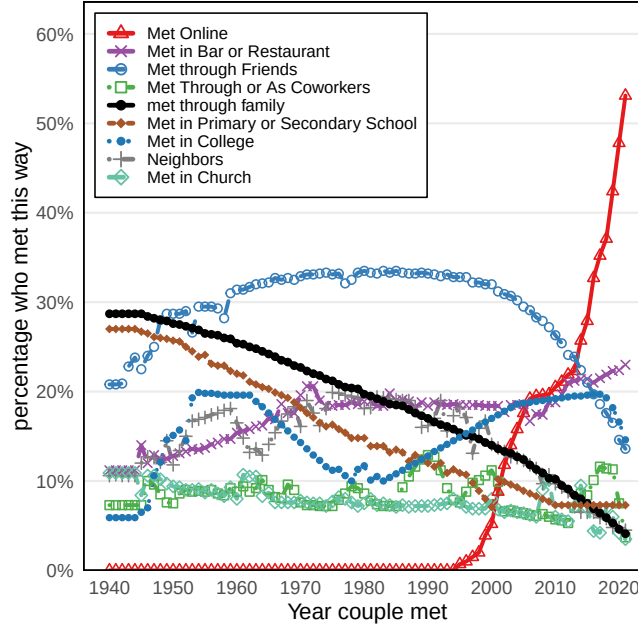


Figure 1: How couples that formed in a given year first met each other.
Source: Rosenfeld et al. (2019).

3 Model

The model builds on work which focuses on a two-sided search model, for example, Seitz (2009), Akin and Platt (2016), Manea (2017), Beauchamp et al. (2018), and Shephard (2019), and, also, on work which includes cohabitation as a choice, for example, Brien et al. (2006) and Drewianka (2006). This paper presents a finite horizon, non-stationary, two-sided search market equilibrium model for analysing marital relationship formulation including cohabitation. The environment of the game is theoretically described in the following section, while its detailed empirical specification is provided in Section 9 and Online Appendix B.2.

utilized multiple methods in their search for a partner.

3.1 Environment

The economy consists of continua of men and women with observable finite discrete types. The type of an individual is specified by various dimensions. A man's type is indexed by $i \in \mathcal{I} = \{i_1, i_2, \dots, i_I\}$, and a woman's type is indexed by $j \in \mathcal{J} = \{j_1, j_2, \dots, j_J\}$. A discrete type consists of time-variant characteristics such as the number of children, and time-invariant characteristics such as race. I explain the specific types in more detail in Section 9 and Online Appendix B.2.

I confine my attention to a non-stationary economy, where situations an individual faces change through time periods. Especially, stocks of individuals in the economy change through time periods. For the discussion under a non-stationary assumption, I explicitly introduce an additional dimension, $t = \{1, 2, \dots, T\}$, which implies a time period or age.

Also, the technology exposure level is denoted as $\kappa \in \mathcal{K} = \{\kappa_1, \kappa_2, \dots, \kappa_K\}$. It represents a technology level that people in the economy can access, and is determined when an individual enters into the marriage market. The whole economy is assumed to be completely divided into sub-economies denoted by κ . This κ corresponds to segregated sub-marriage markets for individuals. Associated with the segregated technology-dependent sub-economies, two important assumptions are also made: An individual's technology level, κ , does not evolve during the dynamic marriage game, and people who live in different κ worlds cannot meet with each other. Accordingly, we can think of κ in the following way: The technology level index, κ , is not about the attribute that an individual has, but rather about the segregated world that the individual enters into. Assuming the technology level, κ , is common and constant within each cohort makes the model computationally tractable.⁸

Specifically, the first cohort (NLS 72) is assumed to have no access to advanced communication technologies during their marriageable years. In contrast, people in the second cohort (NLSY 97) are assumed to have access to these technologies during their marriageable years.⁹

⁸At first glance, restricting possible meetings seems somehow a strong assumption. However, the significant time gap between the two cohorts (NLS 72; $\kappa = 1$ and NLSY 97; $\kappa = 2$) partially justifies this restriction because people in the different cohorts rarely match with each other.

⁹Note that $\kappa = 2$ is common to everyone in the cohort and constant during lifetime. This setting implicitly assumes that, dating styles in the society have changed already at $t = 1997$, or that, say, Tinder, exists from the beginning at the cohort. This assumption is necessary for computational tractability, but is strong. In Online Appendix D, I assess the robustness of my findings of the setting that $\kappa = 2$ is *fixed from the beginning* in the NLSY 97 cohort at $t = 1997$. The analysis demonstrates that my main conclusions are robust even when this assumption is relaxed to allow for the stochastic arrival and heterogenous adoption of new technologies. See Online Appendix D

An individual's marriage market is assumed to be limited to same-race individuals. The same-race marriage market assumption is also motivated by computational burdens and by the low rates of interracial cohabitation and marriage.

Let $\Lambda_{it\kappa}^{SM}$ be the stock of type i single men with κ at time t , $\Lambda_{it\kappa}^{SW}$ be for women, $\Lambda_{ijt\kappa}^C$ be the stock of cohabiting couples by a type i man and a type j woman with κ at time t , and $\Lambda_{ijt\kappa}^M$ be the stock for married couples. I assume a bounded support for each stock. This is a reasonable assumption given the need for stocks to be non-negative and the finite size of each cohort. Let $\Lambda_{t\kappa}$ be a vector of stocks of individuals in the economy given time t under κ , $\Lambda_{t\kappa} = \{\Lambda_{it\kappa}^{SM}, \Lambda_{jt\kappa}^{SW}, \Lambda_{ijt\kappa}^C, \Lambda_{ijt\kappa}^M\}_{ij}^{IJ}$, and assume that it is defined on the topological vector space, $\mathbb{R}_+^{2IJ+I+J}$. Let Λ_κ be a vector of stocks of individuals in the economy under κ , $\Lambda_\kappa = \{\Lambda_{t\kappa}\}_t^T$, and assume it is defined on the topological vector space, $\mathbb{R}_+^{(2IJ+I+J)T}$. The stocks of individuals, Λ_κ , are determined endogenously in the model, which I discuss in more detail in Section 6.

The market has the following aggregate matching technology: Let $\Lambda_{t\kappa}^{SM}$ and $\Lambda_{t\kappa}^{SW}$ be an aggregate stock of single men and an aggregate stock of single women at time t under κ in the economy respectively, and $\Lambda_{t\kappa}^{SM} = \sum_i^I \Lambda_{it\kappa}^{SM}$ and $\Lambda_{t\kappa}^{SW} = \sum_j^J \Lambda_{jt\kappa}^{SW}$. Let $M_{t\kappa}$ be the total number of meetings happening at time t under κ . Let $z_\kappa(\cdot)$ be an aggregate matching function with

$$M_{t\kappa} = z_\kappa(\Lambda_{t\kappa}^{SM}, \Lambda_{t\kappa}^{SW}) \quad (1)$$

(see, for example, Petrongolo and Pissarides (2001)).

3.2 Decision timing, game type and information assumption

The decision timing, game type, and information structure are specified next. Let $\alpha_{ijt\kappa}^M(\Lambda_{t\kappa}) \in [0, 1]$ be a type i man's probability of meeting a type j woman under a level of technology κ at time t . Similarly, let $\alpha_{ijt\kappa}^W(\Lambda_{t\kappa}) \in [0, 1]$ be a type j woman's probability of meeting a type i man under a level of technology κ at time t . A meeting probability is a continuous function mapped from $\Lambda_{t\kappa}$.

At the beginning of each period, all of the following happen: A type i man m and a type j woman w meet based on their meeting probability if they are single. If they meet, they draw a

for more detail.

flow match value, $s_{match} \in \mathbb{R}$, from the conditional distribution $F_{s|ijt\kappa}$ conditional on observable types, i, j, t and κ , which is introduced in more detail in Section B.2. If they are already matched, they redraw a new match value at every period. I assume that there is no during-the-match search in the model.¹⁰

Utility flows from a match are perfectly transferable with side payments within the couple. After meeting, singles divide the value of their match under a Nash bargaining procedure based on the woman's bargaining weight, $\phi \in [0, 1]$. They jointly decide whether to remain single, cohabit, or get married. If they already are matched, they jointly decide whether to continue being in the same relationship or dissolve at every decision time with an option to renegotiate if their situation changes (limited commitment). If a single does not meet with anyone, the individual stays single in the time period. Note that separation is endogenous and occurs when the value of a match is too low for a couple to remain mutually beneficial. All individuals play this game until they reach the terminal time period, T .

I assume that an individual has perfect information about his/her potential partner only after meeting. However, an individual is assumed to know distributions of others who the individual has not met yet in the economy (search frictions).

There are many theoretical studies and several empirical studies about the marriage market using non-cooperative game frameworks also (for example, Wong (2003), Del Boca and Flinn (2012, 2014), Friedberg and Stern (2014), and Ke et al. (2025)), where there is no commitment device assumed. However, in this research, I employ a cooperative generalized Nash bargaining approach. Under a Nash bargaining framework, side payments between players can be incorporated into the model, which makes each player's decision process more straightforward. I return to this point in more detail in Section 4.

Another possible model setting is to include a directed search aspect to take into account an individual's choice about, for example, whether he or she uses online dating sites. Mathematically, it is not difficult to include the aspect into my fixed point argument shown in Online Appendix E.1. More importantly, however, in reality, people use multiple platforms: Not only do people use online dating sites, but also they go to bars. I do not have the information about an individual's actual use of a particular dating platform. Therefore, this paper employs

¹⁰In the data, almost all individuals match with a different partner only after returning to single first.

a random search model without considering a directed search aspect. Rather than modeling individual-level choices across specific dating platforms, I abstract from such details to focus on the market-level consequences of a fundamental shift in social availability of these technologies.

3.3 Flow match value

In this section, I introduce a flow match value. Before doing that, I introduce several variables first. Let $d_{mwt\kappa} = 1, 2$, or 3 be a mutually exclusive choice at time t by a decision unit of a man m and a woman w under technology level κ denoted by m, w, t and κ : $d_{mwt\kappa} = 1$ represents staying single, $d_{mwt\kappa} = 2$ represents cohabiting, and $d_{mwt\kappa} = 3$ represents being married.

Let $u_{ijt\kappa}^m + u_{ijt\kappa}^w \in \mathbb{R}$ be a deterministic part of the flow match value conditioned on observables, i, j, t and κ . It represents the deterministic part of the flow match value caused by a match itself between an i type man and a j type woman under κ at time t . Note that κ in the subscript of $u_{ijt\kappa}^m + u_{ijt\kappa}^w$ captures an additional benefit/loss caused by a level of technology κ . We can think of the effect of technology on a match itself in the following way: People would meet with more/less compatible partners due to the technology advances, conditioned on the same observables, i, j and t . There is a marriage bonus, $\mathbb{M}_{ijt\kappa} \in \mathbb{R}$, which a couple by a type i man and a type j woman receives under κ at time t when $d_{mwt\kappa} = 3$. This research includes separation costs. A couple incurs separation costs when going back to single from a match. Let $\mathbb{C}_{ijt\kappa}^C \in \mathbb{R}$ be a cohabitation separation cost between a type i man and a type j woman at time t with κ . Similarly, let $\mathbb{C}_{ijt\kappa}^M \in \mathbb{R}$ be a divorce cost between a i type of man and a j type of woman at time t with κ .

The flow match value that a type i man, m , and a type j woman, w , with state variables, t and κ , receive, $s_{mwt\kappa} \in \mathbb{R}$, is

$$\begin{aligned} s_{mwt\kappa} = & \mathbf{X}_{ijt}^u \boldsymbol{\beta}_\kappa \mathbb{I}[d_{mwt} \neq 1] + (\mu_\kappa^{Cm} + \mu_\kappa^{Cw}) \mathbb{I}[d_{mwt} = 2] + \mathbf{X}_{ijt}^M \boldsymbol{\beta}_\kappa^M \mathbb{I}[d_{mwt} = 3] + (\mu_\kappa^{Mm} + \mu_\kappa^{Mw}) \mathbb{I}[d_{mwt} = 3] \\ & + \epsilon_{mwt\kappa} \mathbb{I}[d_{mwt} \neq 1] - \mathbb{C}_{ijt\kappa}^C \mathbb{I}[d_{mwt-1} = 2, d_{mwt} = 1] - \mathbb{C}_{ijt\kappa}^M \mathbb{I}[d_{mwt-1} = 3, d_{mwt} = 1], \end{aligned} \quad (2)$$

where $\mathbb{I}[\cdot]$ is an indicator function. In equation (2), $\epsilon_{mwt} \in \mathcal{E} = \mathbb{R}$ is a stochastic part of the flow match value, which is match-specific. The stochastic component, ϵ_{mwt} , represents a match value shock independent of technology after conditioning on observables i, j and κ . I assume that, after

a match, it starts exhibiting serial correlation. I provide a detailed specification of the dynamics of ϵ_{mwt} in Online Appendix B.2.

Additionally, there are two persistent unobserved heterogeneity terms: Let $\mu_{m\kappa}^C$ and $\mu_{m\kappa}^M$ be cohabitation- and marriage-specific unobserved heterogeneity for a man m , and $\mu_{w\kappa}^C$ and $\mu_{w\kappa}^M$ for a woman w , which an individual draws when the individual enters into the marriage game. They are observed by a couple but not by econometricians.

Their variance depends on the technology level κ as follows: Let $\boldsymbol{\mu}_\kappa^M = [\mu_{m\kappa}^C, \mu_{m\kappa}^M]$ and $\boldsymbol{\mu}_\kappa^W = [\mu_{w\kappa}^C, \mu_{w\kappa}^M]$ with their joint distribution

$$\boldsymbol{\mu}_\kappa^M \sim iidN(0, \boldsymbol{\Gamma}_{\mu(\kappa)}^M); \quad (3)$$

$$\boldsymbol{\mu}_\kappa^W \sim iidN(0, \boldsymbol{\Gamma}_{\mu(\kappa)}^W). \quad (4)$$

Note that the technology effect on the stochastic part of the flow match value is captured by allowing the variances, $\boldsymbol{\Gamma}_{\mu(\kappa)}^M$ and $\boldsymbol{\Gamma}_{\mu(\kappa)}^W$, to differ depending on technology level κ . Intuitively, even conditional on the same observables, i, j, t and κ , an individual might meet with an extremely good/bad partner based on the technology level κ .¹¹

3.4 Value functions

In the following sections, I suppress the notation κ for simpler notation.¹² With the above flow match value, I can write down the value functions of being single, cohabiting and getting married in the game. Let $U_{it}^S \in \mathbb{R}$ be a type i man's value function of staying single at time t . Similarly, let $U_{jt}^S \in \mathbb{R}$ be the same for a j type of woman. Denote $W_{ijt}^C(\epsilon_{mwt}) \in \mathbb{R}$ as a match value caused by cohabitation between a type i man m and a type j woman w at time t with a realization of the stochastic part of the flow match value, ϵ_{mwt} . Similarly, denote $W_{ijt}^M(\epsilon_{mwt}) \in \mathbb{R}$ for marriage.¹³ Let $i' \in \mathcal{I}$ and $j' \in \mathcal{J}$ be the next period's type of a man and the next period's type of a woman respectively. For example, i' and j' include the number of children in a household,

¹¹For identification of $\boldsymbol{\Gamma}_{\mu(\kappa)}^M$ and $\boldsymbol{\Gamma}_{\mu(\kappa)}^W$, since my game setting is a cooperative one, only $\boldsymbol{\Gamma}_{\mu(\kappa)}^M + \boldsymbol{\Gamma}_{\mu(\kappa)}^W$ is identified. This is because, under a cooperative game setting, only the sums, $\mu_{m\kappa}^C + \mu_{w\kappa}^C$ and $\mu_{m\kappa}^M + \mu_{w\kappa}^M$, matter.

¹²As I describe in Section 3.1, κ is fixed when an individual enters into the game.

¹³Note that, for simpler notation, I deliberately represent the value functions without the notion of $\boldsymbol{\mu}_\kappa^M$ and $\boldsymbol{\mu}_\kappa^W$.

which can change over time.

Denote a sum of the amounts of marriage surpluses caused by marriage between a type i man m and a type j woman w at time t with ϵ_{mwt} as $Z_{ijt}^{SM}(\epsilon_{mwt}) \in \mathbb{R}$. Namely,

$$Z_{ijt}^{SM}(\epsilon_{mwt}) = W_{ijt}^M(\epsilon_{mwt}) - U_{it}^S - U_{jt}^S. \quad (5)$$

Similarly, a sum of the amounts of cohabitation surpluses, $Z_{ijt}^{SC}(\epsilon_{mwt}) \in \mathbb{R}$, is denoted as

$$Z_{ijt}^{SC}(\epsilon_{mwt}) = W_{ijt}^C(\epsilon_{mwt}) - U_{it}^S - U_{jt}^S. \quad (6)$$

Let $\varsigma \in (0, 1)$ be a discount factor. Let the deterministic part of the flow utility of being single be normalized to 0. Then, the value functions for being single, cohabiting and getting married are

$$U_{it}^S = \varsigma \sum_j \mathbb{E}_{t+1|t} [\alpha_{ij't+1}^M(\Lambda_{t+1}) \max\{U_{i't+1}^S, U_{i't+1}^S + (1 - \phi)Z_{ij't+1}^{SC}(\epsilon_{mwt+1}), U_{i't+1}^S + (1 - \phi)Z_{ij't+1}^{SM}(\epsilon_{mwt+1})\} \\ + (1 - \sum_j \alpha_{ij't+1}^M(\Lambda_{t+1}))U_{i't+1}^S]; \quad (7)$$

$$U_{jt}^S = \varsigma \sum_i \mathbb{E}_{t+1|t} [\alpha_{ij't+1}^W(\Lambda_{t+1}) \max\{U_{j't+1}^S, U_{j't+1}^S + \phi Z_{ij't+1}^{SC}(\epsilon_{mwt+1}), U_{j't+1}^S + \phi Z_{ij't+1}^{SM}(\epsilon_{mwt+1})\} \\ + (1 - \sum_i \alpha_{ij't+1}^W(\Lambda_{t+1}))U_{j't+1}^S]; \quad (8)$$

$$W_{ijt}^C(\epsilon_{mwt}) = s_{mwt} + \varsigma \mathbb{E}_{t+1|t} \max\{U_{i't+1}^S + U_{j't+1}^S, W_{ij't+1}^C(\epsilon_{mwt+1}), W_{ij't+1}^M(\epsilon_{mwt+1})\}; \quad (9)$$

$$W_{ijt}^M(\epsilon_{mwt}) = s_{mwt} + \varsigma \mathbb{E}_{t+1|t} \max\{U_{i't+1}^S + U_{j't+1}^S, W_{ij't+1}^M(\epsilon_{mwt+1})\}. \quad (10)$$

In equations (7) and (8), the surplus division occurs through a generalized Nash bargaining over potential gains represented by spouses' value functions. I assume that the bargaining weight, ϕ , is the same in each bargaining case, cohabitation and marriage.¹⁴ In the model, the ideal of the search cost is captured by the discount factor and the periods left for a searcher (see, for example,

¹⁴Note that I take ϕ as exogenously given. The value of the bargaining weight ϕ determines the distribution of assets or resources between spouses during their match. Therefore, a different value of ϕ might lead to a big difference in their lifetime utility. However, in focusing only on the dynamics of the marital formulation/dissolution, the value of ϕ does not matter by the assumption of the cooperative game with transferable utilities employed in this research. Following the previous literature, I set ϕ to 0.5.

Hall and Rust (2021)).

Individuals in the economy are assumed to have rational expectations on stocks of individuals and meeting probabilities in the future marriage market. Following the typical search literature with a continuum of agents, in equations (7) and (8), we can treat the future meeting probabilities as exogenous for each individual.¹⁵

The duration dependence, for example, caused by the increased emotional connection or the investment between spouses, is captured by changes in i and j , and through the evolution of ϵ_{mwt} in s_{mwt} . Because the law of motion for ϵ_{mwt} exhibits serial correlation, I need to take the conditional expectation of $\max\{\cdot\}$ conditional on its previous realization, ϵ_{mwt} , after a match.¹⁶ The separation costs in s_{mwt} , C_{ijt}^C and C_{ijt}^M , would promote a long-term relationship to some extent. I include them for the model to explain some typical household behaviours that require long-term commitment.

Separation is endogenous and occurs when the value of a match is too low for a couple to remain mutually beneficial. When a couple separates, they move to single. Note that, in equation (10), transitions from marriage to cohabitation do not happen.¹⁷

In this research, the terminal period for all individuals, T , is assumed to be 45. I treat the state of an individual at time T as an absorbing state, which means that, after the period, an individual does not change their marital status.¹⁸ As Wolpin (1992) points out, if we have a discount factor in a model, the specification of the terminal period is not so important in deciding a *current* decision. See, for example, McKenzie (1986) about turnpike theorems accommodating non-stationarity of an economy.

¹⁵An individual needs to take into account the aggregate future stocks and meeting probabilities only. A player does not need to take expectations on other players' actions in the economy due to a continuum of players. The aggregate stocks of individuals and meeting probabilities work as a sufficient statistics for each individual in making their decision in this model. See, for example, Dubey and Kaneko (1984) and Stokey and Lucas (1989) about exogenous treatment of endogenous whole-market-related objects under general/market equilibrium frameworks.

¹⁶Note that an individual considers law of motion of i' and j' . As shown in Online Appendix B.2, i' and j' include the number of children in a household, and it is assumed to follow an exogenous stochastic process.

¹⁷This is because the separation cost from marriage is assumed to occur also in going back to cohabitation. Then, no couple wants to return to cohabitation from marriage, if $M_{ijt} > 0$.

¹⁸This assumption is partially justified because it becomes more difficult for a marital status to change as an individual becomes older. See, for example, Santos and Weiss (2016).

3.5 Law of motion of stocks

To close the market equilibrium model, I describe the law of motion for the endogenous aggregate stocks. Let D_t be a vector of mappings associated with the law of motion of aggregate stocks at time t , which is determined by the whole interactions of the model. Then, the law of motion of the stocks in this economy is described as,

$$\Lambda_{t+1} = D_t \cdot \Lambda_t. \quad (11)$$

Particularly, let $Tr_{ijt}^{a \rightarrow b}$ denote the transition rate from state a to state b for a couple formed by a type i man and a type j woman at time t , where $a, b \in \{SM, SW, C, M\}$. Given a particular type of $i' = \tilde{i} \in \mathcal{I}$ and $j' = \tilde{j} \in \mathcal{J}$ at time $t + 1$, D_t is specified as

$$\Lambda_{\tilde{i}t+1}^{SM} = \sum_{i,j|i'=\tilde{i}} \left[\sum_{a \in \{SM, C, M\}} Tr_{ijt}^{a \rightarrow SM} \Lambda_{ijt}^a \right]; \quad (12)$$

$$\Lambda_{\tilde{j}t+1}^{SW} = \sum_{i,j|j'=\tilde{j}} \left[\sum_{a \in \{SW, C, M\}} Tr_{ijt}^{a \rightarrow SW} \Lambda_{ijt}^a \right]; \quad (13)$$

$$\Lambda_{\tilde{i}\tilde{j}t+1}^C = \sum_{i,j|i'=\tilde{i}, j'=\tilde{j}} \left[\sum_{a \in \{SM, SW, C\}} Tr_{ijt}^{a \rightarrow C} \Lambda_{ijt}^a \right]; \quad (14)$$

$$\Lambda_{\tilde{i}\tilde{j}t+1}^M = \sum_{i,j|i'=\tilde{i}, j'=\tilde{j}} \left[\sum_{a \in \{SM, SW, C, M\}} Tr_{ijt}^{a \rightarrow M} \Lambda_{ijt}^a \right]. \quad (15)$$

The specifications are still highly abstract. The transition rates depend on reservation values, which are formally introduced in Section 5. At this point of the discussion, note that, from the above specifications, I assume that death does not happen. The economy itself is non-stationary such that Λ_t changes across time periods.

4 Decision process

Under a setting with transferable utilities with side payments between spouses, we can simplify the decision process of a couple after meeting and during a match. Namely, we focus only on the sum of the match surpluses of a couple. In this section, I describe a decision criteria under a Nash

bargaining framework setting even with three-alternatives; single, cohabitation, and marriage.¹⁹

In the situation only with the two alternatives, single and marriage, a couple of a type i man m and a type j woman w compares $Z_{ijt}^{SM}(\epsilon_{mwt}) \leq 0$. Similarly, with the three alternatives, a couple of a type i man m and a type j woman w compares the net values of cohabitation with the net values of marriage and 0. In other words,

$$d_{mwt} = 1, \text{ if } \operatorname{argmax}\{0, Z_{ijt}^{SC}(\epsilon_{mwt}), Z_{ijt}^{SM}(\epsilon_{mwt})\} = 0;$$

$$d_{mwt} = 2, \text{ if } \operatorname{argmax}\{0, Z_{ijt}^{SC}(\epsilon_{mwt}), Z_{ijt}^{SM}(\epsilon_{mwt})\} = Z_{ijt}^{SC}(\epsilon_{mwt});$$

$$d_{mwt} = 3, \text{ if } \operatorname{argmax}\{0, Z_{ijt}^{SC}(\epsilon_{mwt}), Z_{ijt}^{SM}(\epsilon_{mwt})\} = Z_{mwt}^{SM}(\epsilon_{mwt}).$$

Remember that, because i and j include endogenous state variables such as a duration of a match, i and j change depending on a couple's actions. This implies that $Z_{ijt}^{SC}(\epsilon_{mwt})$ and $Z_{ijt}^{SM}(\epsilon_{mwt})$ change during a match. Under the limited commitment assumption, a couple is assumed to renegotiate their allocation and decide whether to change their marital status. This creates a dynamic endogenous marital behavior, including transitions from cohabitation to marriage also.

5 Definition, existence and uniqueness of reservation values

Let ϵ_{ijt}^{*SC} be a reservation match value such that $Z_{ijt}^{SC}(\epsilon_{ijt}^{*SC}) = 0$. Namely, ϵ_{ijt}^{*SC} is a reservation match value with which a couple of a type i man and a j type woman at time t is indifferent between staying single and moving to cohabiting. Similarly, ϵ_{ijt}^{*SM} is defined for single to marriage. Let ϵ_{ijt}^{*Si} be a reservation match value such that $Z_{ijt}^{SC}(\epsilon_{ijt}^{*Si}) = Z_{ijt}^{SM}(\epsilon_{ijt}^{*Si})$. Namely, ϵ_{ijt}^{*Si} is a reservation match value with which a couple of a type i man and a j type woman at time t is indifferent between moving to cohabitation or marriage from single. Hereafter, I refer to ϵ_{ijt}^{*Si} as a *single-crossing point*. Let ϵ_{ijt}^{*CS} be a reservation match value such that $Z_{ijt}^{SC}(\epsilon_{ijt}^{*CS}) = 0$. It means that ϵ_{ijt}^{*CS} is a reservation match value with which a couple of a type i man and a j type woman at time t is indifferent between staying in cohabitation and returning to single. Similarly, ϵ_{ijt}^{*MS}

¹⁹See, for example, Shimer and Smith (2000) for a basic search-matching-bargaining model with the two-alternatives, single and marriage.

is defined the same from marriage to single. Let ϵ^* be a vector of reservation match values, $\epsilon^* = \{\epsilon_{ijt}^{*SC}, \epsilon_{ijt}^{*SM}, \epsilon_{ijt}^{*Si}, \epsilon_{ijt}^{*CS}, \epsilon_{ijt}^{*MS}\}_{ijt}^{IJT}$,

In the following, I propose several things. First, there exist the reservation match values, $\epsilon_{ijt}^{*SC}, \epsilon_{ijt}^{*SM}, \epsilon_{ijt}^{*CS}, \epsilon_{ijt}^{*MS}$ in a finite range, $[\underline{\epsilon}, \bar{\epsilon}]$, and they are unique given arbitrary values of stocks.

Theorem 1. *The reservation values, $\epsilon_{ijt}^{*SC}, \epsilon_{ijt}^{*SM}, \epsilon_{ijt}^{*CS}, \epsilon_{ijt}^{*MS} \in [\underline{\epsilon}, \bar{\epsilon}]$ for all, i, j and t exist and are unique given arbitrary values of stocks.*

Proof. For all i, j and $t = 1, 2, \dots, T$, the values of a match are strictly increasing,

$$\frac{\partial W_{ijt}^C(\epsilon_{mwt})}{\partial \epsilon_{mwt}} = 1 + \varsigma \partial \left[\mathbb{E}_{t+1|t} \max \{ U_{i't+1}^S + U_{j't+1}^S, W_{i'j't+1}^C(\epsilon_{mwt+1}), W_{i'j't+1}^M(\epsilon_{mwt+1}) \} \right] / \partial \epsilon_{mwt}, \quad (16)$$

and

$$\frac{\partial W_{ijt}^M(\epsilon_{mwt})}{\partial \epsilon_{mwt}} = 1 + \varsigma \partial \left[\mathbb{E}_{t+1|t} \max \{ U_{i't+1}^S + U_{j't+1}^S, W_{i'j't+1}^M(\epsilon_{mwt+1}) \} \right] / \partial \epsilon_{mwt}. \quad (17)$$

The value of being single is constant with respect to ϵ_{mwt} , and its value is not $-\infty$ or ∞ because value functions $\forall t = t, t+1, \dots, T$ are bounded.²⁰ The support of ϵ_{mwt} is continuous and unbounded. Therefore, the value functions cross somewhere with the value of being single. The slopes of $\frac{\partial W_{ijt}^C(\epsilon_{mwt})}{\partial \epsilon_{mwt}}$ and $\frac{\partial W_{ijt}^M(\epsilon_{mwt})}{\partial \epsilon_{mwt}}$ are strictly greater than 1. See, Brien et al. (2006) and Moriya and Stern (2025) also. Since the reservation thresholds exist and, therefore, are finite, there exist constants $\underline{\epsilon}$ and $\bar{\epsilon}$ such that $\epsilon_{ijt}^{*l} \in [\underline{\epsilon}, \bar{\epsilon}]$ for all i, j, t and $l \in \{SC, SM, CS, MS\}$. $\square$

The theorem ensures the existence of the reservation values. With the reservation values, now, I provide the following two propositions.

Proposition 1 *An individual's value functions, with realizations of $\mu_m^C, \mu_m^C, \mu_w^M$, and μ_w^M , satisfying, $\forall \epsilon_{mwt} \in \mathbb{R}$,*

$$\frac{\partial W_{ijt}^M(\epsilon_{mwt})}{\partial \epsilon_{mwt}} > \frac{\partial W_{ijt}^C(\epsilon_{mwt})}{\partial \epsilon_{mwt}}, \quad (18)$$

have the single-crossing property.

Proof. See, for example, Milgrom and Shannon (1994) and Brien et al (2006). $\square$

²⁰Because I assume that ϵ_{ijt} follows a normal distribution, the integral of $\epsilon_{ijt} \in \mathbb{R}$ is bounded for all t given parameter values. The parameter space is assumed to be compact. Therefore, value functions are bounded for all t .

Equation (18) indicates $\partial W_{ijt}^M(\epsilon_{mwt})$ is steeper. Note that equation (18) is constructed through endogenous variables, $\epsilon_{ijt}^{*SC}, \epsilon_{ijt}^{*SM}, \epsilon_{ijt}^{*Si}, \epsilon_{ijt}^{*CS}$, and ϵ_{ijt}^{*MS} for all i, j and $t + 1, t + 2, \dots, T$. However, at time t , they are already represented only by exogenous primitives by backward induction given arbitrary values of stocks. Therefore, equation (18) is a valid sufficient condition in the sense that it is described only by exogenous primitives.

Next, the following proposition is about the existence of the single-crossing point, ϵ_{ijt}^{*Si} , above the value of being single, that is, $Z_{ijt}^{SC}(\epsilon_{ijt}^{*Si})(= Z_{ijt}^{SM}(\epsilon_{ijt}^{*Si})) > 0$.

Proposition 2 *Given parameter values with $C_{ijt}^M > C_{ijt}^C$ and arbitrary values of stocks, some individuals should exist who satisfy the above single-crossing condition (eq (18)) and $Z_{ijt}^{SC}(\epsilon_{ijt}^{*Si})(= Z_{ijt}^{SM}(\epsilon_{ijt}^{*Si})) > 0$, with measure 1.*

Proof. The support of $\mu_m^C, \mu_w^C, \mu_m^M$ and μ_w^M is continuous and unbounded on the real line, $\mu_m^C, \mu_w^C, \mu_m^M, \mu_w^M \in \mathbb{R}$. As shown in equations (9) and (10), the value of cohabitation and marriage continuously changes in $\mu_m^C, \mu_w^C, \mu_m^M$, and μ_w^M , and they are modeled in separately additive way in equations (9) and (10). Therefore, under my setting with a continuum of players with different values of $\mu_m^C, \mu_w^C, \mu_m^M$, and μ_w^M , individuals should exist who satisfy the single-crossing condition (eq (18)), and the value of cohabitation and marriage at the point is greater than the value of single, with probability 1. $\square$

This discussion about the existence of a single-crossing point (Proposition 2) ensures cohabitation and marriage coexist. This coexistence plays a key role in the following identification discussion. I will discuss this in more detail in Section 8.

6 Equilibrium

The whole economy is assumed to be completely divided into several non-stationary sub-economies, depending on their level of technology, κ . Namely, there are multiple segregated non-stationary economies possibly existing simultaneously at different technology levels κ , and each sub-economy has its own equilibrium.

6.1 Requirements, definition and existence of equilibrium and uniqueness

The equilibrium conditions consist of two requirements; an *optimality condition* and a *rational expectations condition*. The *optimality condition* requires, at equilibrium, each individual behaves optimally given their perception (*belief*) about the future dynamics of the marriage market (stocks of individuals in the economy). The *rational expectations condition* requires, at equilibrium, each individual's given perception about the dynamics of the marriage market should match with the actual aggregate dynamics of the marriage market derived by aggregating each individual's decision in the economy.

I propose the formal definition of the market equilibrium in the model, and state that, under my construction of the model, the equilibrium exists.

Definition 1. Denote $\epsilon^{**} = \{\epsilon_{ijt}^{**SC}, \epsilon_{ijt}^{**SM}, \epsilon_{ijt}^{**Si}, \epsilon_{ijt}^{**CS}, \epsilon_{ijt}^{**MS}\}_{ijt}^{IJT}$ as a vector of equilibrium reservation match values, $\Lambda^{**} = \{\Lambda_{it}^{**SM}, \Lambda_{jt}^{**SW}, \Lambda_{ijt}^{**C}, \Lambda_{ijt}^{**M}\}_{ijt}^{IJT}$ as a vector of equilibrium stocks of individuals in the economy.²¹

Definition 2. A non-stationary market equilibrium is defined by $(\epsilon^{**}, \Lambda^{**})$ such that:

- Each individual optimizes their behavior, given their own perception about future stocks of individuals (*Optimality condition*);
- The given perception of the stocks of individuals are consistent with the actual aggregate dynamics of the economy (*Rational expectations condition*).²²

Theorem 2. Given parameter values, there exists a market equilibrium.

Proof. See Online Appendix E.1. □

I use Brouwer's fixed point theorem to prove the existence of the market equilibrium. The existence of the equilibrium stocks is key to separately identifying the mating preference and matching technology, where I explicitly use the notion of equilibrium stocks. See Section 8.

²¹Note that ϵ^* refers to the reservation match value that constitutes an individual's best response to a given market environment (e.g., future stocks Λ). In contrast, ϵ^{**} denotes the specific reservation match value that satisfies the market equilibrium conditions.

²²Note that, due to the continuum of agents assumption, the law of large numbers holds. The distributions of stocks degenerate.

Theorem 3. *Given parameter values, an equilibrium is locally unique with probability 1.*

Proof. See Online Appendix E.2. □

Online Appendix E.2 shows that the set of parameter values yielding singular Jacobians has Lebesgue measure zero (by Sard’s theorem). Hence, local uniqueness holds with probability 1.²³ Theorem 3 proves local uniqueness of equilibrium. Almost all previous literature of a two-sided equilibrium search model assume existence of equilibrium and its local uniqueness.²⁴ Practically, Theorem 3 is important for estimation: Since local uniqueness holds, equilibrium depends continuously on parameters. Accordingly, for almost every Ω , a damped Newton method with fixed-point iteration, when started from a suitable initial value, tends to converge locally to $\Lambda^{**}(\Omega)$. However, a formal discussion of global uniqueness is analytically intractable with this level of generality of the model.

7 Data

This paper uses the two data sets, the National Longitudinal Study of High School Class of 1972 (NLS 72) and the National Longitudinal Survey of Youth 1997 (NLSY 97). The NLS 72 is assumed to represent the cohort before the advances in the communication technology occur, and the NLSY 97 is assumed to represent the cohort after the advances in the communication technology occur. In addition to detailed demographic information, both data sets have relationship type information including cohabitation from early ages of respondents.²⁵ Compared with other national representative surveys, the two data sets are unique in that they contain detailed retrospective relationship history information during sample periods. Accounting for the fact that cohabitation spells are typically shorter than marriages, I track individuals’ relationship status transitions every six months as in Brien et al. (2006).

²³The local uniqueness is a *necessary* condition for using a gradient-based estimation method employed in this research, and the validity of counterfactuals: A small change in the parameter values does not cause a jump from the equilibrium of interest to a different one. It is, mathematically, equivalent that derivatives of equilibrium objects with respect to parameters are well-defined.

²⁴For example, Guvenen and Rendall (2015) and Goussé et al. (2017) provide discussion about existence and local uniqueness numerically.

²⁵This solves an initial conditions issue. This is because every observation in the data is single at the beginning of the sampling periods.

In the NLS 72, 22,650 students are first interviewed when they are leaving high school in the spring of 1972 with follow-up interviews in 1973, 1974, 1976 and 1979, and, for a limited group, 1986. I use the subset of the whole sample which answers the 1986 follow-up interview. As Lillard et al. (1995) and Brien et al. (2006) mention, the limited group of the whole sample, which answers the 1986 interview, does not represent the whole population composition. Therefore, I apply appropriate weights to the observations to correct for the choice-based sampling problem (see Manski and Lerman (1977), Hellerstein and Imbens (1999), and Nevo (2003)). As used in Weiss and Willis (1997), I use the weights discussed in Tourangeau (1987).²⁶

Particularly, on each cohort, I focus on the data associated with relationship status transitions of an individual, conditioning on an individual's gender, race, education level, a partner's education level if matched, whether there is a child from a previous relationship, whether there is a child from a current relationship, match durations, and age which is assumed equivalent to time effects.^{27, 28}

8 Identification

I explain my identification strategy and how it works in detail. The identification argument uses a similar idea to Friedberg and Stern (2014). Friedberg and Stern (2014) conceptually divide their identification argument into parts to provide intuitively clear identification sources for each parameter to readers.²⁹

Let β^u be a vector of the parameters associated with the deterministic part of the flow match value, β^M be a vector of the parameters associated with the marriage bonus, β^{C^C} be a vector of parameters associated with the cohabitation separation cost and β^{C^M} be a vector of the parameters associated with the divorce cost. We can rewrite the set of the primitives explicitly with β^u ,

²⁶Weighting is necessary from the perspective of proper use of aggregate stocks as a set of moment conditions also. For this reason, it is necessary to apply appropriate weights to the observations in the NLSY97 cohort as well.

²⁷The NLS 72 is basically a single age cohort in which every observation has the same age. So, the effects of respondents' age and the calendar time on marital behaviors cannot be separately identified. Accordingly, I interpret the date of cohabitation or marriage as the elapsed time since the start of the sampling period.

²⁸There is variation in age given t for individuals in the NLSY 97 cohort. However, I assume that the effects on marital behaviors from the age difference are not significant. For computation issues as well, I do not want to deal with age and the calendar time as different state variables.

²⁹The key identification strategy is that each moment included in this section provides distinct information for identifying a specific parameter, holding other parameters hypothetically fixed.

β^M, β^{CC} and β^{CM} , as

$$\{\{u_{ijt}^m + u_{ijt}^w(\beta^u), M_{ijt}(\beta^M), C_{ijt}^M(\beta^{CM}), C_{ijt}^C(\beta^{CC})\}_{ijt}^{IJT}, F_\epsilon, F_{\tilde{\epsilon}|\epsilon}, F_{\mu^C, \mu^M}, \varsigma, \phi\}. \quad (19)$$

Although the meeting probabilities are *endogenously* decided in the model, they are also represented as a function having primitives as its arguments, β^{α^M} and β^{α^W} .³⁰ Namely, the meeting probabilities are

$$\{\alpha_{ijt}^M(\beta^{\alpha^M}), \alpha_{ijt}^W(\beta^{\alpha^W})\}_{ijt}^{IJT}. \quad (20)$$

Let

$$\Omega = \{\beta^u, \beta^M, \beta^{CM}, \beta^{CC}, F_\epsilon, F_{\tilde{\epsilon}|\epsilon}, F_{\mu^C, \mu^M}, \varsigma, \phi, \beta^{\alpha^M}, \beta^{\alpha^W}\} \quad (21)$$

be the parameter set in the model. Note that I give a value for the discount factor, ς , and the bargaining weight, ϕ , exogenously.

For the following argument, it is important to remember that each equilibrium reservation match value is represented as a function of all parameters in the economy (equilibrium meeting probabilities also). A vector of equilibrium reservation values, $\epsilon^{**} = \{\epsilon_{ijt}^{**SC}, \epsilon_{ijt}^{**SM}, \epsilon_{ijt}^{**Si}, \epsilon_{ijt}^{**CS}, \epsilon_{ijt}^{**MS}\}_{ijt}^{IJT}$ is explicitly represented as $\epsilon^{**}(\Omega) = \{\epsilon_{ijt}^{**SC}(\Omega), \epsilon_{ijt}^{**SM}(\Omega), \epsilon_{ijt}^{**Si}(\Omega), \epsilon_{ijt}^{**CS}(\Omega), \epsilon_{ijt}^{**MS}(\Omega)\}_{ijt}^{IJT}$ in this section.

Assume the existence of equilibrium reservation match values and an equilibrium single-crossing point.³¹ I can neatly divide the support of ϵ_{mwt} into three parts with the thresholds, the equilibrium reservation match value from single to cohabitation, ϵ_{ijt}^{**SC} , and the equilibrium single-crossing point, ϵ_{ijt}^{**Si} .

Let $h_{ijt}^{SM(m)}$ be a type i man's hazard rate out of being single to being married with a type j woman at time t . It is given by

$$h_{ijt}^{SM(m)} = \alpha_{ijt}^{M**} (1 - F_\epsilon(\epsilon_{ijt}^{**Si})). \quad (22)$$

³⁰The parameters β^{α^M} and β^{α^W} are associated with the underlying matching function. I show how the underlying aggregate matching function is specified in Section 10.1.

³¹For some parameter values and realizations of persistent unobserved heterogeneity, the single-crossing condition may fail and cohabitation may not be optimal for certain men and women. However, as the sample size grows, Proposition 2 is satisfied.

Similarly, the single to cohabitation hazard is

$$h_{ijt}^{SC(m)} = \alpha_{ijt}^{M^{**}} \left[F_{\epsilon}(\epsilon_{ijt}^{**Si}) - F_{\epsilon}(\epsilon_{ijt}^{**SC}) \right]. \quad (23)$$

Equations (22) and (23) include the unknown parameters, Ω . However, at this point of discussion, ϵ_{ijt}^{**SC} and ϵ_{ijt}^{**Si} are assumed *hypothetically* represented by a function of the following unknown parameters associated with the mating preference and marriage bonus, β^u and β^M only. So, at this point of discussion, only the parameters, β^u and β^M , are assumed left. Accordingly, I can rewrite equations (22) and (23) as

$$h_{ijt}^{SM(m)} = \alpha_{ijt}^{M^{**}} (1 - F_{\epsilon}(\epsilon_{ijt}^{**Si}(\{\beta^u, \beta^M\}))); \quad (24)$$

$$h_{ijt}^{SC(m)} = \alpha_{ijt}^{M^{**}} \left[F_{\epsilon}(\epsilon_{ijt}^{**Si}(\{\beta^u, \beta^M\})) - F_{\epsilon}(\epsilon_{ijt}^{**SC}(\{\beta^u, \beta^M\})) \right]. \quad (25)$$

First, remember that the equilibrium meeting probability, $\alpha_{ijt}^{M^{**}}$, is *hypothetically* fixed at this point of the argument. I have $2 \times IJT$ equations because I get two equations (24) and (25) for each i, j and t .³² The moment conditions (24) and (25) identify β^u and β^M .³³

Let h_{ijt}^{CS} be a hazard rate out of cohabitation by a type i man and a type j woman to single at time t . Remember ϵ_{ijt}^{**CS} is an equilibrium reservation match value with which a couple is indifferent between continuing to cohabit and returning to single.

The hazard rate is represented as, using the notation of β^{CC} ,

$$h_{ijt}^{CS} = \int_{\epsilon_{t-1}} F_{\epsilon|\epsilon_{t-1}} \left(\epsilon_{ijt}^{**CS}(\{\beta^{CC}\}) \right) dF_{\epsilon_{t-1}}. \quad (26)$$

In equation (9), I emphasize that the equilibrium reservation match values, ϵ_{ijt}^{**CS} , depends on the parameters associated with the cohabitation separation cost, β^{CC} . Remember there is serial correlation in the law of motion of ϵ_{mwt} . Therefore, in taking the integral, I take into account the previous draw, ϵ_{mwt-1} , which is written as ϵ_{t-1} in equation (26) for shorter notation. The para-

³²Note that the total number of conditional moment conditions is substantially larger than $2IJT$, as these equations hold for each value of the underlying explanatory variables that define the agent types.

³³Theoretically, I can identify β^u and β^M with moments (24) and (25). However, the estimators might be unstable, as pointed out by Keane (1992). To strengthen identification, I also put an *exclusion restriction*: There is an explanatory variable, whether a couple has children, which belongs only to either $u_{ijt}^m + u_{ijt}^w$ or $\mathbb{M}_{ijt}$. See Online Appendix B.2 for the empirical specification in more detail.

parameters, β^C , are mainly identified by matching the sample moments constructed from the data, $\hat{h}_{ijt}^{CS}$, to the corresponding theoretical moments in the same way as the parameters associated with the mating preference and the marriage bonus case.

Similarly, let h_{ijt}^{MS} be a hazard rate out of marriage by a type i man and a type j woman to single at time t . Similarly, define ϵ_{ijt}^{**MS} as an equilibrium reservation match value with which a couple is indifferent between remaining married and returning to single. The hazard rate is

$$h_{ijt}^{MS} = \int_{\epsilon_{t-1}} F_{\epsilon|\epsilon_{t-1}} \left(\epsilon_{ijt}^{**MS}(\{\beta^{CM}\}) \right) dF_{\epsilon_{t-1}}. \quad (27)$$

I emphasize that ϵ_{ijt}^{**MS} depends on β^{CM} . It is mainly identified by matching the sample moments constructed from the data, $\hat{h}_{ijt}^{MS}$, to the corresponding theoretical moments in the same way as before.

The parameters associated with the unobserved terms, F_{ϵ} , F_{μ^C, μ^M} , and $F_{\tilde{\epsilon}|\epsilon}$, are identified through second sample moments calculated by generalized residuals. Note that, for example, in a standard ordinary least squares model, we can easily calculate its residuals to construct its second sample moments. However, in a discrete choice model, it is not so straightforward to calculate its generalized residuals due to the unobservable latent variable. See, for example, Gourieroux et al. (1987), Goeree (2008), and Friedberg and Stern (2014) for more detail about how to construct them.

In the last step, I pin down the parameters associated with α_{ijt}^M , α_{ijt}^W , β^{α^M} , and β^{α^W} . So far, they have been taken as given. They are identified by using the equilibrium condition, the *rational expectations* condition, which we need to satisfy at equilibrium discussed in Section 6. Let D_t^{**} be a vector of equilibrium operators associated with the law of motion for equilibrium stocks at time t (equations (12) - (15)). We have, given t ,

$$\hat{\Lambda}_{t+1}^{**} = D_t^{**}(\Omega) \hat{\Lambda}_t^{**}. \quad (28)$$

I treat the observed stocks as equilibrium stocks, $\hat{\Lambda}_t^{**}$. I have explicitly proven the existence of the equilibrium stocks in the economy. Theoretically, it is ensured that there are the equilibrium stocks satisfying equation (28). Therefore, we can use the equations as a set of new moments.

The same as the previous arguments, I emphasize the dependence of the matching technology parameters, β^{α^M} and β^{α^W} , and rewrite it as,

$$\hat{\Lambda}_{t+1}^{**} = D_t^{**}(\{\beta^{\alpha^M}, \beta^{\alpha^W}\})\hat{\Lambda}_t^{**}. \quad (29)$$

The set of moments mainly identifies β^{α^M} and β^{α^W} without imposing a constant-returns-to-scale restriction on the matching technology. See, Section 10.1 for more detail.^{34, 35}

Note that, theoretically, the equality needs to hold exactly because of the law of large numbers. However, in practice, the data sets consist of a finite number of individuals/observations, meaning the theoretical law of large numbers, which assumes a continuum of agents, does not hold exactly. Therefore, in constructing moments associated with the rational expectations condition, I assume that there is still a sampling or measurement error to make the moment conditions work properly in estimation.

Additionally, I emphasize there are possible applications of the identification strategy employed in this research.

Remark 1 *Under a setting with a continuum of agents model with an aggregate matching technology, we can use a similar identification strategy. Examples include not only a marriage market but also a firm and a worker market, a housing market and so on.*

9 Estimation

So far, particular functional forms for model primitives have not been explicitly specified. I give a detailed specification of the economy for estimation purposes in Section 10 and Online Appendix B.2. Meanwhile, assume that it is appropriately specified according to the theoretical requirements discussed in Section 6.

³⁴Note that, after employing a market equilibrium framework, the stocks of individuals, naturally, show up as an important aspect in the model. Stocks under an equilibrium model setting with an aggregate matching function works as a new type of constraints. They affect the meeting probabilities through an aggregate matching function but does not affect other model primitives.

³⁵To strengthen the identification argument, I also include *exclusion restrictions*: To separately identify the parameters associated with the preference (match value), β^u , and those of the matching technology, β^{α^M} , there are explanatory variables which belong only to either of the two parameter specifications. See Section B.2, for a more detailed discussion.

9.1 Indirect Inference

This paper uses indirect inference for estimation (Gourieroux et al. (1993)). The underlying concept of indirect inference involves three steps: first, I select a set of moments, which are also called auxiliary statistics. They are assumed to represent the characteristics of the real data (observed moments); second, simulating the structural economic model and calculating the corresponding simulated moments several times with different values of parameters;³⁶ third, picking parameter values such that the simulated moments closely replicate the observed moments. Indirect inference gives us consistent estimates of the parameters (Gourieroux et al. (1993)). I use a gradient-based method in minimizing the distance. The use of a gradient-based method is guaranteed by Theorem 3.³⁷

The reason why I use indirect inference in this research is that the NLS 72 data set is highly confidential with time and computation code restrictions. Its disclosure policy does not allow me to use any estimation method for which others might be able to identify an individual in the sample. Indirect inference requires only aggregate moments' information in a particular data set.

Following previous studies with indirect inference, I select a set of auxiliary statistics which are easy to compute and are able to capture a variety of patterns in the data. Also, the set of moments should be informative for the underlying structural model.³⁸ I evaluate the distance between the simulated and actual data sets using the Euclidean distance through the lens of the auxiliary statistics.

The set of auxiliary statistics is constructed from three primary sources: (A) coefficients from non-structural ordered linear probability models describing transitions between relationship states; (B) the covariance matrix capturing the persistence of individual relationship statuses

³⁶Given a set of parameter values, there exist equilibrium reservation values proven in this model in Theorem 2. Through the equilibrium reservation values calculated, I can obtain simulated moments.

³⁷For how to proceed estimation in more detail, see B.1 in Online Appendix.

³⁸As, for example, van der Klaauw and Wolpin (2008) and Collard-Wexler (2013) say that auxiliary statistics do not need to have any interpretation. They are just required to describe the characteristics of data as much as possible. As Hall and Rust (2021) mention, a variety of moments can be used as a possible set of auxiliary statistics (first, second, third and fourth moments, covariances and quantiles, etc.). In general, it does not matter which auxiliary statistics I use as long as I can properly measure the distance between the simulated and actual data through the lens of the statistics (see, for example, Gourieroux et al. (1993) and Alan (2006)). However, there should be a moment which is sensitive to a change in each underlying structural model parameter (*invertibility or identifiability*).

over time; and (C) the aggregate stocks of individuals in each relationship status over time. A detailed description of these statistics is provided in Online Appendix B.4.

9.2 Objective function

With the set of auxiliary statistics, I can construct the objective function I need to minimize. Let R be the number of simulations, $\hat{\Xi}(\Omega)$ be a vector of the auxiliary statistics calculated by using R simulated data sets with Ω , $\bar{\Xi}$ be a vector of the auxiliary statistics calculated by the actual data, Γ be a weighting matrix. Let $\hat{\Omega}$ be a vector of estimates of the structural parameters that satisfies

$$\hat{\Omega} = \arg \min_{\Omega} [[\hat{\Xi}(\Omega) - \bar{\Xi}]' \Gamma [\hat{\Xi}(\Omega) - \bar{\Xi}]]. \quad (30)$$

Following much of the literature (for example, Altonji and Segal (1996)), I do not use an optimal weighting matrix. Instead, I use a *diagonal* weighting matrix, while adjusting scales of each moment: The weighting matrix is constructed to ensure each group ((A), (B), and (C) introduced in Section 9.1) and Online Appendix B.4 of moments contributes equally to the objective function. To this end, each moment is weighted by the inverse of its group size. This prevents the estimation from being unduly influenced by more numerous moment groups (See Sauer and Taber (2018) and Guvenen et al. (2021)).³⁹

I use a set of moments to match for 30 time periods. For the detailed simulation algorithm, see Online Appendix B.1.

10 Estimation process, functional specification, results, and fit

This paper takes the following two steps to estimate the structural model: First, I estimate the non-structural childbirth probability function outside the main structural model to reduce the computational costs.⁴⁰ Then, taking the estimates of the childbirth probability function as given, I estimate the main structural model. The standard errors (SE) of the estimates are calculated by

³⁹As discussed in Sauer and Taber (2018), I choose the diagonal matrix in a somewhat ad hoc way. However, if the number of observations is large enough, estimates are consistent with respect to a choice of a weighting matrix.

⁴⁰However, as mentioned in Ge (2011), the assumption that the error terms in the childbirth estimations are independent of the error terms in the main structural model is required to get consistent estimators. It probably does not hold, and my estimators may be biased.

100 nonparametric block bootstrap replications, using individual level clusters (see, for example, Baum-Snow and Pavan (2012) and Kaplan (2012)).

I introduce functional specifications and provide parameter estimates, especially, highlighting those associated with the matching technology. I provide the other functional specifications and parameter estimates in detail in Online Appendix B.2. The age and duration variables in the following tables are measured in 6-month time periods.

10.1 Meeting probability specification

Let $\delta_{ijt\kappa}(\cdot)$ be a matching efficiency between a type i man and a type j woman at time t under κ . The matching efficiency is mapped from a vector of observable characteristics for a type i man, $\mathbf{X}_{it}^{m,\alpha}$, and a vector of observable characteristics for a type j woman, $\mathbf{X}_{jt}^{w,\alpha}$. Denote $\mathbf{X}_{ijt}^{\alpha} = [\mathbf{X}_{it}^{m,\alpha}, \mathbf{X}_{jt}^{w,\alpha}]$. The matrix of the variables, $\mathbf{X}_{ijt}^{\alpha}$, includes race, education level, and age. Recall that $\beta_{\kappa}^{\alpha^M}$ and $\beta_{\kappa}^{\alpha^W}$ are vectors of coefficients for the matching efficiency associated with a man's observable type and a woman's observable type, while emphasizing their dependence on κ . Denote $\beta_{\kappa}^{\alpha} = [\beta_{\kappa}^{\alpha^M}, \beta_{\kappa}^{\alpha^W}]$.

The meeting probabilities, $\alpha_{ijt\kappa}^M$ and $\alpha_{ijt\kappa}^W$, are specified as, with the general aggregate matching function specification $z_{\kappa}(\cdot : \delta_{ijt\kappa}(\cdot))$ introduced in equation (1), while emphasizing its dependence on the matching efficiency, $\delta_{ijt\kappa}(\cdot)$,

$$\alpha_{ijt\kappa}^M = \left[z_{\kappa}(\Lambda_{t\kappa}^{SM}, \Lambda_{t\kappa}^{SW} : \delta_{ijt\kappa}(\mathbf{X}_{ijt}^{\alpha})) \frac{\Lambda_{it\kappa}^{SM}}{\Lambda_{t\kappa}^{SM}} \frac{\Lambda_{jt\kappa}^{SW}}{\Lambda_{t\kappa}^{SW}} \right] / \Lambda_{it\kappa}^{SM}; \quad (31)$$

$$\alpha_{ijt\kappa}^W = \left[z_{\kappa}(\Lambda_{t\kappa}^{SM}, \Lambda_{t\kappa}^{SW} : \delta_{ijt\kappa}(\mathbf{X}_{ijt}^{\alpha})) \frac{\Lambda_{it\kappa}^{SM}}{\Lambda_{t\kappa}^{SM}} \frac{\Lambda_{jt\kappa}^{SW}}{\Lambda_{t\kappa}^{SW}} \right] / \Lambda_{jt\kappa}^{SW}. \quad (32)$$

I give a more specific functional form on $z_{\kappa}(\cdot : \delta_{ijt\kappa}(\cdot))$. Then, $\alpha_{ijt\kappa}^M$ and $\alpha_{ijt\kappa}^W$ change to

$$\alpha_{ijt\kappa}^M = \left[\delta_{ijt\kappa}(\mathbf{X}_{ijt}^{\alpha}) z(\Lambda_{t\kappa}^{SM}, \Lambda_{t\kappa}^{SW}) \frac{\Lambda_{it\kappa}^{SM}}{\Lambda_{t\kappa}^{SM}} \frac{\Lambda_{jt\kappa}^{SW}}{\Lambda_{t\kappa}^{SW}} \right] / \Lambda_{it\kappa}^{SM}; \quad (33)$$

$$\alpha_{ijt\kappa}^W = \left[\delta_{ijt\kappa}(\mathbf{X}_{ijt}^{\alpha}) z(\Lambda_{t\kappa}^{SM}, \Lambda_{t\kappa}^{SW}) \frac{\Lambda_{it\kappa}^{SM}}{\Lambda_{t\kappa}^{SM}} \frac{\Lambda_{jt\kappa}^{SW}}{\Lambda_{t\kappa}^{SW}} \right] / \Lambda_{jt\kappa}^{SW}, \quad (34)$$

where the matching efficiency is specified as

$$\delta_{ijt\kappa}(\mathbf{X}_{ijt}^{\alpha}) = \frac{\exp(\mathbf{X}_{ijt}^{\alpha} \boldsymbol{\beta}_{\kappa}^{\alpha})}{1 + \exp(\mathbf{X}_{ijt}^{\alpha} \boldsymbol{\beta}_{\kappa}^{\alpha})}. \quad (35)$$

Following much of previous studies, the aggregate matching technology, $z(\Lambda_{t\kappa}^{SM}, \Lambda_{t\kappa}^{SW})$, is assumed to have the functional form,

$$z(\Lambda_{t\kappa}^{SM}, \Lambda_{t\kappa}^{SW}) = \Lambda_{t\kappa}^{SM^{0.5}} \Lambda_{t\kappa}^{SW^{0.5}}.$$

The specific reason why I explicitly model $\delta_{ijt\kappa}(\cdot)$ is that it would be the case that, even if the stocks are the same, an individual with a certain type tends to meet with an individual with a certain type more or less.⁴¹ To capture the idea, I explicitly include $\delta_{ijt\kappa}(\cdot)$.^{42, 43}

Remember that, because I assume non-stationarity of the economy, I can explicitly include the effects of changes in stocks on marital behaviors through time. If I assume a stationarity of the economy used commonly in previous studies, I cannot control for the effects of changes in stocks through time. Because, in reality, an economy should be non-stationary, without the non-stationarity setting, the estimates are biased.⁴⁴

10.2 Matching technology and second moments estimates

Table 1 shows the parameter estimates associated with the matching technology, $\boldsymbol{\beta}_{\kappa}^{\alpha}$, shown in equation (35). Holding other things constant, blacks meet potential partners more efficiently than whites (#10). If individuals are in a school, they have more opportunities to meet (#3). After 10 years, people suffer a bit less meeting opportunities (#4). There is less of a premium from higher education in meeting opportunities (#5 – #8). Some of the estimates show high

⁴¹People who share common characteristics might share similar life/working styles. It might boost the number of potential meetings.

⁴²Note that, as mentioned in Bobba et al. (2022), there is another way to normalize the aggregate matching function by normalizing the Total Factor Productivity (TFP) (here, $\delta_{ijt\kappa}(\cdot)$) to 1, instead of normalizing the Cobb-Douglas parameter to 0.5.

⁴³Note that, based on the specifications of $\alpha_{ijt\kappa}^M$, $\alpha_{ijt\kappa}^W$, and $\delta_{ijt\kappa}(\cdot)$, changes in stocks affect $\alpha_{ijt\kappa}^M$ and $\alpha_{ijt\kappa}^W$ through $\delta_{ijt\kappa}(\cdot)$. Therefore, every element in $\boldsymbol{\beta}_{\kappa}^{\alpha}$ including a constant term in $\mathbf{X}_{ijt}^{\alpha} \boldsymbol{\beta}_{\kappa}^{\alpha}$ can be identified.

⁴⁴Note that, from an identification perspective, unlike in a stationary environment, this non-stationary framework allows me to use the variation in stocks over time as a new source for identification.

Table 1: Parameters associated with matching technology

#	Parameter	NLS 72		NLSY 97	
		Estimates	SE	Estimates	SE
1	Constant	0.653*	0.002	2.204*	0.003
2	Age (time)	-0.012	0.028	-0.013	0.070
3	Age spline ≤ 5 years	0.688*	0.000	0.684*	0.000
4	Age spline ≥ 10 years	-0.019*	0.001	-0.001	0.002
5	Man education (High school)	0.225*	0.002	0.121*	0.010
6	Man education (College degree)	0.728*	0.002	0.429*	0.001
7	Woman education (High school)	0.429*	0.002	0.322*	0.004
8	Woman education (College degree)	0.672*	0.000	0.577*	0.001
9	Education difference	-0.147*	0.000	-0.341*	0.001
10	Black	0.203*	0.002	0.204*	0.001

* SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72).

* indicates significance at the 5% level.

* Age has piecewise linear effects with nodes at 5 and 10 years from the beginning of the sampling period.

standard errors, for example, age (#2). However, eliminating the term from the model results in a serious deterioration in fit.⁴⁵

One of the main focuses of this research is how the matching technology changes during the two cohorts. Table 2 provides values of $X_{ijt}^\alpha \beta_\kappa^\alpha$ in each cohort, which depends on an individual's characteristics.⁴⁶ Under the specification of the matching efficiency in equation (35), $\delta_{ijt\kappa}(X_{ijt}^\alpha)$ is increasing in $X_{ijt}^\alpha \beta_\kappa^\alpha$. Therefore, Table 2 indicates that people in the NLSY 97 cohort tend to have more opportunities to meet their potential partners because $X_{ijt}^\alpha \beta_\kappa^\alpha$ is larger in the NLSY 97 cohort. As a simple directional external validity check, the estimated rise in matching efficiency aligns with the advances in communication technology shown in HCMST (see Figure 1).

Table 3 provides the parameter estimates associated with the second moments. The standard deviations of the cohabitation- and marriage-specific unobserved heterogeneity terms (#12 – #13) are large. For distributions with large variance, Proposition 2 is more likely to hold in finite samples. The large standard deviation of the stochastic part of a flow match value (#15)

⁴⁵One can see similar arguments in, for example, Rust and Phelan (1997) and Iskhakov and Keane (2021).

⁴⁶Because the age effects (#2 – #4) are almost the same across the two cohorts, I exclude them from the table.

Table 2: Attribute-specific terms in $X_{ijt}^\alpha \beta_k^\alpha$ in equation (35)

Combination of characteristics	NLS 72	NLSY97
Man: High school / Woman: High school / Race: White	1.307	2.647
Man: High school / Woman: High school / Race: Black	1.510	2.851
Man: High school / Woman: College degree / Race: White	1.403	2.561
Man: High school / Woman: College degree / Race: Black	1.606	2.765
Man: College degree / Woman: High school / Race: White	1.663	2.614
Man: College degree / Woman: High school / Race: Black	1.866	2.818
Man: College degree / Woman: College degree / Race: White	2.053	3.210
Man: College degree / Woman: College degree / Race: Black	2.256	3.414

Table 3: Parameters associated with second moments

#	Parameter	NLS 72		NLSY 97	
		Estimates	SE	Estimates	SE
11	ρ (Coef AR1)	0.772*	0.003	0.687*	0.048
12	Standard deviation of cohabitation unobserved heterogeneity	1.672*	0.000	1.673*	0.024
13	Standard deviation of marriage unobserved heterogeneity	2.153*	0.001	1.707*	0.014
14	Covariance of cohabitation and marriage unobserved heterogeneity	1.534*	0.000	1.317*	0.006
15	Standard deviation of match value when single	2.973*	0.001	2.921*	0.017

* SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

* * represents that the estimate is significant at the 5%.

is consistent with Brien et al. (2006). As noted in Section 10.3, for computational tractability, I employ very parsimonious specifications with fewer explanatory variables. Therefore, variations that are supposed to be captured by observable explanatory variables are explained by unobserved terms.⁴⁷ The larger standard deviations partly stem from the parsimonious specifications of my model. For example, compared with Keane and Wolpin (2010), who report putting an unobserved type to a marriage utility specification is redundant, the unobserved heterogeneous types are important in both cohabitation and marriage (#12 – #13). This may be because, in this research, I do not control for, for example, detailed labor market information or tax system changes for computational reasons, while Keane and Wolpin (2010) do.

⁴⁷Previous literature often includes regions, religions, and labor market conditions as exogenous explanatory variables. This paper does not control for any of them as exogenous characteristics of individuals.

10.3 Within-sample fit

Figures 2-3 compare the simulated and actual proportions of stocks of individuals of single, cohabitation and marriage in the NLS 72 cohort and the NLSY 97 cohort over time. My model predicts the changes well. Some parts of the simulated stocks exhibit kink points, for example, around $t = 1987$ in the NLS 72 cohort. These arise because my functional specification has the spline modifications as shown in Online Appendix C.2. I also provide an out-of-sample predictive validation of non-targeted moments in Online Appendix C.3. The current model fits reasonably well in both within- and out-of-sample validations, although I would probably get a better fit by employing a more flexible functional specification with more explanatory variables.⁴⁸

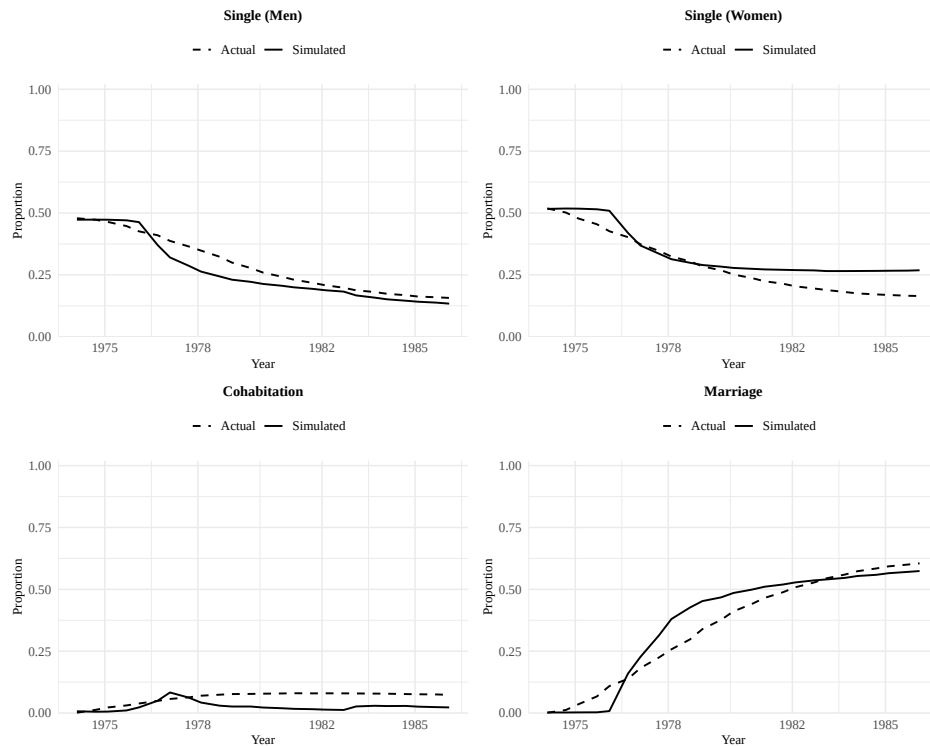


Figure 2: Actual vs. Simulated Proportions - NLS 72 (1974–1986)

⁴⁸The number of total parameters in each cohort is only 47. Compared with previous literature, the number is small. I employ the very parsimonious specifications due to concerns for overfitting and computational burdens.

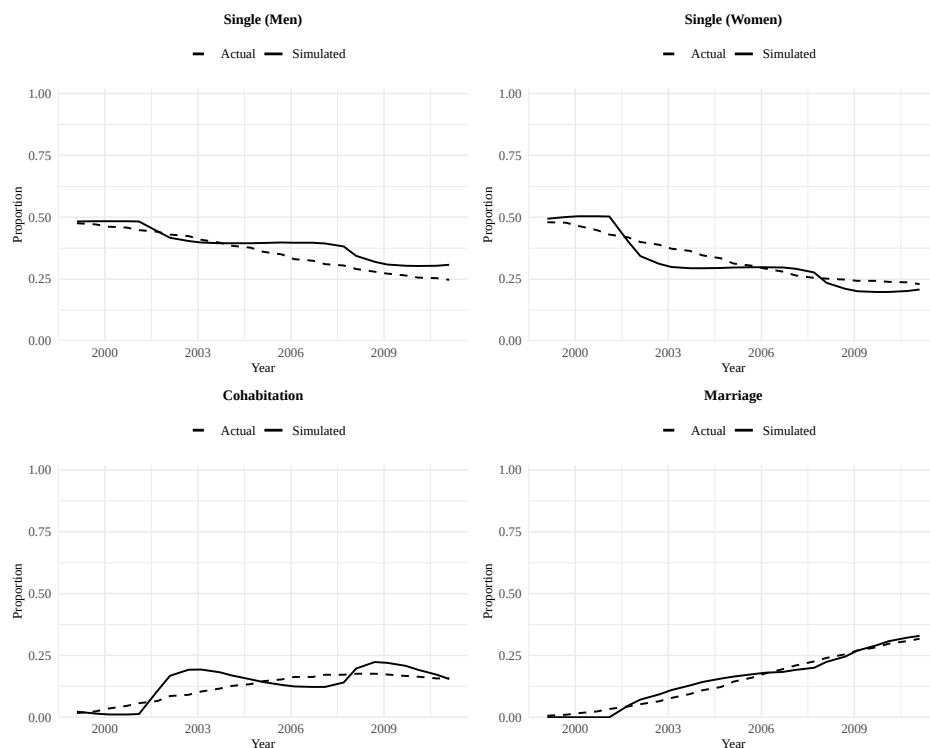


Figure 3: Actual vs. Simulated Proportions - NLSY 97 (1999–2011)

11 Counterfactuals

Based on the parameter estimates, the natural questions I want to ask are whether changes in communication technology impact marital behaviors and welfare, and to what extent changes in marital behavior are explained only by the change in the communication technology. This section addresses these questions.

The structural approach taken in this research enables me to evaluate welfare implications of the communication technology advances. I conduct three types of experiments. First, I assess welfare changes across the two cohorts. This is done by comparing the welfare of the NLS 72 cohort and that of the NLSY 97 cohort. Second, I decompose the channels driving changes in marital behaviors between the NLS 72 and NLSY 97 cohorts, while altering an exogenous environment and isolating the effects of each specific channel.⁴⁹ These counterfactual experiments assess the contributions of specific parameters to observed marital patterns, accounting for equilibrium effects. Third, I evaluate how the value of a match changes by reverting the technology

⁴⁹Under each experiment below with different parameter values, I recalculate the equilibrium stocks based on the parameter values with using a fixed point algorithm.

parameters to those in the previous world. Focusing on the NLSY 97 cohort, I change parameters related to technological advancements—specifically, matching technology and second moments parameters—to their corresponding NLS 72 values, while holding all other parameters constant at NLSY 97 estimates. This is exactly what would happen if matching technology goes back to 1970s levels.

I calculate an individual’s expected lifetime welfare before Nature assigns exogenous characteristics and realizations of marriage- and cohabitation-specific unobserved heterogeneity to a player (Low and Pistaferri (2015) and Abbott et al. (2019)). This is equivalent to welfare from an ex-ante perspective.

Childbearing probability parameters are assumed invariant throughout these experiments. Following most of bargaining literature, I exogenously set the bargaining parameter of a woman to 0.5 in the counterfactuals.

11.1 Experiment 1: Total welfare comparison

In calculating welfare, I apply adjusted weights to each individual in the datasets to accurately represent the true composition of the economy. I calculate ex-ante lifetime welfare for individuals in both cohorts by normalizing the flow value of being single to 0 and the standard deviation of sequential match values during a match to 1 (equation (41) in Online Appendix B.2). A procedure to generate the ex-ante lifetime welfare begins by fixing values of all structural parameters of interest. For each cohort, realized simulated histories are generated with simulated error terms given individuals’ initial exogenous characteristics and cohabitation- and marriage-specific unobserved heterogeneity. Each simulated history is assigned a value by summing up the discounted flow values of all relationship choices until the terminal period. Then, I average the calculated lifetime welfare across individuals to obtain an ex-ante lifetime welfare. The ex-ante expected lifetime welfare is 0.1008 in the NLS 72 cohort and 0.0801 in the NLSY 97 cohort. This result represents a change of $(0.0801 - 0.1008)/0.1008 = -0.2051$, and, therefore, shows that people in the NLSY 97 put less value on being matched relative to being single.

The ex-ante lifetime welfare is small in both cohorts. The results are basically consistent with the data we observe. As I discuss in more detail in Section 11.2, almost all individuals stay single

during the relatively early stages of their lives. Any deviation from being single has less effects on his lifetime welfare after several time periods because of discounting.^{50, 51}

I now examine the dispersion in lifetime welfare outcomes resulting from the realization of cohabitation- and marriage-specific unobserved heterogeneity as shown in Figure 4.⁵² The distributions of lifetime welfare do not significantly differ. One explanation is that during the early stages of their sampling periods, the marital behaviors of the NLS 72 and NLSY 97 cohorts are not significantly different, as I show in Figure 5 in Section 11.2. Although significant changes in marital behavior occur later in life, the discount factor mitigates the impact of these differences on overall lifetime welfare.

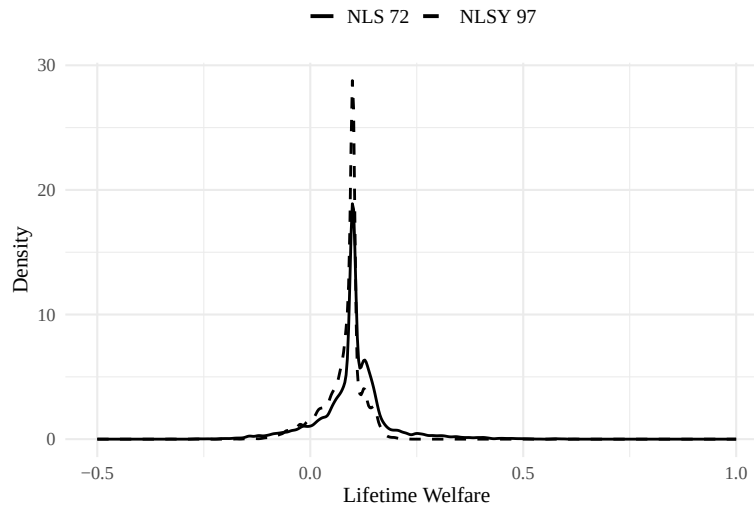


Figure 4: **Distribution of Lifetime Welfare: NLS 72 vs. NLSY 97.** This figure compares the distributions of simulated lifetime welfare between the NLS 72 and NLSY 97 cohorts.

Other model primitives change across the two cohorts. This makes it challenging to identify

⁵⁰Note that the flow value of single is normalized to 0. Recall that I set the discount factor as 0.9 for each period. The annual discount factor is 0.81.

⁵¹Note that the results depend on the terminal values of the value functions. Any cardinal interpretation of lifetime welfare is sensitive to the choice of terminal values, which are theoretically arbitrary: The model's optimal decision rules are, by construction, invariant to a uniform shift in these values. I impose a same specification across cohorts. The ordinal finding — that lifetime welfare for the NLSY 97 cohort is lower than that for the NLS 72 cohort — is robust for specifications of the terminal values.

⁵²The negative lifetime welfare values observed at the lower tail of the distribution in Figure 4 represent rare simulation outcomes. In the model, individuals make decisions to maximize their expected lifetime utility, and they can always choose to remain single for a normalized flow utility of zero. However, a specific realized path of random shocks can be sufficiently unfavorable (e.g., experiencing costly separations after low-quality matches) such that the discounted sum of realized utilities becomes negative. These outliers, resulting from a finite number of simulations for each agent, do not contradict the forward-looking, rational behavior of the agents in the model.

the precise sources of welfare level changes across the two cohorts. In the following sections, I decompose the channels through which marital behavior changes between the NLS 72 and NLSY 97 cohorts.

11.2 Experiment 2: Equilibrium impact and decomposition

I compare the simulated equilibrium marital patterns derived from the NLS 72 setting with those from the NLSY 97 setting, modifying one element at a time: First, I re-calculate the NLS 72 equilibrium marital patterns using the initial individual distributions from the NLSY 97 while keeping all other factors constant in the NLS 72 setting (Stage 1). This exercise reveals how much the changes in the individuals' distributions in the economy contribute to differences in marital behaviors across the two cohorts. Next, I sequentially change the matching technology (Stage 2) and second moments parameters (Stage 3) from those of the NLS 72 estimates to those of the NLSY 97 estimates.⁵³ This analysis quantifies the impact of technological advances on an individual's marital behavior. My goal is to identify which primitive changes critically influence changes in marital behavior across the NLS 72 and NLSY 97 cohorts.

Figure 5 illustrates the differences in the simulated marital behavior under each counterfactual, as a function of age. These experiments share common features in the resulting changes in marital behavior. As age increases, the differences in marital behavior between the two cohorts become more pronounced. A point of emphasis is that Stages 1 and 2 result in marital patterns that do not shift far from the NLS 72 benchmark. This is expected to some extent; either the change in the initial individuals' distribution or the matching technology induces changes in meeting probabilities. Suppose the Stage 1 and/or Stage 2 experiments induce an increase in meeting probabilities. Individuals in the economy have two possible responses. Individuals are more likely to meet, which induces more matches, but, at the same time, the value of being single also increases due to higher future meeting probabilities. These two effects offset each other, resulting in a relatively small overall effect in the marital patterns. A similar discussion

⁵³As Shorrocks (2013) and Taber and Vejlín (2020) note, the results of a decomposition analysis can depend on the order of implementation. To check for this, I performed the analysis in two different orders: (A) changing the matching technology parameters (Stage 2) then the second-moment parameters (Stage 3), and (B) swapping this order. The results were very similar, confirming that my findings on the impact of these factors are robust. In both scenarios, the initial population composition (Stage 1) was adjusted first, as it is not the central focus of my analysis.

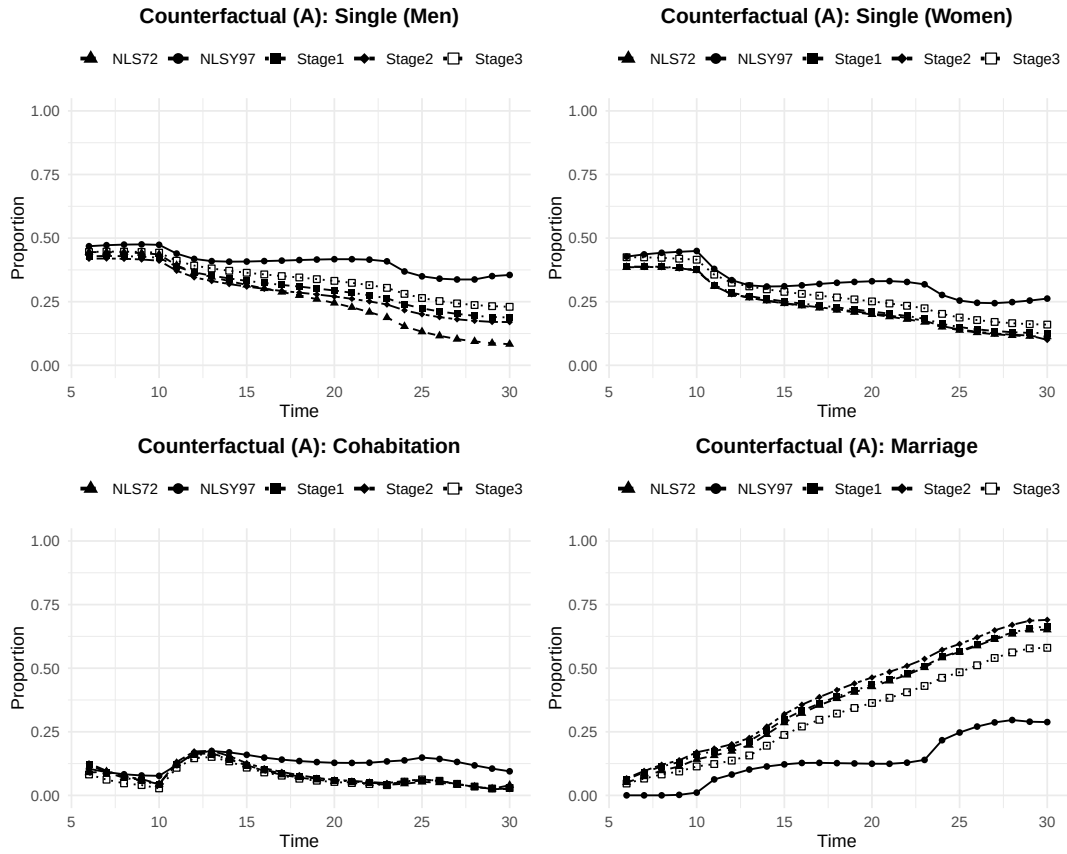


Figure 5: Each panel shows the proportion of individuals who are single, cohabiting, or married by time periods since the initial period. Lines compare the outcomes across counterfactuals (Stage1-Stage3).

can be found in Santos and Weiss (2016) and Blasutto (2024).

Changes in the second moments play a more critical role in determining changes in marital behavior (Stage 3). This aspect has received less empirical attention in previous literature. However, the results with larger changes are theoretically consistent with Weitzman's Pandora's Box argument (Weitzman (1979)) in that changes in the second moments induce changes in optimal stopping rules.

Furthermore, the decomposition results indicate that changes in preferences (such as cohabitation match surplus and marriage bonus) and separation costs are also important factors in explaining the differences in marital behavior between the NLS 72 and NLSY 97 cohorts. This is because, even after Stage 1, 2, and 3 implementations, deviations remain between the simulated aggregate stocks and those of the NLSY 97 cohort.

11.3 Experiment 3: Revert the technology

I examine whether advances in communication technology alter the value of forming a partnership in society. I let the parameters associated with technological advances revert to the NLS 72 situation, while all else remains the same as in the NLSY 97 situation.

The ex-ante lifetime welfare under this experiment is 0.0851. The increase is $(0.0851 - 0.0801) / 0.0801 = 0.062$. This suggests that, if we revert to 1980s technology while keeping other factors the same as in the 2010s, ex-ante lifetime welfare increases. It means that a match has a greater value compared with being single.

This result remains intuitive from a theoretical perspective. As demonstrated in Section 10.2, the matching technology improves in the NLSY 97 cohort. As Shimer and Smith (2000) point out, the relative difference in values between being single and being matched expands when search frictions become larger. This implies that ex-ante lifetime welfare becomes larger.

This result indicates that most of the reduction shown in Section 11.1 is not due to the technological changes but rather to changes in preferences and the separation costs. This implies that people's perception of the value of cohabitation and marriage has changed.

11.4 Analysis of Lifetime Relationship Dynamics

The preceding analysis focused on aggregate trends in marital status stocks. This section shifts the focus to the micro-level dynamics of individuals' relationship histories to investigate how lifetime relationship experiences have evolved across cohorts. A key question is whether the effects of technological advancement are heterogeneous across individuals. I investigate whether the technology has led to a polarization in lifetime relationship outcomes by analyzing the distribution of relationship turnover. Specifically, this investigates whether technological advancements have created a more polarized market, where some individuals experience a greater number of partners while others remain persistently single.

Figures 6 and 7 present the simulated distributions of the number of lifetime relationships for the NLS 72 and NLSY 97 cohorts respectively. The NLSY97 distribution is shifted to the right and has thinner tails. A notable pattern emerges: the NLSY 97 cohort exhibits higher relationship turnover than the NLS 72 cohort. While a larger fraction of individuals in the NLSY

97 cohort experiences multiple partnerships, there is also a greater proportion who never marry. This observation raises a critical question: What is the primary driver of this shift in lifetime relationship dynamics?

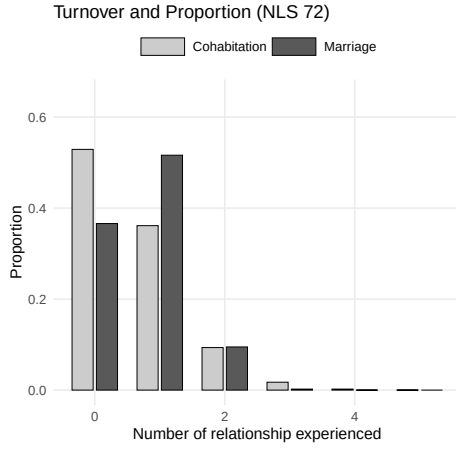


Figure 6: Marital experience and proportion (NLS 72)

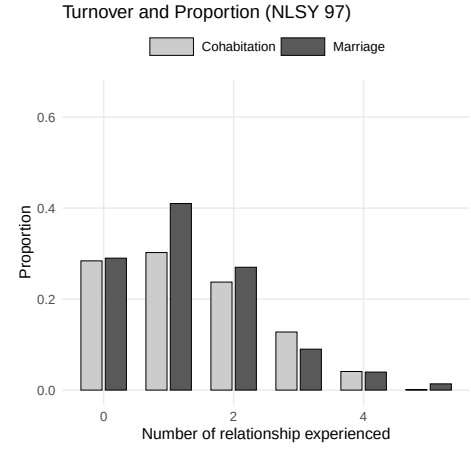


Figure 7: Marital experience and proportion (NLSY 97)

To disentangle the underlying mechanisms, I conduct a decomposition analysis similar with that of Section 11.2. I introduce parameter changes of the corresponding matching technology parts to the NLS 72 baseline model. The results, depicted in Figure 8, indicate that these technology-related factors alone cannot account for the full extent of the observed increase in relationship turnover. While they contribute partially, a significant gap remains when compared to the NLSY 97 distribution. This finding suggests that other primitive changes, namely those related to preferences and separation costs, are the dominant force. To test this hypothesis directly, we conduct a final counterfactual experiment where only the separation cost parameters from the NLSY 97 cohort are applied to the NLS 72 baseline. The change in separation costs alone replicates a substantial portion of the increased turnover observed in the NLSY 97 data (Figure 9).

My structural estimates indicate that the rise in relationship turnover across cohorts is driven primarily by lower separation costs, not by advances in matching technology. To my best knowledge, this is the first evidence within the structural estimation literature documenting how changes in separation costs reshape relationship-formation dynamics.⁵⁴

⁵⁴Reductions in separation (divorce) costs have two main effects: they encourage entry into marriage and, simultaneously, increase incentives to switch partners.

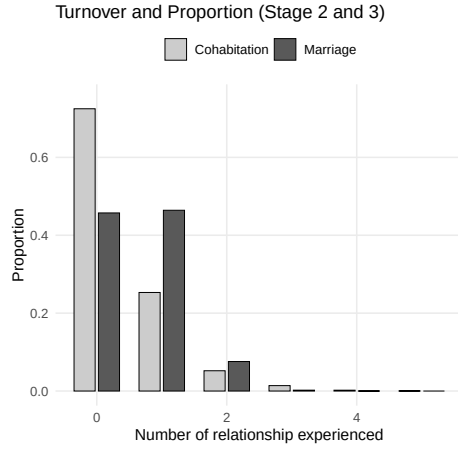


Figure 8: Marital experience and proportion (Stage 2 and 3 together)

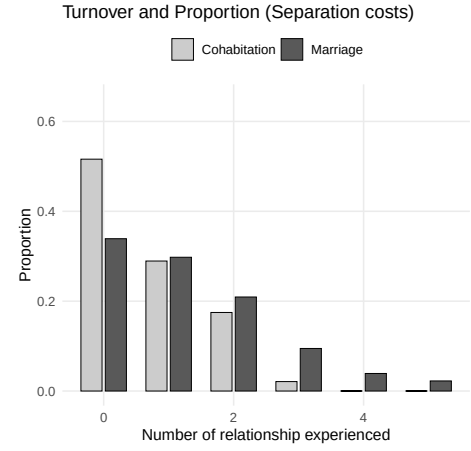


Figure 9: Marital experience and proportion (Separation costs)

12 Conclusion

This paper introduces an empirical, non-stationary, two-sided search market equilibrium model with broad applicability to various contexts. Using the model, this paper evaluates the quantitative impacts of the advances in communication technology on marital behavior including cohabitation and on welfare. The findings reveal that advancements in technology facilitate partner search efficiency. However, the study finds that changes in the observed marital patterns and welfare are driven more by changes in the preferences and separation costs rather than by technological improvements.

One of key departures from a standard model setup is employing a non-stationary market equilibrium aspect. The endogenous market equilibrium stocks work as a new source of identification, which enables me to separately identify the mating preference and the matching technology. Identifying them is crucial in discussing desirable policy interventions. The appropriate policy response may vary depending on the identified channels, for example, whether the government should provide subsidies to individuals to promote marriage or instead restrict technological advancement.

Under the limited data situation, to isolate the technology advances effects, this paper attempts to control for as many other changes occurring between the two cohorts as possible, using microeconomic theory and structural estimation. However, the estimators may remain somewhat biased. Access to richer data sets with explicit information about an individual's ac-

cess to advanced communication technology would allow us to directly measure the effects of the advances in the communication technology.

The model remains stylized, even with the introduction of a non-stationary market equilibrium via an aggregate matching function. Due to a market equilibrium setting, multiplicity of equilibria is a concern. Although the existence of equilibrium and its local uniqueness are proven in the model, sufficient conditions for its global uniqueness cannot be established, despite numerical evidence suggesting uniqueness.

Despite these concerns, this paper's innovation is to leverage a microeconomic-theory equilibrium concept to identify primitives when such variation is absent, which is widely applicable to other fields. I believe that the model setting this paper provides is an important step toward fully understanding what the overall effects of the communication technology advances on the whole society.

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Supplemental Appendix (Online Appendix)

**Communication Technology Advances and
Consequences: Using Two-sided Search Model**

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5th November 2025

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A Appendix A: Data

A.1 Detailed sample selection criteria

Tables A1 and A2 show the sample selection criteria for this research. Originally, I have 22,650 observations in the NLS 72.¹ I use the 12,840 respondents who answer the fifth follow-up interview. I further restrict the sample left to 10,790 black and white individuals. I delete observations for whom I cannot identify their basic demographic variables, leaving 10,720 observations. I drop individuals whose marital history cannot be identified, leaving 10,400 observations. Because a partner's information is important in this research, I drop respondents who do not report their partner's education level or whether their partner has children from their previous relationship, leaving 9,920 observations.² I drop observations who serve in the military, leaving 9,160 observations.³ Lastly, I restrict the sample to individuals who did not cohabit before age 18, leaving 7,410 observations.⁴

Table A1: Sample Selection Criteria (NLS 72)

Selection Criteria	Obs. Left	Obs. Eliminated
Whole Population	22,650	
Fifth Follow-up	12,840	9,810
Black or White	10,790	2,050
With Basic Demographics	10,720	70
With Complete Marital History	10,400	320
With Partner Info	9,920	480
Not Military	9,160	760
Cohab. After Age 18	7,410	1,750
Final Sample	7,410	

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72).

¹For the NLS 72 data, the U.S. Department of Education's National Center for Education Statistics requires rounding numbers to the nearest 10 to ensure disclosure protection.

²Since my main objective in this research is to investigate individual marital dynamics, I remove individuals who miss or refuse to answer questions associated with their marital history during the survey periods. Even though there are several imputation methods to deal with missing data issues (for example, Keane and Sauer (2010)), I remove them for simplification of the analysis. This is partly justified because I still have enough observations left for estimation.

³I remove individuals who serve in the military because their marital behavior differs from that of the rest of population.

⁴Individuals who cohabited before age 18 are excluded, as their marital behavior is likely to differ from that of the main population.

Table A2: Sample Selection Criteria (NLSY 97)

Selection Criteria	Obs. Left	Obs. Eliminated
Whole Population	8,984	
Black or White	7,622	1,362
Age ≥ 16 with Basic Demographics	6,776	846
With Marital History	6,776	0
With Partner Info	6,776	0
Not Military	6,348	428
Final Sample	6,348	

In the NLSY 97, originally, I have 8,984 observations. I restrict the sample to 7,622 black and white individuals. I delete observations for whom I cannot identify their basic demographic variables, leaving 6,766 observations. For the NLSY 97 sample, even if a respondent misses an annual interview about his/her relationship, I impute the missing answer consistent with his/her answer for the periods before and after the period.⁵ For a partner's education level, many responses do not annually report the information. In this research, as long as a partner has the same unique id, I fill in the missing partner's information during the corresponding spells with the partner's highest level of education.⁶ I drop observations who serve in the military, leaving 6,348 observations.

A.2 Descriptive statistics of initial (time 0) individuals' distributions and marital status dynamics

For a non-stationary model, the initial (time 0) individuals' distributions in the economy is also an exogenous model primitive.⁷ Table A3 shows the composition of individuals in the economy,

⁵Suppose an individual misses or refuses to answer an annual interview. However, if he reports that he cohabits/marries with the partner with the same unique id for periods before and after the period, I assume that he also cohabits/marries with the same person during the missing period. I can do this imputation only for the NLSY 97 sample because, for the NLSY 97, the sampling period is annual. On the other hand, the NLS 72 asks the retrospective questions only once in 1986. So, if an individual misses or refuses to answer the interview at 1986, I need to remove the individual.

⁶As Guvenen and Smith (2014) do, I should check the robustness of this imputing filling-in method on the estimates. However, assessing the robustness of the estimates lies beyond the scope of this paper.

⁷In a non-stationary model, the initial (time 0) individuals' distributions are also an exogenous model primitive: If we observe the difference in equilibrium outcomes of the two cohorts, it might come just from the difference in the initial distributions, not from the difference of the other model primitives. However, in my model setting, I can overcome this issue by controlling for the differences in stocks in estimation. See, Section 10.1 for the empirical specification of meeting probabilities.

which comes from U.S. Census Bureau data. Note that the information is expressed as stocks because I use a special case of a Pissarides' style matching function whose arguments are stocks rather than proportions, as discussed in Section 10.1.⁸

Table A3: U.S. resident population ages 16–18 by sex and race: 1980 vs. 2000

	1980				2000			
	Male		Female		Male		Female	
	White	Black	White	Black	White	Black	White	Black
Population ages 16–18	4,731,000	847,000	4,505,000	834,000	4,925,000	869,000	4,813,000	850,000

Table A4: Educational attainment by gender and race: 1980 vs 2000

	1980				2000			
	Male		Female		Male		Female	
	White	Black	White	Black	White	Black	White	Black
Without High School Degree	10.9%	25.3%	10.8%	21.7%	7.9%	8.8%	5.5%	13.4%
High School Degree	62.3%	64.2%	66.0%	65.9%	60.7%	77.7%	57.0%	66.6%
College and Above	26.8%	10.5%	23.2%	12.4%	31.4%	13.5%	37.5%	20.0%

In terms of the exogenous characteristics shown in Table A4, one significant change is that, in the NLSY 97 cohort, a smaller proportion is without a high school degree.⁹ The percentage is 26% in the NLS 72, and it declines to 13% in the NLSY 97 cohort. Other than the educational attainment difference, a change is the total population, which changes from 9,236,000 to 9,738,000. As shown in Table A4, educational attainment by gender and race differs significantly between the two cohorts. Both White and male individuals tend to have higher educational attainment.

Next, we show simple descriptive statistics of marital dynamics to point out that quantitative differences exist at an aggregate level before and after the advances in the communication technology. Certainly, we can observe, to a non-negligible extent that their marital behaviors differ

⁸Given the Pissarides' style matching function, not only the ratio but also the amount of stocks matters (*market thickness*).

⁹Note that an education level is assumed exogenous in this paper. For example, Guvenen and Rendall (2015) endogenize an education choice in their model. I treat education exogenously for simplicity.

during the two cohorts, which is shown in Figures A1-A2. Aggregate proportions evolve in a different way in each cohort. Compared with the NLS 72 cohort, people in the NLSY 97 cohort get married less and cohabit more. As shown in Figure A1, during the NLS 72 sample periods, the proportion of individuals who are cohabiting is at most 10%. For people in the NLSY 97, cohabitation is a more popular choice. When the sample in the NLS 72 reaches age 32, which is 1986 as shown in Figure A1, over 60% of them are married. In the NLSY 97 cohort, the proportion of married individuals is much lower than in the NLS 72 cohort throughout the sampling period.

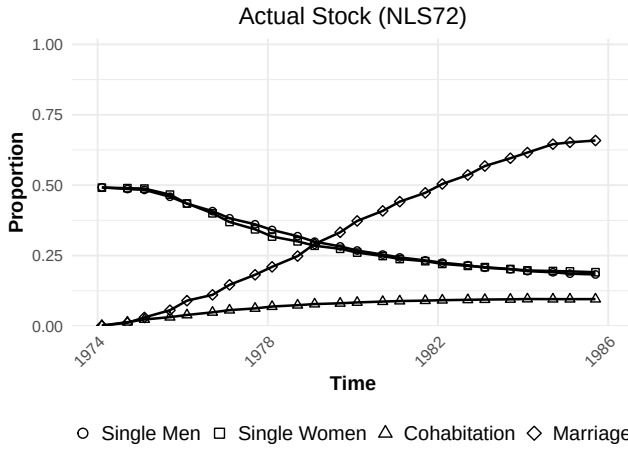


Figure A1: Actual stock over time: NLS 72 cohort

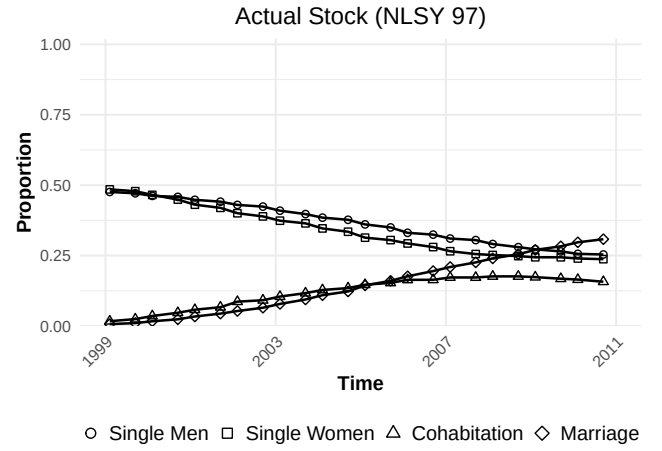


Figure A2: Actual stock over time: NLSY 97 cohort

B Appendix B: Model simulation and empirical specification

B.1 Model simulation

In constructing the simulated data, I perform simulation as follows: Given a set of values of the structural parameters, I solve the dynamic programming problem. With an individual's observable initial exogenous characteristics and the permanent unobserved heterogeneity terms drawn, I simulate an individual decision by drawing shocks, and update the individual's state variables using the decision rules driven by the dynamic programming. I repeat this procedure until the terminal period.

In this research, I employ a moment-based estimation method (indirect inference), which cre-

ates non-smoothness of the objective function on the parameter space, given the finite number of simulated draws. Let R be the number of simulation. In this research, I set $R = 200$ which makes the number of simulated draws at each iteration approximately 100 million. This number is sufficient to consider the objective function as smooth enough on the parameter space (see, for example, Pakes (1986)), even though there are more sophisticated methods, for example, GHK or Stern (1992). In this research, additionally, acknowledging the possibility that my objective function is non-smooth, I use a derivative-based optimization method for a non-smooth function (Lewis and Overton (2013)). In the actual estimation process, numerical derivatives are well defined.¹⁰

B.2 Empirical specification

I need to give specific functional forms to the model primitives described in Section 3. Also, I impose some practical restrictions on dynamics of explanatory variables and marriage markets feasible for players. In this section, I provide parsimonious functional forms of the model primitives and some restrictions imposed on dynamics of a part of explanatory variables and marriage markets. Particular focus of this section are the following model primitives: flow match value, $u_{ijt}^m + u_{ijt}^w$, marriage bonus, $\mathbb{M}_{ijt\kappa}$, cohabitation separation cost, $\mathbb{C}_{ijt\kappa}^C$, divorce cost, $\mathbb{C}_{ijt\kappa}^M$, childbearing probability, $P_{t\kappa}^b$, and the stochastic part of a match value, $F_{\tilde{\epsilon}|\epsilon}$. I also introduce restrictions on dynamics of stocks of children, evaluation of match durations and explain possible marriage market restrictions.

B.2.1 Flow match value, marriage bonus, separation costs and unobserved terms

Most of the model primitives are assumed to be approximated by a function of a linear index in parameters. Let $\mathbf{X}_{it}^{m,u}$ and $\mathbf{X}_{jt}^{w,u}$, be a matrix of explanatory variables indicating an observable type for an i man and an observable type for a j woman at time t in the specification of

¹⁰By Theorem 3, for almost every parameter vector Ω , equilibrium stocks, $\Lambda^{**}(\Omega)$, are differentiable with respect to Ω . This guarantees that (i) the gradient of simulated moments with respect to Ω exists, validating my gradient-based search/Newton updates in indirect inference with damping, and (ii) conceptually, local counterfactual responses $\frac{\partial \Lambda^{**}}{\partial \Omega}$ are well-defined, so (small) perturbations in primitives translate smoothly into equilibrium objects without equilibrium switching. See Online Appendix E.2 for the Sard/implicit-function argument establishing local uniqueness and smoothness.

$u_{ijt}^m + u_{ijt}^w$, and denote $\mathbf{X}_{ijt}^u = [\mathbf{X}_{it}^{m,u}, \mathbf{X}_{jt}^{w,u}]$.¹¹ Let $\beta_\kappa^{m,u}$ and $\beta_\kappa^{w,u}$ be vectors of coefficients for the flow match value associated with a man's observable type and a woman's observable type, and denote $\beta_\kappa^u = [\beta_\kappa^{m,u}, \beta_\kappa^{w,u}]$. The variable κ emphasizes that the parameter values change depending on the communication technology level. Then, $u_{ijt}^m + u_{ijt}^w$ is specified as

$$u_{ijt}^m + u_{ijt}^w = \mathbf{X}_{ijt}^u \beta_\kappa^u.$$

Let $\mathbf{X}_{it}^{m,\mathbb{M}}$ and $\mathbf{X}_{jt}^{w,\mathbb{M}}$, be a matrix of explanatory variables in the specification of $\mathbb{M}_{ijt}$, $\mathbf{X}_{ijt}^{\mathbb{M}} = [\mathbf{X}_{it}^{m,\mathbb{M}}, \mathbf{X}_{jt}^{w,\mathbb{M}}]$, and letting $\beta_\kappa^{m,\mathbb{M}}$ and $\beta_\kappa^{w,\mathbb{M}}$ be vectors of marriage bonus coefficients associated with a man's type and a woman's type with $\beta_\kappa^{\mathbb{M}} = [\beta_\kappa^{m,\mathbb{M}}, \beta_\kappa^{w,\mathbb{M}}]$, $\mathbb{M}_{ijt\kappa}$ is specified as,

$$\mathbb{M}_{ijt\kappa} = \mathbf{X}_{ijt}^{\mathbb{M}} \beta_\kappa^{\mathbb{M}}. \quad (36)$$

The match surplus between a type i man m and a type j woman w at time t under κ presented in equation (2) in Section 3 is specified as

$$\begin{aligned} s_{mwt\kappa} = & \mathbf{X}_{ijt}^u \beta_\kappa^u \mathbb{I}[d_{mwt} \neq 1] + (\mu_\kappa^{Cm} + \mu_\kappa^{Cw}) \mathbb{I}[d_{mwt} = 2] + \mathbf{X}_{ijt}^{\mathbb{M}} \beta_\kappa^{\mathbb{M}} \mathbb{I}[d_{mwt} = 3] + (\mu_\kappa^{Mm} + \mu_\kappa^{Mw}) \mathbb{I}[d_{mwt} = 3] \\ & + \epsilon_{mwt\kappa} \mathbb{I}[d_{mwt} \neq 1] - \mathbf{C}_{ijt\kappa}^C \mathbb{I}[d_{mwt-1} = 2, d_{mwt} = 1] - \mathbf{C}_{ijt\kappa}^M \mathbb{I}[d_{mwt-1} = 3, d_{mwt} = 1] \end{aligned} \quad (37)$$

where the costs, $\mathbf{C}_{ijt\kappa}^C$ and $\mathbf{C}_{ijt\kappa}^M$, are specified in a similar fashion as,

$$\mathbf{C}_{ijt\kappa}^C = \mathbf{X}_{ijt}^{C^C} \beta_\kappa^{C^C}; \quad (38)$$

$$\mathbf{C}_{ijt\kappa}^M = \mathbf{X}_{ijt}^{C^M} \beta_\kappa^{C^M}, \quad (39)$$

and

$$\epsilon_{mwt\kappa} \sim iidN(0, \sigma_f^2(\kappa)) \text{ if a couple first meets;} \quad (40)$$

$$\epsilon_{mwt\kappa} = \rho_\kappa \epsilon_{mwt-1\kappa} + \eta_{mwt} \text{ after a match;}$$

¹¹See Section C.2 for detail descriptions of the observable types.

$$\eta_{mwt} \sim iidN(0,1). \quad (41)$$

For the specification of cohabitation- and marriage- specific unobserved heterogeneity terms for a man and a woman, see equations (3) and (4) in the main text. Because I assume the continuous support for $\mu_{\kappa}^{Cm}, \mu_{\kappa}^{Cw}, \mu_{\kappa}^{Cm},$ and $\mu_{\kappa}^{Cm},$ for numerically solving the model recursively, I discretize each support with 3 grid points, and interpolate each support with a B-spline method.¹²

B.3 Law of motion of explanatory variables

B.3.1 Childbearing probability

A part of uncertainty in the model also arises from the imperfect control women have over childbirth. Let $\mathbf{p}_{\kappa}^m$ and $\mathbf{p}_{\kappa}^w$ be a vector of childbearing coefficients associated with a man's type and a woman's type, and denote $\mathbf{p}_{\kappa}^b = [\mathbf{p}_{\kappa}^m, \mathbf{p}_{\kappa}^w]$. The probability of giving birth is denoted as P_{κ}^b . Let $\mathbf{X}_{it}^{m,p}$ and $\mathbf{X}_{jt}^{w,p}$, $\mathbf{X}_{ijt}^p = [\mathbf{X}_{it}^{m,p}, \mathbf{X}_{jt}^{w,p}]$ be a matrix of explanatory variables in the specification of P_{κ}^b . Then, P_{κ}^b is specified as

$$P_{\kappa}^b = \frac{\exp^{\mathbf{X}_{ijt}^p \mathbf{p}_{\kappa}^b}}{1 + \exp^{\mathbf{X}_{ijt}^p \mathbf{p}_{\kappa}^b}}. \quad (42)$$

The parameters of this function are estimated outside of the main structural model to reduce the computational cost and treated as given in the estimation of the main structural model. This implicitly assumes that childbearing probabilities are treated as given by players and that birth probability errors are independent of other errors in the model. Otherwise, the estimates are biased.¹³

B.3.2 Children

Women are assumed to inherit children from their current marriage, which is denoted as L_t^c , and from their previous relationship, which is denoted as L_t^{pr} . I denote b_{tk} as whether to have a birth

¹²For a detailed discussion about how to make the discrete grid points, see Brien et al. (2006).

¹³Another important disadvantage of treating the childbearing decision as exogenous is that we implicitly ignore an important household investment decision from the model. The endogenous choice of childbirth is an important part of a household investment because children play an important role for deciding an investment decision within a household. See, for example, Del Boca et al. (2014).

at time t . So, $b_{t\kappa} = 1$ with probability P_{κ}^b . The law of motion of the stock of children evolves as,

$$L_{t+1}^{pr} = \begin{cases} 1 & \text{if } L_t^{pr} = 1 \\ L_t^{pr} + b_{t\kappa} & \text{if } L_t^{pr} = 0, d_{mwt} = 2, 3 \text{ and } d_{mwt-1} = 1 \\ 1 & \text{if } L_t^{pr} = 0, L_t^c = 1, \text{ and } d_{mwt} = 1 \\ L_t^{pr} + L_t^c + b_{t\kappa} & \text{if } L_t^{pr} = 0, L_t^c = 0, \text{ and } d_{mwt} = 1 \end{cases}, \quad (43)$$

and

$$L_{t+1}^c = \begin{cases} 1 & \text{if } L_t^c = 1 \text{ and } d_{mwt} = 2, 3 \\ L_t^c + b_{t\kappa} & \text{if } L_t^c = 0, d_{mwt} = 2, 3 \text{ and } d_{mwt-1} = 2, 3 \\ 0 & \text{if } d_{mwt} = 1 \end{cases}. \quad (44)$$

Note that the above law of motion implies that, if a couple separates at time t , their children from the match become the stock of children from a previous relationship upon entering time $t + 1$. If a birth happens at time t and the individual is single, the child becomes part of the stock of the previous children even if the individual gets matched at the end of time t .¹⁴

B.3.3 Match duration evolution

The state space for the dynamic programming is too large without imposing some restrictions on evolutions of some part of state variables. Particularly, I put an upper bound for the match durations at time t . Let Q_t be a duration with any partner at time t . It evolves as,

$$Q_{t+1} = \begin{cases} 8 & \text{if } Q_t = 8 \text{ and } d_{mwt} = 2, 3 \\ Q_t + 1 & \text{if } Q_t \leq 7 \text{ and } d_{mwt} = 2, 3 \\ 0 & \text{if } d_{ijt} = 1 \end{cases}. \quad (45)$$

Note that the implication of this limit is not that individuals cannot experience duration effects for more than 8 sampling periods. Instead, it means that extra years beyond that do not have

¹⁴This specification is basically the same as Beauchamp et al. (2018). However, in this research, for computational simplicity, the upper bound of L_t^c and L_t^{pr} is 1 as in Van der Klaauw (1996), but Beauchamp et al. (2018) make their upper bound 2.

any marginal effects on individual behavior. Also, this specification implies that, even if a couple switches their relationship type from cohabitation to marriage, they inherit their duration effects.

B.4 Auxiliary statistics

Consider an observable type for an i man and an observable type for a j woman at time t represented by a vector of demographic and endogenous state variables, $\mathbf{X}_{it}^m$ and $\mathbf{X}_{jt}^w$, and denote $\mathbf{X}_{ijt} = [\mathbf{X}_{it}^m, \mathbf{X}_{jt}^w]$. The matrix of the variables, $\mathbf{X}_{ijt}$, includes gender, race, education level, whether there is a child from a previous relationship, whether there is a child from a current relationship, match durations, and age. In the following, I suppress the notation representing an individual. Let $y_t^S \in \{1, 2, 3\}$ be a dependent variable representing that a *single* individual at the beginning of time t selects single, $y_t^S = 1$, cohabitation, $y_t^S = 2$, and marriage, $y_t^S = 3$, at the end of time t . Let $y_t^C \in \{1, 2, 3\}$ be a dependent variable representing that a *cohabiting* individual at the beginning of time t selects single, $y_t^C = 1$, cohabitation, $y_t^C = 2$, and marriage, $y_t^C = 3$, at the end of time t . Similarly, y_t^M is defined for transitions from *marriage*. Then, the non-structural ordered linear probability models are

$$y_t^S = \mathbf{X}_{ijt} \gamma^S + e^S; \quad (46)$$

$$y_t^C = \mathbf{X}_{ijt} \gamma^C + e^C; \quad (47)$$

$$y_t^M = \mathbf{X}_{ijt} \gamma^M + e^M, \quad (48)$$

where γ^S, γ^C and γ^M are vectors of coefficients for the non-structural ordered linear probability models for transitions from single, cohabitation and marriage respectively, and e^S, e^C and e^M are specified as $(e^S, e^C, e^M) \sim iidN(0, \mathbf{H})$, where $\mathbf{H}$ is the covariance matrix of (e^S, e^C, e^M) .¹⁵ In particular, as auxiliary statistics, I focus on the coefficients of the relationship status transitions equations, γ^S, γ^C and γ^M .

Let a_t be an individual choice at time t observable in the data, $a_t = \{Single, Cohabitation, Marriage\}$, and $\mathbf{a}$ be a vector of the individual observable relationship choices throughout his/her life, $\mathbf{a} = \{a_1, a_2, \dots, a_T\}$. Through $\mathbf{a}$ across individuals, I construct the sample covariance matrix capturing the persistence of individual relationship statuses over time. I use the sample covariance

¹⁵Note that I do not use $\mathbf{H}$ as auxiliary statistics. Therefore, $\mathbf{H}$ is not specified as whether it is a diagonal matrix or not.

matrix as part of auxiliary statistics, and denote it as $\boldsymbol{\vartheta}$. Aggregate stocks of single, cohabiting, and married individuals, $\boldsymbol{\Lambda}_\kappa = \{\boldsymbol{\Lambda}_{t\kappa}\}_t^T$, also are part of auxiliary statistics. Let Δ be a set of auxiliary statistics,

$$\Delta = \{\gamma^S, \gamma^C, \gamma^M, \boldsymbol{\vartheta}, \boldsymbol{\Lambda}_\kappa\}.$$

Table B1 summarizes the auxiliary statistics used in this research. The set of auxiliary statistics is closely linked to the moments for identification discussed in Section 8.

Table B1: Auxiliary Statistics

#	Auxiliary Statistic	Corresponding parameters
1	Coefficients from ordered linear probability of transitions from single to cohabitation/marriage, conditional on gender, race, educational level, partner's educational level, children from previous relationship, age, and age spline modification.	$\{\beta^u, \beta^M, \beta^{C^M}, \beta^{C^C}\}$
2	Coefficients from ordered linear probability of transitions from cohabitation to single/marriage, conditional on gender, race, educational level, partner's educational level, children from previous relationship, age, age spline modification, children from current relationship, duration, and duration spline modification.	
3	Coefficients from ordered linear probability of transitions from marriage to single, conditional on gender, race, educational level, partner's educational level, children from previous relationship, age, age spline modification, children from current relationship, duration, and duration spline modification.	
4	Sample covariance matrix of individuals' marital history.	$\{F_\epsilon, F_{\bar{\epsilon} \epsilon}, F_{\mu^C, \mu^M}\}$
5	Stocks (single, cohabitation, and marriage).	$\{\beta^{\alpha^M}, \beta^{\alpha^W}\}$

C Estimation results and out-of-sample fit

C.1 Estimates outside of main structural model

Tables C1 and C2 present the parameter estimates of the parsimonious non-structural childbirth probability functions, and their standard errors.¹⁶ Table C1 gives the estimated coefficients of the logistic childbearing probability function when a woman is single. As expected, a woman has a smaller probability of giving birth with more education. Black women are more likely to give birth. Table C2 shows the coefficients of the logistic childbearing probability function when a woman is matched with a man. Overall, if women are matched, the probability of giving birth is higher, *holding everything else constant*. The results in Tables C1 and C2 are consistent with Sheran (2007) and Seitz (2009).

C.2 Estimates of main structural model

Tables C3-C6 show the main structural parameter estimates, and their standard errors estimated via a bootstrap procedure. First, Table C3 provides the coefficient estimates of a flow value of cohabitation relative to being single. Table C4 presents the coefficients for the marriage bonus. The estimates in Table C3 are interpreted as an incremental utility flow from cohabitation when the corresponding explanatory variable changes. For example, the coefficient on *Black* in the NLS 72 cohort (#14), which is -0.085 , means that, holding other variables constant, blacks get less utility from cohabitation relative to being single, than whites.¹⁷ The interpretation of the coefficient estimates in Table C4 is as follows: If a couple selects marriage, they can get their corresponding marriage bonus part in addition to the corresponding cohabitation flow match value based on their characteristics.¹⁸ Note that, as discussed in Footnote 35 in the main text,

¹⁶I can specify the functional forms in a more flexible way with more explanatory variables. Note that not only existence of children within a household, but also their age and the number also matter. However, to limit the size of the state space, the childbirth probabilities are assumed to depend only on the presence of children not on their age, as in van der Klaauw (1996).

¹⁷About its unit, recall that I normalize the flow value of being single to 0 and the standard deviation of a sequential match values during a match to 1 (equation (41) in Online Appendix B.2)

¹⁸For example, in the NLS 72 cohort, if a black couple selects cohabitation, they get -4.281 as their constant part and -0.085 for their match. However, if they select marriage, they get additionally 1.247 as their constant part for their match and -0.038 caused by the *Black* term. Therefore, their flow match value of marriage is the sum of these two constant terms, $-4.281 + 1.247$ as well as, -0.085 (*Black*) for cohabitation and -0.041 (*Black*) for marriage. In sum, a black married couple gets $-4.281 + 1.247 + (-0.085 - 0.041) = -2.908$, holding constant other

Table C1: Parameters associated with the logistic function for childbirth (single woman)

Parameter	NLS 72	NLSY 97
Constant	-3.878* (0.226)	-3.822* (0.167)
Age (time)	-0.036* (0.012)	0.060* (0.008)
Age spline ≤ 5 years	0.028* (0.020)	0.010 (0.015)
Age spline ≥ 10 years	-0.087* (0.013)	-0.045* (0.006)
Education	-0.596* (0.067)	-0.834* (0.054)
Black	2.108* (0.111)	1.334* (0.081)

* SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72).

* * represents that the estimate is significant at the 5%.

* Age has piecewise linear effects with nodes at 5 and 10 years from the beginning of the sampling periods.

Table C2: Parameters estimates associated with the logistic function for childbirth (matched woman)

Parameter	NLS 72	NLSY 97
Constant	-2.691* (0.155)	-1.284* (0.175)
Age (time)	0.042* (0.008)	-0.064* (0.009)
Age spline ≤ 5 years	0.024* (0.010)	-0.004 (0.013)
Age spline ≥ 10 years	-0.006* (0.003)	0.018* (0.004)
Education	-0.060 (0.035)	-0.148* (0.037)
Education partner	-0.012 (0.015)	0.011 (0.008)
Match duration	-0.036* (0.006)	0.026* (0.002)
Match duration spline ≤ 2 years	-0.143* (0.024)	-0.122* (0.034)
Kid from previous relationship	-1.371* (0.131)	0.523* (0.072)
Black	0.180* (0.081)	-0.080 (0.064)

* SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

* * represents that the estimate is significant at the 5%.

* Age has piecewise linear effects with nodes at 5 and 10 years from the beginning of the sampling periods. Duration has piecewise linear effects with a node at 2 year from the beginning of a relationship.

Kid from previous relationship and *Kid from current relationship* work as exclusion restrictions.¹⁹

These estimation results are caused by choosing parameter values to match data features. For example, a result that blacks get a lower utility flow both from cohabitation and marriage than whites is reflected in that the transition rates from single to cohabitation and marriage of blacks

characteristics.

¹⁹Presence of children is typically considered as something to induce a match itself. At the same time, presence of children is also considered as something to measure how much they invest in their relation-specific capital more in marriage relative to cohabitation in the literature. However, the duration is also assumed to be something to capture the degree of household investments (#12 – #13 and #24 – #25). If I add children to try to capture the incentive difference in a relation-specific capital between cohabitation and marriage, they are weakly identified. Therefore, I omitted them from the marriage bonus. See Keane (1992) for a necessity of an exclusion restriction even in a simple discrete choice model.

Table C3: Parameters associated with cohabitation flow match value

#	Parameter	NLS 72		NLSY 97	
		Estimates	SE	Estimates	SE
1	Constant	-4.281*	0.002	-3.527*	0.015
2	Age (time)	-0.285*	0.016	-0.142*	0.071
3	Age spline ≤ 5 years	-1.548*	0.000	-1.087*	0.008
4	Age spline ≥ 10 years	0.126*	0.026	0.109	0.065
5	Man education (High school)	0.043*	0.002	0.045*	0.020
6	Man education (College degree)	0.038*	0.004	0.317*	0.021
7	Woman education (High school)	0.089*	0.002	0.023*	0.024
8	Woman education (College degree)	-0.001	0.003	-0.041	0.015
9	Education difference	-0.653*	0.000	-0.661*	0.008
10	Kid from previous relationship	-0.675*	0.002	-0.685*	0.009
11	Kid from current relationship	0.764*	0.001	0.743*	0.015
12	Match duration	0.542*	0.005	0.117	0.082
13	Match duration spline ≤ 2 years	-1.514*	0.001	-0.105*	0.014
14	Black	-0.085*	0.000	-0.041*	0.002

* SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72).

* indicates significance at the 5% level.

* Age has piecewise linear effects with nodes at 5 and 10 years from the beginning of the sampling periods. Duration has piecewise linear effects with a node at 2 years from the beginning of a relationship.

are lower than whites after controlling for other observable characteristics. This is consistent with Seitz (2006), Sheran (2007), Keane and Wolpin (2010), and Beauchamp et al. (2018).

In both cohabitation and marriage, the structural model estimates indicate positive effects of duration of a match. Marriage has a stronger positive effect than cohabitation (#12 – #13 and #24 – #25). The positive effects imply more investment or accumulation of match-specific capital for a couple in marriage than in cohabitation. The results of the sign of duration effects are consistent with Brien et al. (2006). However, Sheran (2007) and Beauchamp et al. (2018) report opposite results. This might be because our functional specifications are different from each other.

Table C4: Parameters associated with marriage bonus

#	Parameter	NLS 72		NLSY 97	
		Estimates	SE	Estimates	SE
15	Constant	1.247*	0.002	-0.394*	0.036
16	Age (time)	0.012	0.016	-0.027*	0.003
17	Age spline ≤ 5 years	-0.436*	0.000	-0.873*	0.002
18	Age spline ≥ 10 years	-0.038*	0.024	0.042	0.001
19	Man education (High school)	0.111*	0.002	0.130*	0.044
20	Man education (College degree)	-0.403*	0.004	-0.441*	0.030
21	Woman education (High school)	-0.035*	0.002	-0.122*	0.047
22	Woman education (College degree)	-0.271*	0.002	-0.321*	0.046
23	Education difference	-0.319*	0.000	-0.356*	0.015
24	Match duration	0.656*	0.005	0.229	0.221
25	Match duration spline ≤ 2 years	-0.238*	0.001	-0.007	0.014
26	Black	-0.038*	0.000	-0.006*	0.000

* SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72).

* * indicates significance at the 5% level.

* Age has piecewise linear effects with nodes at 5 and 10 years from the beginning of the sampling periods. Duration has piecewise linear effects with a node at 2 years from the beginning of a relationship.

We see that women with more education get lower flow match utility in marriage (#8 and #21 – #22). This may occur because they prefer being single or there might be better employment opportunities for singles, which directly affects the value of their outside options of being single. Remember that I do not include a mechanism for how an individual wage is determined. Nor do I include wages as a part of state variables. I assume that the level of education can partly capture the difference in wages.²⁰

Thinking that getting higher education causes negative effects on a relationship for women is misleading. As I show Table 1 in the main text, women with higher education get more chances

²⁰However, Eckstein et al. (2019) point out that married women earn 18% more than single women, which I cannot find in this research: The education effects are more negative in the NLSY 97 cohort. Eckstein et al. (2019) discuss that controlling for changing labor market opportunities/policies (wage offer distributions) and, a mother's education matters. The difference in the estimates between Eckstein et al. (2019) and mine may come partly from the fact that I do not control for them.

Table C5: Parameters associated with cohabitation separation cost

#	Parameter	NLS 72		NLSY 97	
		Estimates	SE	Estimates	SE
27	Constant	2.555*	0.000	2.530*	0.038
28	Existence of kid	0.348*	0.000	0.348*	0.006
29	Black	0.203*	0.000	0.405*	0.002

* SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

* indicates significance at the 5% level.

Table C6: Parameters associated with marriage separation cost

#	Parameter	NLS 72		NLSY 97	
		Estimates	SE	Estimates	SE
30	Constant	3.588*	0.000	2.839*	0.038
31	Existence of kid	0.562*	0.000	0.565*	0.006
32	Black	0.216*	0.000	0.152*	0.020

* SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

* indicates significance at the 5% level.

to meet their potential partner as pointed out in Ge (2011).

The estimates indicate that couples with children within the household are less likely to separate as other empirical studies suggest (Guvenen and Rendall (2015)). The estimates of having a child within the relationship causes a positive effect (#11), but having a child outside of the relationship causes a negative effect on a relationship (#10). This might be because, if people are assumed to gain emotional satisfaction (Becker (1973)), having children increases emotional attachment and satisfaction between the biological father and mother. However, if children are stepchildren, we do not expect to have such an increase in emotional attachment. Children from previous relationships may be a potential source of conflict within a new relationship, as pointed out in Beauchamp et al. (2018).

Male divorcees are more likely to remarry than female divorcees. As written in Section 9, it is assumed that a custodial parent is the mother after their separation. So, the negative coefficient of the existence of a child from a previous relationship indicates that a woman is less likely to remarry if she has a child.

The divorce cost is higher than the cohabitation separation cost similar to those of Brien et al. (2006). However, when we focus on the NLSY 97 cohort, the difference in the two separation costs is smaller. This comes from the observed data patterns. In the NLSY 97 cohort, more married couples get divorced and return to single. As discussed in Matouschek and Rasul (2008), the lower divorce cost induces higher turnover of relationships. This is also consistent with the data patterns we observe. See Figures 6-7 in the main text. Note that, even though I do not impose $C_{ijt}^M > C_{ijt}^C$ to hold in estimation, the estimation results imply $C_{ijt}^M > C_{ijt}^C$ and support Proposition 2.

C.3 Out-of-sample validation

To further investigate the validity of the model, I perform an out-of-sample prediction. As in Wolpin (1992), Fu and Wolpin (2018), and Hall and Rust (2021), I use data from a different year beyond the time periods used for estimation to perform out-of-sample validation. The NLS 72 covers the periods only from 1972 to 1986, which I already used for estimation. Therefore, I use the Current Population Survey (CPS) and the American Community Survey (ACS) for periods beyond 1986 in this out-of-sample exercise. For the NLS 72 cohort, I focus on $t = 1988$, and, for the NLSY 97 cohort, I focus on $t = 2013$.²¹

For the NLS 72 cohort, at $t = 1988$, the simulated fraction of married individuals is 0.688, the simulated fraction of cohabiting individuals is 0.033, and the simulated fraction of single individuals is 0.279. The corresponding actual fraction of married individuals is 0.662, that of cohabiting individuals is 0.055, and that of single individuals is 0.283. The simulated model fits reasonably well given the parsimonious specifications.

Table C7: Out-of-sample prediction (NLS 72 cohort)

Marital status	Simulated proportion	Actual proportion
Single	0.279	0.283
Cohabitation	0.033	0.055
Marriage	0.688	0.662

For the NLSY 97 cohort, at $t = 2013$, the simulated fraction of married individuals is 0.279, the simulated fraction of cohabiting individuals is 0.052, and the simulated fraction of single individuals is 0.669. The corresponding actual fraction of married individuals is 0.312, actual fraction of cohabiting individuals is 0.089, and the actual fraction of single individuals is 0.599. Although some deviations remain between the actual and the simulated patterns, the simulated model fits reasonably well given the parsimonious specifications.

²¹Remember that my model is explicitly assumed to be non-stationary, which, theoretically, allows the environment to change drastically. I select the nearby year on purpose, trying not to include unexpected changes happening that might cause non-negligible changes in individuals' behavior in the non-stationary environment. See similar discussions about out-of-sample validation in a non-stationary environment in, for example, Giacomini and Rossi (2010) and Inoue et al. (2017).

Table C8: Out-of-sample prediction (NLSY 97 cohort)

Marital status	Simulated proportion	Actual proportion
Single	0.669	0.599
Cohabitation	0.052	0.089
Marriage	0.279	0.312

D Robustness: Stochastic arrival and heterogenous adoption

My main findings suggest that shifts in mating preferences, rather than advances in communication technology, are the primary drivers of the observed changes in marital behavior between the NLS 72 and NLSY 97 cohorts. This conclusion, however, is derived from a model that rests on a strong assumption: Technology has already changed *at the beginning* of the cohort at time $t = 1997$.

I conduct a robustness check by replacing the assumption with a more plausible rational expectations framework. In this setting, individuals are assumed to know a probability distribution governing its arrival. They optimize their life-cycle decisions based on this uncertainty.

This paper models the technological advance as an aggregate shock that shifts the whole economy from $\hat{\Omega}_{\kappa=1}$ (estimated from NLS 72) to a new parameter regime $\hat{\Omega}_{\kappa=2}$ (estimated from NLSY 97). The arrival time of the shock, T^h , is a random variable following a geometric distribution, implying a constant annual hazard rate, h , of the technology jump. Let κ_t^{NLSY97} be a technology level at time $t = 1997, 1998, \dots$, in the NLSY 97 cohort. Then, the stochastic technology adoption is modeled as

$$\kappa_t^{NLSY97} = 1 + \mathbb{I}\{t \geq T^h\}, \quad \text{where } \Pr(T^h = t) = (1 - h)^{t-1}h. \quad (49)$$

I assess the impact of relaxing to the rational expectations setting as follows: First, I set the baseline annual hazard rate h to 0.13. The choice of the hazard rate is calibrated to ensure the cumulative probability of the shock's arrival reaches a high value (95.5%) by 2014, coinciding with the widespread adoption of Tinder. Second, I conduct a counterfactual simulation under uncertainty, which starts with the $\hat{\Omega}_{\kappa=1}$ regime and faces a 13% probability in each time period

of permanently switching to the $\hat{\Omega}_{\kappa=2}$ regime. This stochastic arrival scenario is simulated 50 times, and the outcomes are averaged.

The results of the simulation under this uncertainty demonstrate a remarkable robustness when compared to the benchmark model I employ in the main text. The reason the outcomes do not change significantly under this stochastic arrival is the following: First, as shown in the counterfactual analysis in Section 11.2, the difference in marital behavior between the two cohorts is small in the early time periods. Second, with a hazard rate of 0.13, the cumulative probability of the technology shock occurring reaches nearly 75% by the 10th time period. Therefore, the stochastic case where I start simulation with the NLS 72 parameters switches to the original NLSY 97 parameters reasonably quickly.

Note that, without doing the above exercise, theoretically, the estimated technological effects in the baseline setting in the main text can be interpreted as an upper bound. The model assumes that the technological change occurs with the start of the period. Therefore, compared to a probabilistic scenario in which technology adoption is delayed, my estimates theoretically represent the maximum potential impact of the technological change.

Furthermore, I relax the assumption of homogeneous adoption to allow for heterogeneous adoption. Allowing the matching technology κ to vary at the individual level would cause computations to be impossible. Therefore, instead, I assume that adoption occurs simultaneously within individuals within same characteristics.

Specifically, I partition the population into a finite set of coarse groups $g \in G$, based on attributes, education, gender, and race. Each group g is assigned its own stochastic adoption time T_g^h . Writing the adoption hazard for group g as h_g , the distribution of the adoption time is given by:

$$\Pr(T_g^h = t) = (1 - h_g)^{t-1} h_g, \quad t = 1, 2, \dots \quad (50)$$

The technology state for group g at time t , $\kappa_{g,t}$, is

$$\kappa_{g,t} = 1 + \mathbb{I}\{t \geq T_g^h\}.$$

Thus, when the shock T_g^h arrives, only individuals belonging to that group switch their parameter vector from $\hat{\Omega}_{\kappa=1}$ to $\hat{\Omega}_{\kappa=2}$, while other groups where the shock has not yet arrived remain

in the $\kappa = 1$ regime. The parameter vector for group g at time t is therefore:

$$\Omega_{g,t} = \begin{cases} \hat{\Omega}_{\kappa=1} & \text{if } \kappa_{g,t} = 1 \\ \hat{\Omega}_{\kappa=2} & \text{if } \kappa_{g,t} = 2 \end{cases}.$$

This specification models a gradual technological adoption process across groups while maintaining computational tractability. Here, the same as the stochastic case above, I assume $h_g = 0.13$ for all g .

By allowing for such heterogeneous adoption, the overall adoption of technology in the economy becomes necessarily more gradual compared to the benchmark case with (stochastic) homogeneous adoption. This reinforces the interpretation, as discussed earlier, that the estimated technological effects in the baseline setting in the main text represent an upper bound on the contribution of improved matching technology.

E Appendix E: Proofs

E.1 Proof of Theorem 2

Proof. First, I formally introduce the mathematical objects which are used in the proof, although some of them have been already provided in previous sections. In this proof, I suppress the notation κ . Let $\Lambda = \{\Lambda_{it}^{Sm}, \Lambda_{jt}^{Sw}, \Lambda_{ijt}^C, \Lambda_{ijt}^M\}_{ijt}^{IJT}$ be a vector of stocks of individuals in the economy. Each stock is assumed to be located in a closed and bounded region between 0 and $\bar{\Lambda}$, which is denoted as an upper bound. Let $\mathcal{M}$ be a convex Euclidean product space for the stocks of individuals in the economy, $\mathcal{M} = \{\Lambda\}$. The space, $\mathcal{M}$, is compact as well because, by Tychonoff's theorem, a finite product of compact spaces is compact. Next, let $\alpha = \{\alpha_{ijt}^m, \alpha_{ijt}^w\}_{ijt}^{IJT}$ be a vector of meeting probabilities. Each meeting probability is bounded between 0 and 1. Let $\mathcal{P}$ be a convex and compact Euclidean product space for meeting probabilities, $\mathcal{P} = \{\alpha\}$.²² Moreover, let $\mathbf{U} = \{U_{it}^{Sm}, U_{jt}^{Sw}\}_{ijt}^{IJT}$ be a vector of value functions for being single. Define a vector space for $\mathbf{U}$ as $\mathcal{U} = \{\mathbf{U}\}$. Let $\mathbf{W} = \{W_{ijt}^C(\epsilon), W_{ijt}^M(\epsilon)\}$ be a vector of value functions for a match, where ϵ is a shorthand notation for a realization of ϵ_{ijt} . Define a vector space $\mathcal{W}$ which is continuously dif-

²²Its compactness comes also from Tychonoff's theorem.

ferentiable functions with respect to ϵ , $\mathcal{W} = \{W\}$.²³ Let $V = \{U, W\}$, and let $\mathcal{V}$ be a vector space for value functions, $\mathcal{V} = \{V\}$. Furthermore, let $\epsilon^* = \{\epsilon_{ijt}^{*SC}, \epsilon_{ijt}^{*SM}, \epsilon_{ijt}^{*Si}, \epsilon_{ijt}^{*CS}, \epsilon_{ijt}^{*MS}\}_{ijt}^{IJT}$ be a vector of reservation match values. I can say that each reservation match value is located in a closed and bounded region between $\underline{\epsilon}$ and $\bar{\epsilon}$, which is a lower bound and an upper bound respectively. This is because, as I have proved in Section 5, each reservation match value does not go to $-\infty$ or $+\infty$. Let $\mathcal{E}^*$ be a convex and compact Euclidean product space for reservation match values, $\mathcal{E}^* = \{\epsilon^*\}$. Finally, let $\tau = \{\tau_{ijt}^{SS(m)}, \tau_{ijt}^{SS(w)}, \tau_{ijt}^{SC(m)}, \tau_{ijt}^{SC(w)}, \tau_{ijt}^{SM(m)}, \tau_{ijt}^{SM(w)}, \tau_{ijte}^{CS}, \tau_{ijte}^{CC}, \tau_{ijte}^{Si}, \tau_{ijte}^{MS}, \tau_{ijte}^{MC}, \tau_{ijte}^{MM}\}$ be a vector of transition probabilities, where ϵ is a shorthand notation for a realization of ϵ_{ijt-1} . Each transition probability is located in a closed and bounded region between 0 and 1. Let $\mathcal{T}$ be a convex and compact Euclidean product space for marital status transition probabilities, $\mathcal{T} = \{\tau\}$.

I construct a mapping $\Phi : \mathcal{M} \rightarrow \mathcal{M}$ composing the paths of the whole endogenous interactions of the model: The map Φ is decomposed as,

$$\mathcal{M} \xrightarrow{\omega} \mathcal{P} \xrightarrow{\iota} \mathcal{V} \xrightarrow{\psi} \mathcal{E}^* \xrightarrow{\xi} \mathcal{T} \xrightarrow{\varrho} \mathcal{M}. \quad (50)$$

These mappings are specified as:

- $\omega : \mathcal{M} \rightarrow \mathcal{P}$ describes a mapping from stocks of individuals to meeting probabilities. Given a structure about a meeting probability as mentioned in equation (1), it is assumed to be a continuous in Λ .
- $\iota : \mathcal{P} \rightarrow \mathcal{V}$ describes a mapping from meeting probabilities to the space for the value functions. Each value function is uniquely mapped given a set of meeting probabilities. The structure of a value functions described in equations (7)-(10) ensures that value functions change continuously in α .
- $\psi : \mathcal{V} \rightarrow \mathcal{E}^*$ describes a mapping from value functions to reservation match values. Note that, again, this ψ is also a function. This is because, given $V \in \mathcal{V}$, the reservation match values, $\epsilon_{ijt}^{*SC}, \epsilon_{ijt}^{*SM}, \epsilon_{ijt}^{*Si}, \epsilon_{ijt}^{*CS}, \epsilon_{ijt}^{*MS}$, are uniquely decided, as I have proven in Theorem 1.

²³The values of value functions and the reservation values, which will be introduced below, depend also on $\mu_i^C, \mu_i^M, \mu_j^C$ and μ_j^M . However, for simple notation, I suppress the notation of them in this section.

From the Implicit Function Theorem, ϵ^* is continuous in V .

- $\xi : \mathcal{P} \times \mathcal{E}^* \rightarrow \mathcal{T}$ denotes a mapping from meeting probabilities and reservation match values to transition probabilities. This is a continuous function in α and ϵ^* .
- $\varrho : \mathcal{T} \times \mathcal{M} \rightarrow \mathcal{M}$ describes a function from transition probabilities and the stocks of individuals to stocks of individuals in the economy. This mapping, conceptually, corresponds to a mapping from *stocks of individuals (belief)* to *stocks of individuals (actual)*.

Again, by Tychonoff's theorem, the space $\mathcal{M}$ is compact. It is also convex. The function $\omega(\Lambda)$ is continuous in Λ , the function $\iota(\alpha)$ is continuous in α , the function $\psi(V)$ is continuous in V , the function $\xi(\alpha, \epsilon^*)$ is continuous in α and ϵ^* . The function $\varrho(\tau, \Lambda)$ is continuous in τ and Λ . In sum, the whole mapping Φ is a function and continuous in Λ . So, by Brouwer's fixed point theorem, there exists Λ^{**} , which is a vector of equilibrium stocks satisfying the equilibrium conditions. Therefore, there also exist $\alpha^{**} = \omega(\Lambda^{**})$, $V^{**} = \iota(\alpha^{**})$, $\epsilon^{**} = \psi(V^{**})$ and $\tau^{**} = \xi(\alpha^{**}, \epsilon^{**})$.²⁴ □

E.2 Proof of Theorem 3

Proof. The key is to use Sard's theorem (1946) in my non-stationary, two-sided market setting.²⁵ As I introduce in Appendix E.1, the market equilibrium Λ_κ^{**} is a fixed point of the mapping, $\Phi_\Omega : \mathcal{M} \rightarrow \mathcal{M}$, that is $\Lambda_\kappa^{**} = \Phi_\Omega(\Lambda_\kappa^{**})$.²⁶ From equation (50), the composition mapping is decomposed into

$$\Phi_\Omega = \omega_\Omega \circ \iota_\Omega \circ \psi_\Omega \circ \xi_\Omega \circ \varrho_\Omega. \quad (51)$$

In equation (51), I also explicitly represent each mapping with Ω . Similarly, as in Online Appendix E.1, from my functional specifications, the function $\omega_\Omega(\Lambda_\kappa)$ is C^∞ differentiable in Ω , the function $\iota_\Omega(\alpha)$ is C^∞ differentiable in Ω , the function $\psi_\Omega(V)$ is C^∞ differentiable in Ω , the function $\xi_\Omega(\alpha, \epsilon^*)$ is C^∞ differentiable in Ω . The function $\varrho_\Omega(\tau, \Lambda_\kappa)$ is C^∞ differentiable in Ω .^{27,28}

²⁴Note that, if the fixed point mapping is assumed to start from the space $\mathcal{V}$, I need to use Schauder's fixed point.

²⁵One can see a similar argument in Debreu (1970).

²⁶Here, with Φ_Ω , for the convenience of the following discussion, I emphasize that the fixed point depends on parameter values, Ω .

²⁷Actually, what I need to have is C^1 differentiability for Φ_Ω for Sard's theorem.

²⁸The mapping $\iota(\cdot)$ has $\max\{\cdot\}$ in value functions. Without imposing any assumption, we typically face non-differentiability. However, I integrate $\max\{\cdot\}$ in value functions with the random variable following the normal

In sum, the whole mapping Φ_Ω is C^∞ differentiable in Ω . With the same argument, Φ_Ω is C^∞ differentiable in Λ_κ . I rewrite the fixed point mapping equation

$$\Phi_\Omega(\Lambda_\kappa) - \Lambda_\kappa = \mathbf{0},$$

as, using $G_\Omega(\cdot)$,

$$G_\Omega(\Lambda_\kappa) = \mathbf{0}.$$

Let $|\Omega|$ be the number of parameters in the model and Θ be the space for Ω , $\Omega \subset \Theta = \mathbb{R}^{|\Omega|}$. Now, we define a set $Z \subset \mathbb{R}_+^{(I+J+2IJ)T} \times \mathbb{R}^{|\Omega|}$ such that

$$Z = \{(\Lambda_\kappa, \Omega) : G_\Omega(\Lambda_\kappa) = \mathbf{0}\}, \quad (52)$$

or, more simply,

$$Z = \{(\Lambda_\kappa^{**}, \Omega)\}. \quad (53)$$

The set Z is, intuitively, the values of Λ_κ which satisfies the equilibrium condition in Section 6 given a set of values of Ω . I introduce a new mapping π , $\pi : Z \rightarrow \Theta$, which is a (linear) projection mapping to the space Θ from the space Z such that

$$\Omega = \pi(Z). \quad (54)$$

Let JG_{Λ_κ} be a Jacobian matrix of $G_\Omega(\Lambda_\kappa)$ at Λ_κ . Let $Z_{reg} \subseteq Z$ be a set of regular points, which is mathematically equivalent to,

$$Z_{reg} = \{(\Lambda_\kappa, \Omega) : G_\Omega(\Lambda_\kappa) = \mathbf{0} \wedge \det [JG_{\Lambda_\kappa}] \neq 0\}. \quad (55)$$

Therefore, in a neighborhood of any $(\Lambda_\kappa^*, \Omega) \in Z_{reg}$, the implicit function theorem implies that $\Lambda_\kappa = \Lambda_\kappa(\Omega)$ is uniquely (and smoothly in Λ_κ) determined. Hence, the equilibrium Λ_κ^* at Ω is locally unique.

distribution, ϵ_{mwt} . It makes value functions smooth in parameters. See, for example, Brien et al. (2006). Recall that, from Theorem 1, ϵ^* is unique given Ω . Therefore, using the implicit function theorem, ψ is differentiable in Ω .

Let $Z_{sing} \subseteq Z$ be such that

$$Z_{sing} = \{(\Lambda_\kappa, \Omega) : G_\Omega(\Lambda_\kappa) = \mathbf{0} \wedge \det [JG_{\Lambda_\kappa}] = 0\}, \quad (56)$$

Note that $[JG_{\Lambda_\kappa}] = 0$ is equivalent to being a singular point in the sense of Debreu (1970). I now prove the smoothness of π on Z_{reg} . This is directly because π is a linear projection mapping. Because $G_\Omega(\cdot)$ is C^∞ differentiable, π is also C^∞ differentiable. Let λ_Θ be a Lebesgue measure on the parameter space Θ with dimension $|\Omega|$. Let the set of critical points of Z be $Z \setminus Z_{reg} = Z_{sing}$. By Sard's theorem, $\pi(Z_{sing}) \subset \Theta$ is Lebesgue measure zero.

□