A Near Pareto Optimal Auction with Budget Constraints

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Motivation

- Auction theory has paid relatively little attention to the case of budget constrained bidders.
- Budget constraints are relevant when the objects is are of high value for a single bidder to buy all objects.
- Budget constraints introduce important differences: the usual quasi-linear setting breaks down.

Motivation

- Online advertisement auctions are arguably one of setups where budget constraints matter.
- That could be why the advertisers have to specify “a value per click” and “a daily maximum budget” for the advertisement auctions.
- Single item (query), single slot, no quality scores for simplification

Model

- m units of an item to be sold
- n bidders with private values v_i and budgets b_i
- Bidders budget constraints are hard. Bidder i's utility by getting q units of the good and paying p is

$$u_i(q,p) = \begin{cases} qv_i - p & \text{if } p \leq b_i \\ -C & \text{if } p > b_i \end{cases}$$

  where $\infty \geq C > 0$.
- Equilibrium concept is ex-post Nash equilibrium

Objective, Previous work and Contribution

- Design a mechanism (allocation and pricing rule) for this setup
  - with good revenue and efficiency properties
- Dobzinsyki, Lavi, Nisan (2008): There is no truthful and Pareto optimal mechanism in this setup.
- We introduce a new mechanism: Vickrey with Budgets
- It is not truthful: bidders can benefit by misreporting their types
- Misreporting is beneficial to a bidder only when it increases the revenue to the auctioneer.

Vickrey with Budgets

- Take the vectors $(\mathbf{b}, \mathbf{v})$ and sort them in nonascending order of values
- Calculate the unique cut-point c^* according the pricing function:
  - winners (bidders $1, .., j-1$) pay v_j per unit up to a budget of s, then pay v_{j+1} per unit up to a budget of b_{j+1},and so on, until their budgets are exhausted;
  - the partial winner (bidder j) pay v_{j+1} per unit up to a budget of b_{j+1}, then pay v_{j+2} per unit up to a budget of b_{j+2}, and so on, until she spends $b_j - s$.
- Similar to a Vickrey auction adjusted for budgets

Proposition 2 Vickrey with Budgets's revenue, $R^S(\mathbf{b}, \mathbf{v})$ is nondecreasing in $\mathbf{b}$ and $\mathbf{v}$.

Truthfulness

- We consider different deviations from truthful revelations

Propositions 3,4,5 (b_i, v_i) weakly dominates (b_i, v_i^-) , (b_i^-, v_i) and (b_i^-, v_i^-)

Proposition 6 (b_i^-, v_i^+) can be in the set of best responses only when bidder i's utility in her best response is 0.

Proposition 7 Whenever bidding (b_i^+, v_i^-) brings a higher utility to i than bidding (b_i, v_i) , the auctioneer's revenue with (b_i^+, v_i^-) is not lower than the revenue with (b_i, v_i)

Revenue and Efficiency Properties

- Vickrey with Budgets has good revenue and efficiency properties

Definition A *refined equilibrium* is an equilibrium of Vickrey with Budgets where for all bidders i, bidder i does not play $\left(b_i^-, v_i\right)$, $\left(b_i, v_i^-\right)$, $\left(b_i^-, v_i^-\right)$, or $\left(b_i^-, v_i^+\right)$. Moreover, a bidder i plays $\left(b_i^+, v_i^-\right)$ only when
$u_i\left(\left(\mathbf{b}_{-i}, b_i^+\right), \left(\mathbf{v}_{-i}, v_i^-\right)\right) > u_i\left(\left(\mathbf{b}_{-i}, b_i\right), \left(\mathbf{v}_{-i}, v_i\right)\right)$.

Theorem 1 In a refined equilibrium of Vickrey with Budgets, revenue is bounded below by the revenue of Vickrey with Budgets with truthful revelations.

Theorem 2 In any ex-post Nash equilibrium of Vickrey with Budgets where v_j is the announced value of the partial winner j; every bidder $i \neq j$ who has true value $v_i^T > v_j$ is a full winner, and every bidder $i \neq j$ who has true value $v_i^T < v_j$ is a loser.

Market Clearing Price Mechanism and Vickrey with Budgets

- We compare Vickrey with Budgets to a more standard mechanism

Definition *Market Clearing Price Mechanism (MCPM)* is a mechanism that sells m items to all interested bidders at a fixed price.

Proposition 8 Under the MCPM, overstating budget or value is weakly dominated by bidding true types.

Proposition 9 For any announcements $(\mathbf{b}, \mathbf{v})$, $R^M(\mathbf{b}, \mathbf{v}) - R^S(\mathbf{b}, \mathbf{v}) \leq b_{\max}$

Market Clearing Price Mechanism and Vickrey with Budgets

- Denote the revenue of MCPM with the truthful revelation of types by R^*.

Theorem 3 Revenue of any refined equilibrium of Vickrey with Budgets is not lower than $R^* - b_{\max}$.

Example Consider pairs $(b_0, v_0) = (16, 18)$ and $(b_1, v_1) = (8, 9)$.
There are N bidders of each type and a supply of $3N$.
Revenue of MCPM is $16N$ compared to Vickrey with Budgets revenue which is not smaller than $24N - 16$.

Conclusion

- We analyzed the problem of selling multi-units of an item to a set of budget constrained bidders. A problem relevant to advertisement auctions.
- We introduce a mechanism, Vickrey with Budgets, which achieves good truthfulness, revenue, and efficiency properties.
  - in Vickrey with Budgets, there are profitable deviations from truthful revelations of types, but that can only happen in a revenue increasing way
  - in a refined expost equilibrium, the revenue of Vickrey with Budgets is bounded below by $R^* - b_{\max}$
  - the equilibrium allocation is *nearly Pareto efficient* in the sense that full winners and losers are ordered in the right way given the announced value of the partial winner.
- We compared Vickrey with Budgets to MCPM. We show that in MCPM,
  - revenue increasing deviations are dominated
  - the revenue can be much smaller than $R^* - b_{\max}$.

Future Work

- Multiple slots for keyword auctions
- Soft budget constraints
- Multiple items