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A Note on Identification of Match Fixed Effects as Interpretable Unobserved Match Affinity

Suguru Otani
The University of Tokyo

Tohya Sugano
The University of Tokyo

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Suguru Otani[*], Tohya Sugano[†]

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Abstract

We highlight that match fixed effects, represented by the coefficients of interaction terms involving dummy variables for two elements, lack identification without specific restrictions on parameters. Consequently, the coefficients typically reported as relative match fixed effects by statistical software are not interpretable. To address this, we establish normalization conditions that enable identification of match fixed effect parameters as interpretable indicators of unobserved match affinity, facilitating comparisons among observed matches. Using data from middle school students in the 2007 Trends in International Mathematics and Science Study (TIMSS), we highlight the distribution of comparable match fixed effects within a specific school.

Keywords: Match fixed effect, Affinity, Identification

JEL code: C21, J24, J31, I26

[*]suguru.otani@e.u-tokyo.ac.jp, Market Design Center, The University of Tokyo

[†]sugano-tohya1011@g.ecc.u-tokyo.ac.jp, The University of Tokyo

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1 Introduction

Evaluating the quality of matching and affinity between entities is common in empirical research. Affinity is divided into observed and unobserved components. Observed affinity is typically measured by the coefficient of interaction terms involving observable characteristics, while unobserved affinity is captured through match fixed effects using dummy variables. This approach is widely used in labor and education economics to analyze and interpret match quality across pairs such as teacher-student and worker-company relationships. Our study aims to clarify the identification process of match fixed effects.

As an illustrative example, Inoue and Tanaka (2023) investigate the impact of a teacher's major on students' achievement using the following econometric model:

$$Y_{ifj} = \beta Major_{fj} + \delta_f + \eta_{ij} + \varepsilon_{ifj}, \tag{1}$$

where Y_{ifj} represents the science test score of student i in subfield f within class j. The parameter β denotes the coefficient of interest, and $Major_{fj}$ is an indicator variable that indicates whether the teacher's major field in natural science matches the subfield of the student's test score. δ_f denotes the fixed effect specific to subfield f, while η_{ij} represents the student-teacher fixed effects, accounting for any subfield-invariant determinants of science test scores between student i and teacher j, thereby capturing their unobserved affinity. The authors find a significant increase in R-squared upon introducing student-teacher fixed effects, underscoring the importance of match fixed effects in their analysis.

Similar methodologies are utilized in various studies, including pitcher-catcher fixed effects on strikeout likelihood (Biolsi *et al.* 2022), worker-company fixed effects on income (Mittag 2019) and turnover rates (Ferreira and Taylor 2011), student-school fixed effects on student performance (Ovidi 2022), teacher-school fixed effects on student test scores (Jackson 2013), and student-university fixed effects on post-graduation income (Dillon and Smith 2020). Additionally, some studies account for two-way fixed effects by separately considering the fixed effects on each side.

Despite their common use, the interpretation of match fixed effects as affinity indicators for all matches remains unclear. Our paper addresses this by distinguishing between relative and absolute match fixed effects within a standard model. We argue that absolute match fixed effects are not identifiable without specific restrictions, whereas relative match fixed effects are identifiable. We propose location normalization conditions to enable the identification and interpretation of absolute match fixed effects, facilitating comparisons of unobserved match quality.

Applying this approach to the data from Inoue and Tanaka (2023), we demonstrate the distribution of unobserved match quality between students and teachers in a specific school. Our analysis reveals that one teacher excels with high-achieving students but underperforms with lower-achieving ones, while another teacher shows the opposite pattern in terms of unobserved match affinity.

2 Model

We consider the following typical setting. Suppose that we can observe I students indexed by i and J teachers indexed by j at time $t = 1, \cdots, T$. Student i makes some outcome with teacher j at time t. For avoiding later notational complexity, we do not include time fixed effects like a panel regression, but the inclusion does not affect our findings. We consider the following regression model:

$$
\begin{aligned}
Y_{ijt} &= \alpha_i + \beta_j + \mu_{ij} + X'_{ijt}\gamma + \varepsilon_{ijt}, \qquad (2)\\
&= \sum_{i'=1}^{I} \alpha_{i'} 1(i = i') + \sum_{j'=1}^{J} \beta_{j'} 1(j = j') + \sum_{i'=1}^{I}\sum_{j'=1}^{J} \mu_{i'j'} 1(i = i', j = j') + X'_{ijt}\gamma + \varepsilon_{ijt} \qquad (3)
\end{aligned}
$$

where Y_{ijt} is the test score of student i with teacher j at time t, X_{ij} is d-dimensional covariates consisting of observed characteristics of student i and teacher j and its interaction at time t, α_i is student i's fixed effect, β_j is teacher j's fixed effect, and μ_{ij} is (i, j)-match fixed effect which is of our interest, γ is a d-dimensional vector of parameters, $1(\cdot)$ is an indicator function, and ε_{ijt} is an error term assumed to be drawn i.i.d from standard normal distribution. Note that we explicitly decompose affinity between student i and teacher j into two parts, that is, $X'_{ijt}\gamma$ and μ_{ij} as the observed and unobserved affinities. For later discussion, we call μ_{ij} the *absolute* match effect of student i and teacher j. Similarly, we call α_i and β_j the absolute fixed effects.[1]

In the context of the standard fixed effect model, when there are I groups, typically $I - 1$ fixed effects are incorporated, alongside a constant term, to avoid multicollinearity that would arise from including the I-th group. The match fixed effect case is more complex. Similarly, for a student

[1] Using Equation (3), matrix representation is described as $Y = X\gamma + 1_I\alpha + 1_J\beta + 1_{IJ}\mu + \varepsilon = X\gamma + \tilde{X}\delta + \varepsilon$ where Y is $IJT \times 1$, X is $IJT \times d$, γ is $d \times 1$, 1_I is $IJT \times I$, α is $I \times 1$, 1_J is $IJT \times J$, β is $J \times 1$, 1_{IJ} is $IJT \times IJ$, μ is $IJ \times 1$, ε is $IJT \times 1$, and denote $\tilde{X} = [1_I 1_J 1_{IJ}]$ and $\delta = [\alpha^T \beta^T \mu^T]^T$. Then, define I as $IJT \times 1$ one vector and $M = I - \tilde{X}(\tilde{X}^T\tilde{X})^{-1}\tilde{X}^T$ as the annihilator matrix for $\tilde{X}$. The OLS estimator of δ is obtained as $\hat{\delta} = (\tilde{X}^T M\tilde{X})^{-1}(\tilde{X}^T MY)$, so the standard full rank condition for δ is $Rank(\tilde{X}^T M\tilde{X}) = (I + J + IJ)$. See Hansen (2022) Chapter 3.16 for reference. However, the condition does not hold due to multicollinearity.

$i \neq 1$ and teacher $j \neq 1$, Equation (2) can be rewritten as

$$\begin{aligned}
Y_{ijt} = &\underbrace{\alpha_1 + \beta_1 + \mu_{11}}_{\text{constant}} \\
&+ \underbrace{(\beta_j - \beta_1) + (\mu_{1j} - \mu_{11})}_{\text{teacher } j\text{'s relative fixed effect}} \\
&+ \underbrace{(\alpha_i - \alpha_1) + (\mu_{i1} - \mu_{11})}_{\text{student } i\text{'s relative fixed effect}} \\
&+ \underbrace{(\mu_{ij} + \mu_{11} - \mu_{1j} - \mu_{i1})}_{(i,j)\text{'s relative match effect}} + X'_{ijt}\gamma + \varepsilon_{ijt},
\end{aligned} \tag{4}$$

where the first line is a constant parameter normalized to student 1 and teacher 1 which are arbitrarily chosen, the second line is called teacher j's *relative* fixed effect which is the fixed effect relative to teacher 1's fixed effects, the third line is called student i's *relative* fixed effect which is the fixed effect relative to student 1's fixed effects, the third line is called the *relative* match effect of student i and teacher j relative to student 1 and teacher 1. Avoiding multicollinearity, we can identify and estimate these relative fixed effects and γ instead of absolute fixed effects. Statistical software automatically reports the estimates of the relative fixed effects.

Relative fixed effects indicate how a match compares to a specific reference match, rather than categorizing it as "good" or "bad" compared to all other matches. For controlling match fixed effects or obtaining overall affinity without distinguishing observed from unobserved components, relative fixed effects are adequate. However, for measuring and understanding unobserved affinity, such as personality match quality, relative fixed effects are insufficient. In these cases, absolute fixed effects are crucial for meaningful comparison and interpretation of unobserved affinity.

Our central question posed is: "Can we derive the absolute fixed effects from the estimated relative fixed effects without imposing any restrictions?" In mathematical terms, "Can we solve the system of equations involving relative fixed effects for absolute fixed effects without restrictions?" Our conclusion is in the negative: No, we cannot achieve this without imposing constraints. Subsequently, we propose location normalization restrictions that are necessary for identifying the absolute fixed effects μ_{ij}, summarized in Proposition 1.

Proposition 1. *For regression model (2), the following results hold.*

1. *Without any restriction, the absolute match effect μ_{ij} is not identified.*

2. *With $\sum_i \mu_{ij} = \sum_j \mu_{ij} = 0$, the absolute match effect μ_{ij} for all i and j is identified by relative fixed effects.*

3. *With restriction $\sum_i \mu_{ij} = \sum_j \mu_{ij} = \sum_i \alpha_i = \sum_j \beta_j = 0$ for all i and j, the absolute fixed effects α_i, β_j, and absolute match effect μ_{ij} for all i and j are identified by relative fixed*

effects.

See the proof and illustrative example in Appendix A. Intuitively, the conditions outlined are effective for location normalization. The restricted match effect indicates how much better the match is compared to the average match, rather than a specific match. Consequently, it yields positive values when the match is relatively superior and negative values when it is relatively inferior compared to the average match. This approach ensures interpretability across students and teachers, facilitating meaningful comparisons of unobserved affinity.

3 Empirical exercise

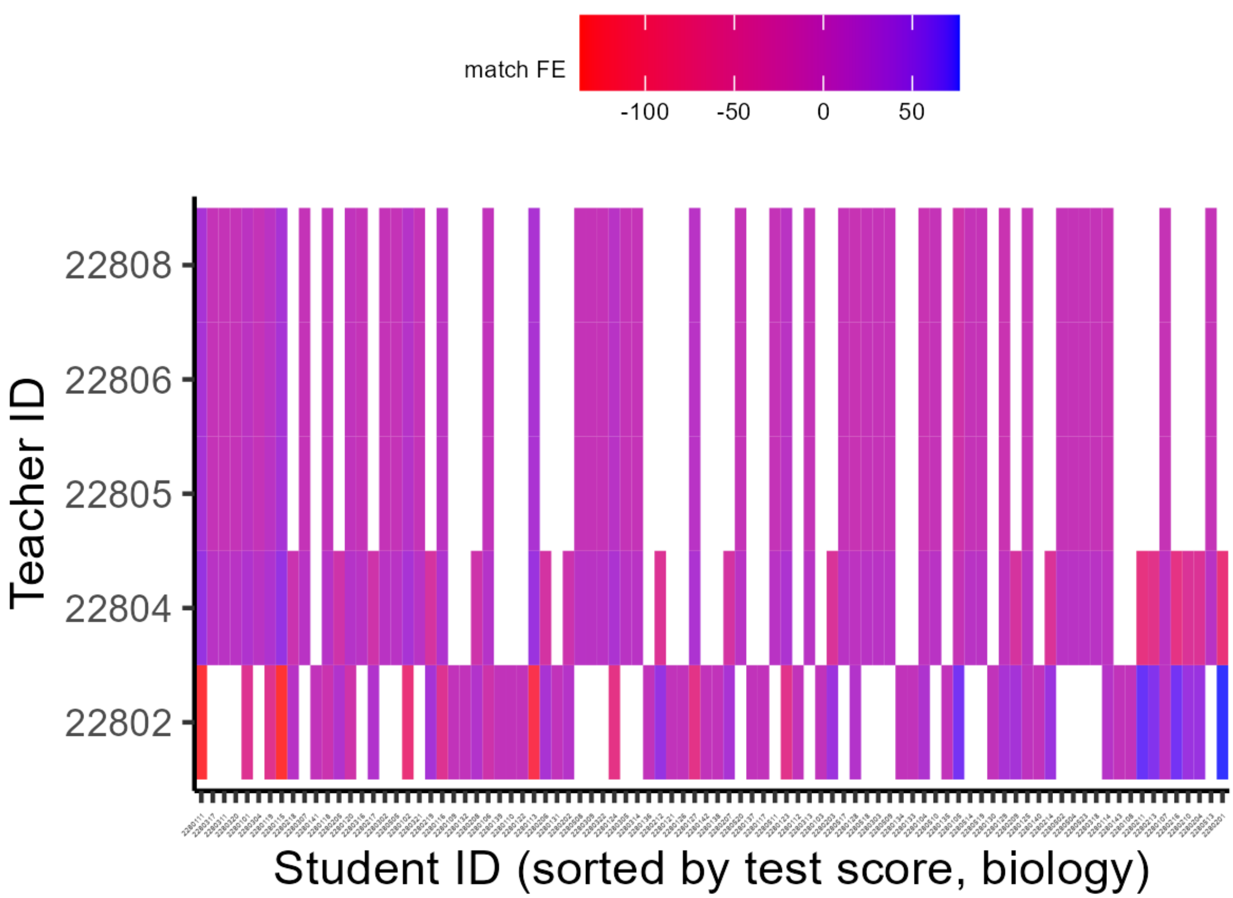

Figure 1: Heatmap of absolute match fixed effects
Notes: The blank cell shows no match in the data.

To illustrate our approach, we use data from the 2007 Trends in International Mathematics and Science Study (TIMSS), as in Inoue and Tanaka (2023). This dataset includes middle school students' test scores in various science subfields (physics, chemistry, biology, and Earth science) and teacher-related variables. For details, refer to Inoue and Tanaka (2023). We focus on data from a specific school (ID 228) with five teachers and 90 students, using Equation (1) to estimate absolute match fixed effects with location normalization. We exclude subfield variables to avoid multicollinearity with student-teacher interaction dummies, as some subjects are taught exclusively by certain teachers. The author's GitHub page provides replication files for our Monte Carlo simulation and empirical exercises.

Figure 1 presents a heatmap of absolute match fixed effects. Teacher ID 22802 shows better effects with high-performing students but poorer effects with low-performing students, while teacher ID 22804 displays the opposite pattern. This suggests that teacher 22802 excels with high achievers but underperforms with lower achievers, whereas teacher 22804 performs better with lower achievers. Furthermore, absolute match effects offer a more comparable measure than relative match effects. For instance, the absolute match effect for student ID 2280201 and teacher ID 22802 is 76.34, compared to 44.57 for student ID 2280115 and teacher ID 22804, indicating that the former match has 1.75 times higher unobserved affinity. Thus, absolute fixed effects are crucial for meaningful comparisons and interpretations of unobserved affinity.

4 Conclusion

We examine the estimation of affinities using match fixed effects, distinguishing between observed and unobserved components. We emphasize that, without proper restrictions, only relative fixed effects—fixed effects relative to normalized values—can be estimated, which do not provide interpretable measures of unobserved affinity. To enable meaningful interpretation of absolute match effects, we introduce theoretical constraints on parameters. Using 2007 TIMSS data on middle school students, we illustrate the distribution of absolute match fixed effects and underscore their significance.

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A Proof (Online appendix)

Proof. Before showing the proof, we provide the overview. The system of linear equations about parameters to be solved without any restrictions is

$$\begin{pmatrix} A \\ B \\ C \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \mu \end{pmatrix} = \begin{pmatrix} \alpha' \\ \beta' \\ \mu' \end{pmatrix}$$

where A, B and C are the coefficient matrix for relative effects and α', β' and μ' are calculated relative fixed effects defined later. Let T denote transpose. Then if $rank\left([A^T \quad B^T \quad C^T]^T\right) = rank\left([\alpha^T \quad \beta^T \quad \mu^T]^T\right) = I+J+IJ$, the absolute fixed effects and match effects are just identified. Thus, we will check the rank conditions to determine whether a system of linear equations is underdetermined (i.e., underidentified), meaning there are fewer equations than unknowns.
Since there are I students and J teachers, there are $I+J$ unknown absolute fixed effect parameters and IJ unknown absolute match effect parameters. The fixed effects of (i,j) relative to (i_0, j_0), $\alpha'_{i_0j_0,i}, \beta'_{i_0j_0,j}, \mu'_{i_0j_0,ij}$ is calculated by

$$\alpha'_{i_0j_0,i} = \alpha_i - \alpha_{i_0} + \mu_{ij_0} - \mu_{i_0j_0} \tag{5}$$
$$\beta'_{i_0j_0,j} = \beta_j - \beta_{j_0} + \mu_{i_0j} - \mu_{i_0j_0} \tag{6}$$
$$\mu'_{i_0j_0,ij} = \mu_{ij} + \mu_{i_0j_0} - \mu_{ij_0} - \mu_{i_0j}. \tag{7}$$

First, we prove that absolute match effects are not identifiable without any restrictions. Let coefficient matrices A, B and C represent coefficient of α_i, β_j and μ_{ij} in Equations (5), (6), and

(7) respectively as follows:

$$
\begin{aligned}
\mathrm{A} &= \begin{pmatrix}
a_1^{11,2} & \cdots & a_I^{11,2} & 0 & \cdots & 0 & c_{11}^{11,2} & \cdots & c_{1J}^{1} & \cdots & c_{I1}^{11,2} & \cdots & c_{IJ}^{11,2} \\
a_1^{11,3} & \cdots & a_I^{11,3} & 0 & \cdots & 0 & c_{11}^{11,3} & \cdots & c_{1J}^{11,3} & \cdots & c_{I1}^{11,3} & \cdots & c_{IJ}^{11,3} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
a_1^{IJ,I-1} & \cdots & a_I^{IJ,I-1} & 0 & \cdots & 0 & c_{11}^{IJ,I-1} & \cdots & c_{1J}^{IJ,I-1} & \cdots & c_{I1}^{IJ,I-1} & \cdots & c_{IJ}^{IJ,I-1}
\end{pmatrix} \\
\mathrm{B} &= \begin{pmatrix}
0 & \cdots & 0 & b_1^{11,2} & \cdots & b_J^{11,2} & c_{11}^{11,2} & \cdots & c_{1J}^{11,2} & \cdots & c_{I1}^{11,2} & \cdots & c_{IJ}^{11,2} \\
0 & \cdots & 0 & b_1^{11,3} & \cdots & b_J^{11,3} & c_{11}^{11,3} & \cdots & c_{1J}^{11,3} & \cdots & c_{I1}^{11,3} & \cdots & c_{IJ}^{11,3} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \cdots & 0 & b_1^{IJ,J-1} & \cdots & b_J^{IJ,J-1} & c_{11}^{IJ,J-1} & \cdots & c_{1J}^{IJ,J-1} & \cdots & c_{I1}^{IJ,J-1} & \cdots & c_{IJ}^{IJ,J-1}
\end{pmatrix} \\
\mathrm{C} &= \begin{pmatrix}
0 & \cdots & 0 & 0 & \cdots & 0 & c_{11}^{11,2} & \cdots & c_{1J}^{11,2} & \cdots & c_{I1}^{11,2} & \cdots & c_{IJ}^{11,2} \\
0 & \cdots & 0 & 0 & \cdots & 0 & c_{11}^{11,3} & \cdots & c_{1J}^{11,3} & \cdots & c_{I1}^{11,3} & \cdots & c_{IJ}^{11,3} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \cdots & 0 & 0 & \cdots & 0 & c_{11}^{IJ,J-1} & \cdots & c_{1J}^{IJ,J-1} & \cdots & c_{I1}^{IJ,J-1} & \cdots & c_{IJ}^{IJ,J-1}
\end{pmatrix} \\
& \qquad \underbrace{\qquad\qquad\qquad}_{I} \quad \underbrace{\qquad\qquad\qquad}_{J} \quad \underbrace{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}_{IJ}
\end{aligned}
$$

where $a_k^{i_0j_0,i}$ and $b_k^{j_0j_0,i}$ represent the coefficients of α_k and β_k, and $c_{kl}^{i_0j_0,i}$ represents the coefficient of μ_{kl} in the equation $\mu'_{i_0j_0,ij} = \alpha_i - \alpha_{i_0} + \mu_{ij_0} - \mu_{i_0j_0}$. Every element in each matrix is either $0, 1$ or -1.

First, consider the matrix C. Assume $c_{kl}^{i_0j_0,ij} = 1$ $(k \neq i_0,\ l \neq j_0)$, then $c_{i_0j_0}^{i_0j_0,ij} = 1$, $c_{i_0l}^{i_0j_0,ij} = c_{kj_0}^{i_0j_0,ij} = -1$ and $c_{k'l'}^{i_0j_0,ij} = 0$ for $k' \notin \{k, i_0\}$, $l' \notin \{l, j_0\}$. Then, $c^{ij,i_0j_0} = c^{i_0j_0,ij}$, $c^{ij_0,i_0j} = -c^{i_0j_0,ij}$, and $c^{i'_0j'_0,ij} = c^{i_0j_0,ij} + c^{i_0j_0,i'_0j'_0} - c^{i_0j_0,i'_0j} - c^{i_0j_0,ij'_0}$ hold. Here fix $i_0 = j_0 = 1$. Then $c^{i'_0j'_0,i'j'}$ can be obtained from $c^{11,ij}$. Thus, $rank(C) = (I-1)(J-1)$. Similarly, because $a^{i_0j_0,i} = a^{ij_0,i_0}$ and $a^{i'_0j'_0,i} = a^{11,i} - a^{11,i'_0} + c^{ij'_0,i'_01}$, $rank\left([A^T \quad C^T]^T\right) = (I-1)+(I-1)(J-1) = (I-1)J$. Also, because $b^{i_0j_0,j} = b^{i_0j,j_0}$ and $b^{i'_0j'_0,j} = b^{11,j} - b^{11,j'_0} + c^{i'_0j,1j'_0}$, $rank\left([A^T \quad B^T \quad C^T]^T\right) = (J-1)+(I-1)J = IJ-1$.

This concludes that $rank\left([A^T \quad B^T \quad C^T]^T\right) = IJ - 1 < IJ$ and absolute match effects are not identifiable. Note that without match fixed effect, then all elements in C is 0, $rank\left([A^T \quad B^T]^T\right) = I + J - 2$, and thus α and β is not identifiable without additional restrictions.

Next, we prove that absolute match effects are identifiable with restrictions $\sum_i \mu_{ij} = \sum_j \mu_{ij} = 1$. Each restriction is independent from every equation above and the number of independent restrictions is $I + J - 1$. Therefore, $rank\left([C^T \quad R_1^T]^T\right) = IJ$ where R_1 is the coefficient matrix for the restrictions $\sum_i \mu_{ij} = \sum_j \mu_{ij} = 1$ as

$$R_1 = \begin{array}{c} \\ \left(\begin{array}{cccccccccccccccc} 0 & \cdots & 0 & 0 & \cdots & 0 & \overbrace{1 & \cdots & 1 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0}^{IJ} \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 1 & \cdots & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 1 & \cdots & 1 \\ 0 & \cdots & 0 & 0 & \cdots & 0 & 1 & \cdots & 0 & 1 & \cdots & 0 & \cdots & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \underbrace{0 & \cdots & 0}_{I} & \underbrace{0 & \cdots & 0}_{J} & \underbrace{0 & \cdots & 1}_{I} & \underbrace{0 & \cdots & 1}_{I} & \cdots & \underbrace{0 & \cdots & 1}_{I} \end{array}\right) \end{array} \begin{array}{l} \left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\} \text{I} \\ \left.\begin{array}{c} \\ \\ \\ \end{array}\right\} \text{J} \end{array}.$$

Then the absolute match effect can be identified since the rank is the same as the number of absolute match effect parameters.

Finally, we prove that absolute match effect and fixed effects are identifiable with restrictions $\sum_i \mu_{ij} = \sum_j \mu_{ij} = \sum_i \alpha_i = \sum_j \beta_j = 0$. Each row in the coefficient matrix for restrictions $\sum_i \alpha_i = \sum_j \beta_j = 0$, R_2 and R_3 defined as

$$\begin{aligned} R_2 &= \begin{pmatrix} 1 & \cdots & 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \end{pmatrix} \\ R_3 &= \begin{pmatrix} \underbrace{0 & \cdots & 0}_{I} & \underbrace{1 & \cdots & 1}_{J} & \underbrace{0 & \cdots & 0}_{IJ} \end{pmatrix} \end{aligned}$$

are independent from every row in A, B, C and R_1 and they have rank of 1. This is because the number of a's in a row which is not zero is 2 while at least I elements remains not zero by adding the rows in R_2, and similar for R_3 independence. Therefore, $rank\left([A^T \quad B^T \quad C^T \quad R_1^T \quad R_2^T \quad R_3^T]^T\right) = IJ + I + J$ which equals to the number of parameters, and thus absolute match effect and fixed effect parameters are identifiable. □

A.1 Illustrative example

Consider an illustrative example with $I = 2$ and $J = 3$. The relative match effect is estimated as

$$\mu'_1 = \mu_{11} + \mu_{22} - \mu_{12} - \mu_{21}$$
$$\mu'_2 = \mu_{11} + \mu_{23} - \mu_{13} - \mu_{21}$$

and all the other relative match effect is calculated from μ'_1 and μ'_2. Without zero sum restrictions, absolute match effect μ_{ij} is not identifiable, but with restrictions $\sum_i \mu_{ij} = \sum_j \mu_{ij} = 0$, the match effect is calculated as[2]

$$\begin{aligned}
\mu_{11} &= \frac{\mu'_1}{6} + \frac{\mu'_2}{6} \\
\mu_{12} &= -\frac{\mu'_1}{3} + \frac{\mu'_2}{6} \\
\mu_{13} &= \frac{\mu'_1}{6} - \frac{\mu'_2}{3} \\
\mu_{21} &= -\frac{\mu'_1}{6} - \frac{\mu'_2}{6} \\
\mu_{22} &= \frac{\mu'_1}{3} - \frac{\mu'_2}{6} \\
\mu_{23} &= -\frac{\mu'_1}{6} + \frac{\mu'_2}{3}.
\end{aligned}$$

[2]You can derive μ_{11} from $\mu'_1 + \mu'_2 = 2\mu_{11} + \mu_{22} + \mu_{23} - \mu_{12} - \mu_{13} - 2\mu_{21} = 6\mu_{11}$ by using the restrictions. And $\mu_{11} = -\mu_{21} = (\mu'_1 + \mu'_2)/6$. Then, from the equations, you get $\mu_{12} = -\mu_{21} = (2\mu_{11} - \mu'_1)/2$ and $\mu_{13} = -\mu_{23} = (2\mu_{11} - \mu'_2)/2$.