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Nonparametric Estimation of Matching Efficiency and Mismatch in Labor Markets via Public Employment Security Offices in Japan, 1972-2024

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Abstract

I identify significant biases in the traditional Cobb-Douglas function under misspecification of nonadditive, time-varying matching efficiency, and evaluate the finite sample performance of the nonparametric estimation method of Lange and Papageorgiou (2020). Additionally, I extend the mismatch index by Şahin *et al.* (2014) to a nonparametric framework and develop a computational methodology. Applying the method, I analyze changes in matching efficiency, elasticities, and mismatch in Japan's labor market using Hello Work data for unemployed workers from January 1972 to April 2024. I find a declining trend in matching efficiency, aligned with decreasing job and worker finding rates. The estimated elasticities range from 0.5 to 0.9 for unemployment and between -0.4 and 0.4 for vacancies. Furthermore, I demonstrate that occupational mismatch is more severe than geographical mismatch, with the Cobb-Douglas mismatch index significantly underestimating the true extent of mismatch.

Keywords: matching efficiency, matching elasticity, matching function, mismatch
JEL code: E24, J61, J62, J64

1 Introduction

Understanding labor market matching efficiency and mismatch—defined as the misalignment between the distribution of vacant jobs and unemployed workers—is essential for analyzing labor

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market dynamics. The matching function, which maps hires to the number of job seekers and vacancies while incorporating matching efficiency and search effort, is central to understanding frictional labor markets. However, as Lange and Papageorgiou (2020) highlight, a critical limitation of existing studies is their reliance on restrictive functional form assumptions, such as the Cobb-Douglas specification, which often lack empirical and theoretical justification. These assumptions can lead to biased estimates of matching efficiency and elasticity. To address this issue, Lange and Papageorgiou (2020) propose a nonparametric approach to identify the matching function and recover matching efficiency without imposing functional form restrictions.

Despite this advancement, the gap between nonparametric and Cobb-Douglas approaches remains, particularly in measuring labor mismatch. This paper aims to bridge this gap by evaluating its finite sample performance of both approaches to address practitioners' concerns, pointing out the bias of the Cobb-Douglas approaches under misspecification, and extending the mismatch index proposed by Şahin *et al.* (2014) to a nonparametric framework.

Several studies have assessed labor mismatch and its contribution to unemployment fluctuations using the Cobb-Douglas specification. Şahin *et al.* (2014) provide a methodology to quantify mismatch as the deviation between observed labor market outcomes and socially optimal outcomes. However, the Cobb-Douglas specification oversimplifies the mismatch index by canceling out time-varying matching efficiency and elasticities. As a methodological contribution, this paper combines the nonparametric approach with the mismatch index, allowing for nonparametrically estimated matching efficiency and elasticities. Additionally, I demonstrate how to compute the index using the Mathematical Programming with Equilibrium Constraint (MPEC) framework proposed by Su and Judd (2012). To address practitioners' concerns, I highlight the bias of the traditional Cobb-Douglas matching function and mismatch index and evaluate the finite sample performance of the proposed estimator via Monte Carlo simulation and empirical application.

My empirical application focuses on the Japanese labor market from 1972 to 2024, yielding two main findings. First, matching efficiency, normalized to 1972, exhibits a declining trend with notable fluctuations, aligning with downward trends in job and worker finding rates. This decline may reflect the increasing role of alternative matching opportunities outside the government-operated Hello Work platform. The implied match elasticity with respect to unemployment ranges from 0.5 to 0.9, consistent with global findings such as Petrongolo and Pissarides (2001) (0.5–0.7) and previous Japanese studies. In contrast, the elasticity with respect to vacancies is more volatile, ranging between -0.4 and 0.4, deviating from estimates by Lange and Papageorgiou (2020) (0.15–0.3) and Japanese studies (0.15–0.6).

Second, substantial mismatches are observed at both the prefecture and occupation levels, with an upward trend over time. The average mismatch across prefectures is 0.3, while across occupations, it ranges from 0.4 to 0.65—significantly larger than reported by Shibata (2020) and Higashi and Sasaki (2023). The proposed nonparametric mismatch index reveals the limitations of the Cobb-Douglas mismatch index, demonstrating its downward bias through both Monte Carlo simulations and empirical application. These findings underscore the importance of accounting for time-varying matching efficiency and elasticity in labor market analysis.

1.1 Related Literature

This paper contributes to the three strands of literature. First, I examine the trend of matching efficiency in Japanese labor markets via Hello Work nonparametrically using a novel approach (Lange and Papageorgiou 2020), which shows how to nonparametrically identify the matching function and estimate the matching function allowing for unobserved matching efficacy, without imposing the usual independence assumption between matching efficiency and search on either side of the labor market, allowing for multiple types of jobseekers. Lange and Papageorgiou (2020) highlight positive correlations between efficiency and market structure such as tightness and so on, which induces a positive bias in the estimates of the vacancy elasticity whenever unobserved matching efficacy is not controlled for, as is the case in the traditional Cobb-Douglas matching function with constant elasticity parameters.[1] I investigate the finite sample performance of the nonparametric approach to address practitioners' concerns regarding sample size, specification, and endogeneity issues, as Lange and Papageorgiou (2020) provides only the theoretical identification proof. I highlight the bias inherent in the Cobb-Douglas matching function when nonadditive matching efficiency is misspecified and confirm the potential advantages of the nonparametric approach. By implementing this method, I contribute updated results derived from the more flexible approach, complementing the existing findings of Kano and Ohta (2005), Kambayashi and Ueno (2006), Sasaki (2008), and Higashi (2018), which relied on the traditional Cobb-Douglas matching function with geographical and occupational category fixed effects to capture regional and occupational heterogeneity. Table 1 summarizes the previous findings. My findings are also useful for comparison with other countries' results reported in Bernstein *et al.* (2022) and Petrongolo and Pissarides (2001).

Second, this paper contributes to the literature on labor market mismatch, building on the foundational work of Şahin *et al.* (2014) extending the earlier analysis by Jackman and Roper (1987). In the static matching model with multiple sectors, Jackman and Roper (1987) demonstrate that aggregate hires are maximized when unemployment is distributed across sectors in a way that equalizes sectoral labor-market tightness. Şahin *et al.* (2014) extend this framework by incorporating a dynamic, stochastic environment with multiple sources of heterogeneity, providing a methodology to construct counterfactual measures of unemployment in the absence of mismatch. Their focus is on mismatch unemployment, defined as unemployed workers searching

[1] Brancaccio *et al.* (2020a, 2023) apply the method to estimate the matching function in exporter-ship transactions in the global bulk shipping market. Brancaccio *et al.* (2020b) summarize practical issues.

Table 1: Estimation results of the aggregate matching function in Japan

Paper	CRS	FE	Estimation	Sample	$\frac{d\log M}{dlogU}$	$\frac{d\log M}{dlogV}$
Kano and Ohta (2005)	yes	yes	OLS	1973–1999	0.56-0.59	0.29-0.3
Kambayashi and Ueno (2006)	no	yes	OLS	1996–2001	0.81	0.30
Sasaki (2008)	yes	no	OLS	1982–2016		0.38-0.45
Shibata (2013)	yes	no	GMM	2000–2013		0.4
Shibata (2020)	yes	no	GMM	2000–2019		0.34-0.4
Kawata *et al.* (2016)	yes	yes	IV	2000–2009		0.51-0.6
Kawata (2019)	yes	yes	OLS	2012–2016		0.52
Higashi (2018)	yes	no	OLS	2008–2016	0.37-0.46	0.17-0.24
Higashi (2020)	yes	no	OLS	2006–2016	0.48	0.17
Higashi (2021)	yes	no	OLS, 2SLS	2008–2018	0.22-0.46	0.15-0.31
Higashi and Sasaki (2023)	yes	yes	OLS	2016–2021	Calibrate	Calibrate
This paper	yes	no	Nonparametric	1972–2024	0.5-0.9	-0.4-0.4

Note: All papers except for this paper use a Cobb–Douglas matching function. Constant-returns-to-scale (CRS) imposes restriction on matching elasticity parameters in estimation. Fixed effect (FE) captures geographical-level and time-level heterogeneity. Estimation Instrument Variable (IV), Two-Stage-Least-Square (2SLS), and Generalized Method of Moments (GMM) based on Borowczyk-Martins *et al.* (2013) use instruments in the specific context. Blank cells mean no reported results.

in the "wrong" sector.[2]

The approach is widely used in a variety of contexts, for example, United Kingdom in Patterson *et al.* (2016). In the context of Japanese labor markets, several papers estimate mismatch at geographical- and occupation-level using Cobb-Douglas specification (Shibata (2013, 2020), Kawata *et al.* (2016), Higashi (2018), Kawata (2019), Higashi (2020, 2021), Higashi and Sasaki (2023)). This paper points out that Cobb-Douglas specification cancels out time variations of matching efficiency which induce bias of the mismatch index. Then, this paper proposes the nonparametric version of the mismatch index, and applies a novel computational method known as Mathematical Programming with Equilibrium Constraint (MPEC) proposed by Su and Judd (2012) and Dubé *et al.* (2012) to calculate the geographical- and occupation-level mismatch index.

Third, this paper contributes to the literature on the impact of COVID-19 on the matching efficiency of the worker-vacancy platform operated by the government. Due to the extensive

[2]Note that a distinct body of literature, such as Eeckhout and Kircher (2011), uses the term "mismatch" to describe employed individuals in suboptimal jobs, where the alignment between worker skills and firm capital is not ideal. Eeckhout (2018) provides an overview of this literature, identifying three primary causes of mismatch: search frictions (Eeckhout and Kircher 2011), stochastic sorting (Chade and Eeckhout 2016), and multidimensional types (Lindenlaub 2017). In addition, Guvenen *et al.* (2020) explores skill mismatch, which quantifies the gap between the skills required by an occupation and the abilities a worker possesses. Another approach, as in Shimer (2012), takes a flow perspective by decomposing unemployment volatility into transition rates between four labor force states: (i) permanent-contract employment, (ii) temporary-contract employment, (iii) unemployment, and (iv) non-participation, providing a foundation for calculating mismatch measures based on employment status.

volume of literature relevant to this topic, I will not provide a comprehensive list of all sources to maintain clarity and conciseness. The most closely related papers in my context are Higashi and Sasaki (2023) and Fukai *et al.* (2021). Higashi and Sasaki (2023) estimate mismatch indices for local labor markets clustered in by occupations vulnerable and not vulnerable to COVID-19 by assuming Cobb-Douglas matching function with calibrated matching elasticity parameters. Fukai *et al.* (2021) utilize the Labor Force Survey, a large-scale government statistic, to estimate the group average treatment effect of COVID-19 on employment status for each month from January to June 2020. To the best of my knowledge, this is the first paper to describe changes in matching efficiency before and after COVID-19 using a novel nonparametric approach.

2 Model

2.1 Nonparametric aggregate matching function

Our main interest is in matching efficiency and matching elasticity with respect to the number of unemployed workers and vacancies in the labor market via Public Employment Security Offices in Japan. A matching function based on search models plays a central role in labor economics.[3] The matching function relies on random search from both sides of the market, that is, individuals seeking jobs represent the supply of labor and recruiters represent the demand for labor. To estimate the matching function and recover matching efficiency, I follow the novel approach proposed by Lange and Papageorgiou (2020).[4] The paper points out the endogeneity problem of matching efficiency (Borowczyk-Martins *et al.* 2013) and the problem of too restrictive specification of a Cobb-Douglas matching function with the fixed matching elasticity then proposes nonparametric identification and estimation of matching efficiency under some conditions introduced later.

Let unscripted capital letters (A, U, V) denote random variables while realizations are subscripted by time t. I consider the matching function $m_t(\cdot,\cdot)$ that maps period-t unemployed workers U_t, per-capita search efficacy/matching efficiency of the unemployed workers A_t, and vacancies V_t into hires H_t. I assume that the underlying data generating process is stationary and that I observe a long enough time-series so that I can treat the joint distribution $G : \mathbb{R}^3_+ \to [0,1]$ of (H_t, U_t, V_t) as observed. Also, denote by $F(A, U)$ the joint distribution of A and U.

I identify the matching function as well as unobserved, time-varying matching efficiency, A. First, I assume that V and A are independent conditional on U, that is, $A \perp V \mid U$. Second, I assume that the matching function $m(AU, V) : \mathbb{R}^2_+ \to \mathbb{R}$ has constant returns to scale (CRS).[5]

[3] See Pissarides (2000), Petrongolo and Pissarides (2001), and Rogerson *et al.* (2005) for reference.

[4] Lange and Papageorgiou (2020) additionally incorporate search effort (Mukoyama *et al.* 2018) and recruitment index (Davis *et al.* 2013). Unfortunately, our Hello Work data does not report the information.

[5] To follow the original model of Matzkin (2003), I can rewrite $H = m(AU, V)$ into $H/U = m(A, V/U)$ under CRS. Here, H/U and V/U are known as a job-finding rate and market tightness. It is known that increasing

Then, by applying nonparametric identification results of Matzkin (2003), Proposition 1 of Lange and Papageorgiou (2020) proves that $G(H, U, V)$ identifies $F(A, U)$ and $m(AU, V) : \mathbb{R}^2_+ \to \mathbb{R}_+$ up to a normalization of A at one point denoted as A_0 of the support of (A, U, V).[6]

2.2 Nonparametric mismatch

I construct a nonparametric version of a mismatch index which is a measure of the fraction of hires foregone due to mismatch. Suppose there are L local markets in time t with a given number of vacancies, unemployed workers, and aggregate matching efficiencies in the economy. The number of hires $H_{\ell t}$ in market g in time t is determined by a matching function $m(A_{\ell t}U_{\ell t}, V_{\ell t})$. Note that, unlike Şahin *et al.* (2014), I allow any specification of m satisfying CRS and independence of $A_{\ell t}$ and $U_{\ell t}$ conditional on $V_{\ell t}$. Given matching efficiency $A_{\ell t}$ and the number of vacancies $V_{\ell t}$, a social planner maximizes the total hires in the economy by distributing a given number of unemployed workers to each labor market.

Following Şahin *et al.* (2014), I define mismatch as the deviation of the hires in the data from the efficient allocation chosen by the social planner. Under the assumptions of homogenous productivity and job separation rate across labor markets, the optimal allocation of unemployed workers satisfies the following equilibrium conditions:

$$\frac{\partial m}{\partial U}\left(A_{1t}U^*_{1t}, V_{1t}\right) = \cdots = \frac{\partial m}{\partial U}\left(A_{Lt}U^*_{Lt}, V_{Lt}\right) \tag{1}$$

where $*$ denotes the planner's allocation. The equilibrium condition means that keeping total unemployed workers $U_t = \sum_{\ell=1}^{L} U_{\ell t}$, the planner allocates more unemployed workers to more effective labor markets, that is, with more vacancies and higher matching efficiency until their marginal contribution to the hires is equalized across markets. The main difference from Şahin *et al.* (2014) is that equilibrium condition (1) does not have an analytical formula like Cobb-Douglas specification. In Section 3.2, I explain how to compute the optimal allocation of unemployed workers numerically.

Using optimal allocation of unemployed workers $U^*_{\ell t}$, the aggregate actual and optimal number

returns to scale in the matching process can create multiple equilibria and the subsequent literature has found scant evidence for increasing returns, as in (Petrongolo and Pissarides 2001).

[6]In Section 4, I investigate finite sample performance with Monte Carlo simulation. Based on simulation results with sample size $T = 50$, the sample size in this paper is enough to recover matching efficiency well. The code is available on the author's Github. Also, see Brancaccio *et al.* (2020b) for practical issues. The approach is used to estimate a matching function in a trade model (Brancaccio *et al.* 2020a, 2023).

of new hires can be expressed as

$$H_t = \sum_{\ell=1}^{L} m(A_{\ell t} U_{\ell t}, V_{\ell t}),$$
$$H_t^* = \sum_{\ell=1}^{L} m(A_{\ell t} U_{\ell t}^*, V_{\ell t}).$$

Using these expressions, I define mismatch index as

$$\mathcal{M}_t = 1 - \frac{H_t}{H_t^*} \tag{2}$$

where $\mathcal{M}_t$ measures the fraction of hires lost in period t because of misallocation. This index accounts for the heterogeneity of matching efficiencies across labor markets. I explicitly refer to the mismatch index (2) under the Cobb-Douglas matching function as the Cobb-Douglas mismatch index.[7]

2.3 Why is the nonparametric mismatch index superior to the Cobb-Douglas mismatch?

To illustrate the potential benefit of nonparametric mismatch index, consider L markets observed over T years. Estimating the Cobb-Douglas mismatch requires specifying a Cobb-Douglas matching function with additive matching efficiency as $H_{\ell t} = A_\ell A_t U_{\ell t}^{1-\gamma} V_{\ell t}^{\gamma}$ with $L+T+1$ parameters: L market fixed effects (A_l), T year fixed effects (A_t), and a constant vacancy share parameter (γ). The equilibrium condition (1) can be expressed as:

$$A_l A_t \left(\frac{V_{lt}}{U_{lt}^\star}\right)^\gamma (1-\gamma) = A_{l'} A_t \left(\frac{V_{l't}}{U_{l't}^\star}\right)^\gamma (1-\gamma).$$

This formulation highlights a critical limitation of the Cobb-Douglas mismatch: the time variations in A_t and the elasticity γ are canceled out in the equilibrium condition. In contrast, the proposed nonparametric approach estimates $T \times L$ time- and market-specific efficiencies and corresponding time-varying elasticities, thereby preserving the temporal variations of A_t and elasticity. This allows for greater flexibility and precision in capturing the dynamics of matching efficiency and mismatch across both time and markets.

While the nonparametric approach offers more flexibility, it also requires a larger sample size to ensure precise estimates. The finite sample performance of this approach is evaluated in Section 4.

[7] See Appendix A.2 for construction of the Cobb-Douglas mismatch index.

3 Estimation

3.1 Matching efficiency and elasticities

Following Lange and Papageorgiou (2020), I begin by estimating $F(A_0|U)$ across the support of U. To this end, I utilize the distribution of hires conditional on unemployed, U, and observed vacancies, V. Specifically, I have

$$F(A_0|\psi U_0) = G_{H|U,V}(\psi H_0|\psi U_0, \psi V_0) \quad \text{for any arbitrary scalar } \psi.$$
$$F(\psi A_0|\lambda U_0) = G_{H|U,V}(\psi H_0|\lambda U_0, \psi V_0) \quad \text{where } \lambda > 0 \text{ is a scaling factor}$$

where $F(A_0|\psi U_0)$ and $G_{H|U,V}$ are conditional distributions. By varying (ψ, λ), I can therefore trace out $F(A|U)$ across the entire support of (A, U).

Given our finite data, I rely on an estimate of $G_{H|U,V}$ for our constructive estimator. Consider an arbitrary point (H_τ, U_τ, V_τ). To obtain $G(H_\tau|U_\tau, V_\tau)$, I compute the proportion of observations with less than H_τ observed hires among observations proximate to (U_τ, V_τ) in (U, V)-space. Practically, this is achieved by averaging across all observations in the data, penalizing those with values (U_t, V_t) using a kernel that weighs down observations distant from (U_τ, V_τ). Consequently, our estimate is given by

$$F(\psi A_0|\lambda U_0) = G_{H|U,V}(\psi H_0|\lambda U_0, \psi V_0)$$
$$\hat{F}(\psi A_0|\lambda U_0) = \sum 1(H_t < \psi H_0)\kappa(U_t, V_t, \lambda U_0, \psi V_0)$$

where $\kappa(.)$ denotes a bivariate normal kernel with bandwidth 0.01.

Having recovered the distribution function $F(A|U)$, I invert $F(A_t|U_t)$ to derive A_t. This is achieved by

$$A_t = F^{-1}(G(H_t|U_t, V_t)|U_t)$$

for all observations (H_t, U_t, V_t) in the dataset. Finally, I recover the matching function as

$$m(A_t, U_t) = G^{-1}(F(A_t|U_t)|U_t).$$

Finally, for calculating matching elasticities, I run a LASSO regression projecting hires on the original and squared numbers of vacancies and unemployed interacted with implied matching efficiency. The estimates approximate the derivatives of the matching function with respect to vacancies and unemployed interacted with implied matching efficiency, that is, an estimate of the elasticity of the matching function.[8]

[8]The matching elasticity with respect to unemployed $\frac{d\log m(AU,V)}{d\log U} = \frac{dm(AU,V)}{dU}\frac{U}{H} = \frac{dm(AU,V)}{dAU}\frac{dAU}{dU}\frac{U}{H} = \frac{dm(AU,V)}{dAU}\frac{AU}{H} = \frac{d\log m(AU,V)}{d\log AU}$ is obtained from the regression coefficient of H on AU and multiplying it by $\frac{AU}{H}$. Concretely, I approximate m by the second order polynomial $m = \beta_1(AU) + \beta_2(AU)V + \beta_3 V + \beta_4(AU)^2 + \beta_5 V^2$

3.2 Mismatch

Unlike the original Cobb-Douglas specification of Şahin *et al.* (2014), equilibrium condition (1) does not have an analytical formula. Instead, I employ the Mathematical Programming with Equilibrium Constraint (MPEC) approach proposed by Su and Judd (2012) and Dubé *et al.* (2012). The constrained optimization problem in time t is defined as follows.

$$\max_{U_{1t},\cdots,U_{Lt}} \sum_{\ell=1}^{L} m\left(A_{\ell t}U_{\ell t}, V_{\ell t}\right), \tag{3}$$

$$\text{subject to } \frac{\partial m}{\partial U}\left(A_{1t}U_{1t}^{*}, V_{1t}\right) = \cdots = \frac{\partial m}{\partial U}\left(A_{Lt}U_{Lt}^{*}, V_{Lt}\right),$$

$$U_t = \sum_{\ell=1}^{L} U_{\ell t}.$$

Maximizing the objective function is equivalent to solving equilibrium conditions (1) so that the central planner's problem represents a nonlinear system of equations with linear equilibrium constraints. I solve the model by using `Ipopt.jl` and `JuMP.jl` which is often used for the MPEC approach. Note that the formulation allows us to incorporate a rich set of additional equilibrium constraints and elements such as the government's costs of moving unemployed workers from some markets to other markets for calibration studies, which is out of the scope of this paper.

4 Simulation

To address practitioners' concerns, I investigate the finite sample performance of our estimator, extending Section 4 of Matzkin (2003) to the aggregate labor market setting. The primary focus is on sample size, robustness to model specification, and sensitivity to endogeneity in estimating time-varying matching efficiency. I simulate 100 datasets with sample sizes $T = 50$, $T = 100$, and $T = 200$, representing aggregate-level data over 50, 100, and 200 months, respectively, and consider correlations between V and A, $\rho^{AV} = 0, 0.1, 0.2, 0.5$. Positive ρ^{AV} induces upward bias. A stationary AR(1) process is generated for the inputs as $U_t = \rho^U U_{t-1} + \varepsilon_t^U$, $V_t = \rho^V V_{t-1} + \varepsilon_t^V$, and $A_t = \rho^A A_{t-1} + \varepsilon_t^A$, with $(\rho^U, \rho^V, \rho^A) = (0.2, 0.2, 0.2)$, and $(\varepsilon_t^U, \varepsilon_t^V, \varepsilon_t^A)$ drawn i.i.d. from the standard normal distribution. If $\rho^{AV} > 0$, A_t is recomputed via Cholesky decomposition to achieve the desired correlation with V_t. Inputs are rescaled to mimic actual data by multiplying U_t and V_t by 100 and A_t by 10, normalizing A_t such that $A_1 = 100$.

Restricting the analysis to the CRS class, I specify matching function $m(A_tU_t, V_t)$ as a Cobb-Douglas function $((A_tU_t)^{\gamma}V_t^{1-\gamma})/10$ and a perfect substitute function $((\gamma A_tU_t + \gamma V_t)/10)$, where

and get $\frac{dm(AU,V)}{dAU} = \beta_1 + \beta_2 V + 2\beta_4(AU)$ and $\frac{d\log m}{d\log U} = \frac{d\log m}{d\log AU} = (\beta_1 + \beta_2 V + 2\beta_4(AU))\frac{U}{H}$ and $\frac{d\log m}{d\log V} = (\beta_2(AU) + \beta_3 + 2\beta_5 V)\frac{V}{H}$. For calculating mismatch, I need $\frac{dm}{dU} = \frac{d\log m(AU,V)}{d\log AU}\frac{H}{U}$ at $U = U^*$ and $H = H^* = m(AU^*, V)$.

elasticity parameter $\gamma = 0.3$. Data for the number of matchings H_t is generated using the true $m(A_tU_t, V_t)$. For nonparametric estimation, a Gaussian kernel with bandwidth $h = 0.1$ is applied. For comparison, a misspecified Cobb-Douglas model is estimated using OLS, i.e., $\log(10H_t) = \hat{A} + \hat{\gamma}\log(U_t) + (1-\hat{\gamma})\log(V_t)$, recovering $A = \exp\left(\frac{\hat{A}}{\hat{\gamma}}\right)$ as the normalized efficiency term.

Bias and root-mean-squared error (RMSE) of matching efficiency A_t are calculated at 50 randomly chosen fixed grid points. The averages of these results, across the 50 points and 100 simulations, are reported in Table 2. The results highlight differences between the nonparametric (NP) and ordinary least squares (OLS) approaches. When the sample size is $T = 200$, the NP method demonstrates robust performance with minimal bias (consistently below 1.5%) and relatively low RMSE, even under varying levels of ρ^{AV}. Conversely, the OLS approach exhibits substantial bias and significantly larger RMSE across all scenarios due to the misspecification of A_t as time-invariant. Furthermore, the inaccuracies in $\hat{A}$ and $\hat{\gamma}$ amplify errors in recovering A_t via log-transformation. These findings underscore the reliability of the NP method in capturing time-varying matching efficiency, particularly for larger sample sizes, and its modest robustness to endogeneity, contrasting with the limitations of the OLS approach.[9]

5 Data

First, I use the Report on Employment Service (*Shokugyo Antei Gyomu Tokei*) for the month-level aggregate data from January 1972 to April 2024. These datasets include the number of job openings, job seekers, and successful job placements, primarily sourced from the Ministry of Health, Labour and Welfare (MHLW) of Japan, which publishes monthly reports and statistical data on the Public Employment Security Office, commonly known as Hello Work. Hello Work is a government-operated institution in Japan that provides job seekers with employment counseling, job placement services, and vocational training, playing a critical role in Japan's labor market. The data is often used as in Kano and Ohta (2005), Kambayashi and Ueno (2006), Sasaki (2008), Kambayashi (2013), and Higashi (2018) estimating the traditional Cobb-Douglas matching function. Using up-to-date Hello Work data, Kawata and Sato (2021) construct a simple framework to quantitatively measure the impacts of an economic shock of COVID-19 on unemployed workers' welfare in their companion project.[10] The period for my dataset is selected to ensure the longest consistent timeframe available at the time of writing this paper. In Appendix A.1, I provide additional analysis using the year-level aggregate data in more extended periods, available from January 1963 to April 2024, instead of month-level one.

We study the labor markets for full-time and part-time workers and jobs. In the Hello Work

[9]Note that endogeneity problem can be resolved by applying the instrumental variable approach proposed by Imbens and Newey (2009), although finding the instruments is usually difficult in the aggregate labor market setting.
[10]`https://www.crepe.e.u-tokyo.ac.jp/material/crepecl12.html`: Accessed 2024 June 6.

Table 2: Monte Carlo simulation results of matching efficiency A_t for $t = 1, \cdots, T$

(a) Cobb-Douglas

Sample size	ρ^{AV}	Bias (NP)	RMSE (NP)	Bias (OLS)	RMSE (OLS)
50	0.0	1.28	14.50	2300.12	2370.36
50	0.1	1.96	14.73	917.18	1008.53
50	0.2	1.14	14.51	181.41	305.90
50	0.5	1.98	13.91	-86.73	93.84
100	0.0	-0.43	13.13	406.54	456.63
100	0.1	-0.92	12.97	68.66	143.96
100	0.2	-0.28	14.24	-29.17	105.86
100	0.5	-0.84	14.67	-94.58	94.93
200	0.0	-0.25	12.20	79.84	118.95
200	0.1	-1.01	11.93	-20.09	77.27
200	0.2	-0.82	12.93	-65.47	71.56
200	0.5	-1.13	10.88	-98.39	98.72

(b) Perfect substitute

Sample size	ρ^{AV}	Bias (NP)	RMSE (NP)	Bias (OLS)	RMSE (OLS)
50	0.0	3.51	9.33	269.67	357.98
50	0.1	2.88	11.27	310.58	441.84
50	0.2	2.93	11.58	-51.56	105.06
50	0.5	3.16	14.00	-98.41	98.75
100	0.0	1.78	7.78	-24.58	86.31
100	0.1	1.75	7.54	-58.95	85.38
100	0.2	1.51	8.92	-79.45	86.61
100	0.5	1.49	13.78	-98.69	99.03
200	0.0	0.90	6.97	-60.63	69.08
200	0.1	0.90	6.58	-69.45	77.17
200	0.2	1.41	7.73	-87.99	88.41
200	0.5	-0.77	13.37	-98.76	99.06

Note: NP = Nonparametric approach, OLS = Ordinary Least Square. The true value of matching efficiency A_t is normalized to 100 at $t = 1$. I calculate bias and root-mean-squared-error (RMSE) of matching efficiency A_t at randomly chosen 50 fixed grid points. The averages of these results, over the 50 points and 100 simulations are reported. The replication code is available on the author's Github.

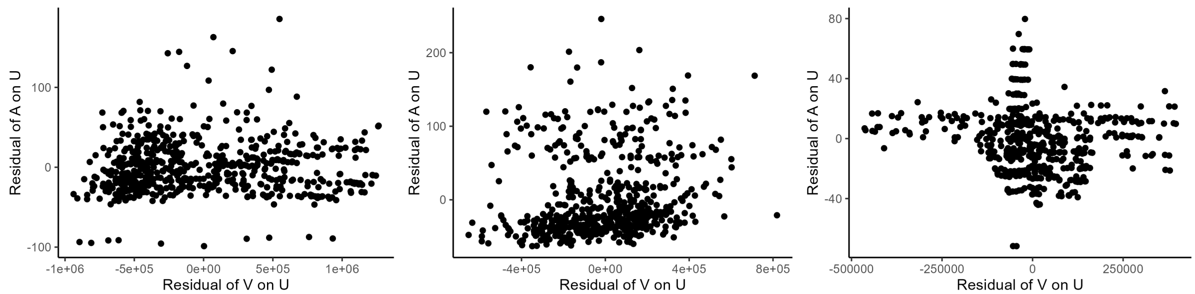
system, part-time workers are defined as those who work fewer hours than regular employees at the same establishment. MHLW classifies part-time workers into two categories: (1) regular part-time workers, who have contracts for an indefinite period or a period exceeding four months, and (2) temporary part-time workers, whose contracts last between one and four months or whose employment is fixed and typically tied to seasonal demand. New graduates and vacancies targeting them are excluded because new graduates are not eligible for the provision of unemployment insurance. While in many countries, part-time and full-time jobs are distinguished by the number of hours worked, in Japan, these terms often imply additional differences, such as variations in responsibilities, benefits, and flexibility. Part-time workers generally have fewer responsibilities and benefits but enjoy more flexible working hours compared to full-time employees. Full-time workers are those who are not part-time workers.

The three types of number of job openings, job seekers, and successful job placements are reported in the data. The first is the number for full-time and part-time workers and jobs. The second is the number for full-time workers and jobs. The third is the number for part-time workers and jobs. The first is the sum of the second and third. Note that data on unemployed, vacancies, and hires exclude workers newly graduated from college, and are used to calculate the unemployment rate for Japan. Using the three types of data, I can decompose the labor market features into full-time and part-time ones.

Second, to estimate worker-job mismatch nonparametrically across regions and occupation categories, I use submarket data from January 2012 to March 2023 for prefecture-level analysis, and up to March 2024 for occupation category-level analysis. The time frame, consistent with Kawata (2019) and Higashi (2018), is constrained by the revision of job classifications before and after 2012, which complicates accurate data connection. Finally, I analyze all 47 prefectures and 67 occupations, treating each as a submarket on a monthly basis.

6 Empirical Results

We apply the above estimation approach for each dataset. Before presenting the results, I test the assumption that vacancies are independent of matching efficiency, conditional on overall labor supply, i.e., $V \perp A \mid U$. Specifically, I use the residuals from a regression of vacancies V on the unemployed U, and similarly, the residuals of implied matching efficiency A on S. For the aggregate data, the correlation between these two residuals is close to zero (-0.04), indicating no systematic relationship between them. Conversely, for the full-time and part-time data, the correlations between the residuals are 0.12 and -0.14. Because the patterns appear to be influenced by some outliers as in Figure 1, the effect of violation of independence seems limited.



(a) Aggregate (b) Full-time (c) Part-time

Figure 1: Residual plot

6.1 Aggregate trends in 1972-2023

Figures 2 (a)-(d) provide monthly data patterns of unemployed individuals, vacancies, labor market tightness (V/U), hires, the (U,V) relationship, and job and worker finding rates (H/U and H/V) in aggregate labor markets.[11] In summary, the numbers of unemployed individuals and vacancies increase with fluctuations, corresponding with market tightness, while the number of hires shows a noticeable decline around the late 1970s, peaks and troughs from the mid-1980s to the late 1990s, peaks around mid-2000s and 2010, and a sharp decline towards recent years. Although the (U,V) relationship does not corresponds with Beveridge curve because these numbers are not divided by the corresponding number of labor force, the prominent rightward shifts are consistent with the U.S. studies (Elsby *et al.* 2015).

Figures 2 (e) and (f) present the estimation results of the matching function along with matching efficiency and elasticities ($\frac{d\ln m}{d\ln AU}$ and $\frac{d\ln m}{d\ln V}$). Notably, matching efficiency (normalized to 1972) shows a declining trend with notable fluctuations, which is consistent with the downward trends of job and worker finding rates. In particular, the significant decline after 2015 is remarkable. This seems due to an increase in matching opportunities outside of the government-operated platform, which is discussed in Otani (2024) using proprietary on-the-job search platform data and Kanayama and Otani (2024) using spot gig work platform data.

The implied match elasticity with respect to unemployment is 0.5-0.9, which is comparable to previous worldwide findings such as Petrongolo and Pissarides (2001) (range: 0.5-0.7) and Japanese studies such as Higashi (2018) (0.38 for 2000-2014 monthly), Kawata (2019) (0.48 for 2012-2017 prefecture-month-level), Kano and Ohta (2005) (0.56 for 1972-1999 prefecture-year-level), Sasaki (2007) (about 0.6 for 1998-2007 prefecture-quarter-level), and Kambayashi and Ueno (2006) (about 0.8 for 1996-2001 prefecture-month-level).[12] On the other hand, the implied match elasticity

[11] Worker finding rates are also known as vacancy yields. Davis *et al.* (2013) define vacancy yields in month t as the number of hires in month t (h_t) divided by the number of vacancies in the previous month (v_{t-1}) instead of the same month (v_t), the two series are very similar.

[12] For reference, Petrongolo and Pissarides (2001) summarize early aggregate studies in many countries based on

with respect to vacancies is 0.1-0.4 before 2015, which is comparable to Lange and Papageorgiou (2020) (range: 0.15-0.3) and Japanese studies such as Higashi (2018) (0.24 for 2000-2014 monthly), Kawata (2019) (0.52 for 2012-2017 prefecture-month-level), Kano and Ohta (2005) (0.3 for 1972-1999 prefecture-year-level), Sasaki (2007) (about 0.2 for 1998-2007 prefecture-quarter-level), and Kambayashi and Ueno (2006) (about 0.3 for 1996-2001 prefecture-month-level). However, the elasticity declines significantly after 2010, reaching close to -0.5 by 2020. The negative elasticity implies that an increase in vacancies results in a decreasing number of matches, which could reflect structural changes in the labor market during COVID-19. The recent sharp decline also highlights a substantial inefficiency in how vacancies are being filled in the labor market.

Figures 2 (g) and (h) illustrate some correlation patterns between matching efficiency and market structure variables such as labor market tightness, worker finding rate, and job finding rate. Consistent with Lange and Papageorgiou (2020), these highlight positive correlations between efficiency and market structure, such as tightness, which induce a positive bias in the estimates of the vacancy elasticity whenever unobserved matching efficiency is not controlled for, as is the case in traditional estimators.

6.2 Decomposition of part-time and full-time trends in 1972-2023

Next, I decompose the aggregate trends into full-time and part-time labor markets' trends. Figure 3 provide full-time and part-time labor markets' trends corresponding Figures 2 (a)-(f). Panel (a) shows that full-time unemployment and vacancies show higher counts and more significant fluctuations compared to part-time data and that part-time unemployment and vacancies exhibit a more gradual and steady trend over the years. Both full-time and part-time labor markets experience periods of high tightness during the same general periods, notably in the early 1990s. Part-time labor market tightness shows more pronounced peaks compared to the full-time labor market. Panel (b) shows that the sharp decline in full-time hires post-2008 suggests greater sensitivity to economic downturns and structural changes in the labor market. The steady rise in part-time hires indicates increasing reliance on part-time employment, possibly due to greater flexibility and adaptability in the labor market. Panel (c) illustrates that full-time employment data covers a wider range of both unemployed and vacancies, indicating higher variability in the full-time labor market in contrast to a more stable part-time labor market. Panel (d) shows that full-time finding rates show higher initial values and more significant declines compared to

a Cobb-Douglas matching function with the flow of hires on the left-hand side and the stock of unemployment and job vacancies on the right-hand side. In short, match elasticity with respect to unemployment is in the range of 0.5–0.7. Bernstein *et al.* (2022) review the recent empirical literature that estimates the matching elasticity in the U.S. by four types of specification; Cobb-Douglas, Cobb-Douglas with endogeneity correction, a constant elasticity of substitution (CES), and nonparametric (Lange and Papageorgiou 2020). The broad range of estimates is partly due to variations in data choices and periods. Higher estimates are typically derived from JOLTS data (Borowczyk-Martins *et al.* 2013, Şahin *et al.* 2014), while lower estimates are often obtained from CPS flows data (Barnichon and Figura 2015) or occupation-level hires from CPS data (Şahin *et al.* 2014).

part-time finding rates. Part-time finding rates demonstrate a more gradual decline and stable fluctuations over time. Full-time finding rates exhibit larger fluctuations and more pronounced declines, indicating greater sensitivity to economic changes. Part-time finding rates, while also showing initial declines, stabilize earlier and display less volatility, suggesting a more resilient part-time labor market.

Given estimation results in each dataset, Figure 4 (e) depicts the trend of implied matching efficiency. The downward trends are common. Full-time matching efficiency fluctuates from 80% to 300%, whereas part-time matching efficiency fluctuates from 60% to 300% compared to January 1972. This concludes that the sharp decline of matching efficiency after COVID-19 shown in the aggregate trend is driven by the decline of matching efficiency for both full-time and part-time workers.

Figure 4 (f) depicts the trends of matching elasticities. The elasticity with respect to unemployment for full-time workers shows moderate fluctuations, particularly between the 1970s and the early 1980s, where it oscillates between 0.4 and 0.8. After the 1980s, this elasticity stabilizes around 0.5. In contrast, the part-time elasticity with respect to unemployment follows a more unstable pattern, fluctuating between 0.1 and 0.9 with significant variations over time. This instability highlights the vulnerability of part-time workers labor markets to economic shocks, particularly during periods of recession, such as the COVID-19 pandemic. The larger fluctuations in elasticity suggest that changes in unemployment among part-time workers have had an uneven and more pronounced impact on the matching process compared to full-time workers.

Figure 4 illustrates some correlation patterns between matching efficiency and market structure variables corresponding with Figures 2 (g)-(h). I find stronger and more consistent relationships in the full-time labor market. This confirms that positive correlations between efficiency and market structure driven mainly by full-time markets.

6.3 Prefecture-level aggregate results in 2012-2024

Figure 5 displays the month-prefecture level matching efficiency, normalized to 2013 January Tokyo, across different regions of Japan from 2012 to 2024. Each panel shows the efficiency trends within a specific region, highlighting notable regional variations. For instance, the Tohoku region (panel b) and Chubu region (panel d), which are rural areas, exhibit relatively high volatility and generally higher efficiency values compared to other regions, suggesting greater variability in matching efficiency. In contrast, urban regions such as Kanto (panel c) and Kansai (panel e) show more stable and lower efficiency levels. The Tohoku area is about twice as efficient as the Tokyo area. These patterns suggest that regional economic factors and labor market conditions significantly influence matching efficiency across Japan.

Figures 6 and 7 illustrate month-prefecture-level matching elasticities with respect to unemployed and vacancies in 2012-2024. There is significant regional heterogeneity within the specific region. The matching elasticities with respect to unemployed and vacancies are 0.6-0.8 and 0.05-0.3 on average.

6.4 Occupation level results in 2012-2024

Figure 8 illustrates significant disparities in matching efficiency across occupations, normalized to 2013 January for clerical jobs. Primary and secondary industries show relatively higher efficiency than tertiary industries. For example, categories like "Agriculture, Forestry, and Fishing" (panel f) and "Construction and Mining" (panel i) in primary and secondary industries exhibit high volatility and generally higher efficiency levels. In contrast, occupations such as "Managerial" (panel a) and "Service" (panel e) display more stable and lower efficiency values. These variations highlight the diverse nature of labor market dynamics across different sectors, influenced by factors such as skill specialization, economic cycles, and industry-specific demand fluctuations.

Figures 9 and 10 illustrate matching elasticities with respect to unemployed and vacancies in each occupation in 2012-2024. As in the prefecture-level results, there is significant regional heterogeneity within the specific job area. The matching elasticities with respect to unemployed and vacancies fluctuates with a range generally between 0.1 and 6.0, which is larger than prefecture-level results. This indicates that matching heterogeneity depends on the definition of markets and matters in occupation level rather than geographical level.

6.5 Mismatch in 2012-2024

Finally, I compute the estimated nonparametric mismatch index and compare it to the Cobb-Douglas mismatch index using either estimated or calibrated matching elasticity.[13] Panel (a) of Figure 11 presents the nonparametric mismatch index across 47 prefectures from 2014 to 2024. The index, capturing the deviation of actual hires from the social planner's optimal level, reveals significant seasonal fluctuations, with an overall increase to around 0.3. It also displays a cyclical pattern, indicating the influence of seasonal factors or regional economic cycles on labor market dynamics. Notably, the mismatch level across prefectures is consistently higher than the Cobb-Douglas mismatch indices reported by Shibata (2020) (0.05–0.1) and Higashi and Sasaki (2023) (0.15–0.25). While I can partially replicate their findings using their calibrated matching efficiency values, the results highlight that the Cobb-Douglas mismatch index underestimates fluctuations and is biased upward due to its inability to account for time-varying matching efficiency and elasticity across prefectures.

Panel (b) of Figure 11 illustrates the mismatch index across occupations during the same period. Unlike the prefecture-based index, it shows a steady upward trend with minimal seasonal variation, reaching 0.6—substantially higher than the levels reported in Shibata (2020) and Higashi

[13] See Appendix A.2 for the Cobb-Douglas matching function estimation by OLS with fixed effects.

and Sasaki (2023). This discrepancy underscores the limitations of the Cobb-Douglas approach, which underestimates mismatch due to its exclusion of time-varying efficiency and elasticity across occupations. Using their calibrated matching efficiency values, I cannot replicate their results, in particular, their levels. Although tons of fixed effects estimated from my more recent data are different from the old estimates in existing studies, Panel (b) suggests that incorporating time-varying efficiency and elasticity plays a critical role in mitigating the downward bias of the Cobb-Douglas mismatch index, particularly at higher levels of disaggregation.[14]

The steady rise in the occupational mismatch index reflects a growing misalignment between the skills provided by job seekers and those demanded by employers. This trend likely stems from evolving industry demands, technological advancements, and a lag in workforce adaptation to new skill requirements. The sharper increase post-2020 may further indicate the exacerbation of skill mismatches due to the COVID-19 pandemic, driven by shifts such as accelerated digital transformation and uneven sectoral impacts.

7 Conclusion

This study highlights significant biases in the traditional Cobb-Douglas function when nonadditive, time-varying matching efficiency is misspecified, and evaluates the finite sample performance of the nonparametric estimation method proposed by Lange and Papageorgiou (2020). By extending the mismatch index of Şahin *et al.* (2014) to a nonparametric framework and developing a computational methodology, I analyze Japan's labor market using Hello Work data for unemployed workers from 1972 to 2024. The findings reveal a declining trend in matching efficiency, consistent with falling job and worker finding rates. Estimated elasticities range from 0.5 to 0.9 for unemployment and between -0.4 and 0.4 for vacancies, indicating greater volatility in vacancy elasticity. Moreover, the results show that occupational mismatch is more pronounced than geographical mismatch, with the traditional Cobb-Douglas mismatch index significantly underestimating the extent of mismatch. These findings underscore the importance of adopting nonparametric approaches to better capture time-varying matching efficiency, elasticity, and mismatch.

Future research should focus on expanding this analysis to other private platforms and exploring individual-level behavior, as discussed in studies like Kambayashi *et al.* (2025) and Roussille and Scuderi (2023), to provide a more comprehensive understanding of labor market dynamics.

[14] This property about the levels of disaggregation is further discussed in online Appendix A.A5 of Şahin *et al.* (2014).

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A Appendix

A.1 Year-level trends in 1966-2023

Figures 12 provide a year-level counterpart to Figure 2. The findings in the main text remain valid.

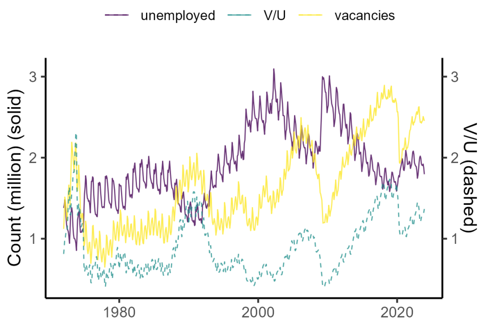
A.2 The Cobb-Douglas mismatch index and matching function

Under the Cobb-Douglas matching function with additive matching efficiency, i.e., $H_{\ell t} = A_\ell A_t U_{\ell t}^{1-\gamma} V_{\ell t}^{\gamma}$, the equilibrium condition (1) can be rewritten as $\frac{V_{\ell t}}{U_{\ell t}^*} = \left(\frac{A_{\ell'}}{A_\ell}\right)^{\frac{1}{\gamma}} \frac{V_{\ell' t}}{U_{\ell' t}^*}$. The optimal nationwide number of new hires is then derived as $H_t^* = A_t U_t^{1-\gamma} V_t^{\gamma} \left[\sum_{\ell=1}^{L} A_\ell \left(\frac{U_{\ell t}}{V_t}\right)^{1-\gamma} \left(\frac{V_{\ell t}}{V_t}\right)^{\gamma}\right]$. Substituting this into the rewritten equilibrium condition yields $H_t^* = \bar{A}_t A_t U_t^{1-\gamma} V_t^{\gamma}$, where $\bar{A}_t = \left[\sum_{\ell=1}^{L} A_\ell^{\frac{1}{\gamma}} \left(\frac{V_{\ell t}}{V_t}\right)\right]^{\gamma}$. Using this, the Cobb-Douglas mismatch index takes an analytical form as $\mathcal{M}_t = 1 - \frac{H_t}{H_t^*} = 1 - \sum_{\ell=1}^{L} \left(\frac{A_\ell}{A_t}\right) \left(\frac{U_{\ell t}}{U_t}\right)^{1-\gamma} \left(\frac{V_{\ell t}}{V_t}\right)^{\gamma}$. When matching efficiencies are identical across labor markets, the Cobb-Douglas mismatch index is equivalent to the conventional mismatch index proposed by Jackman and Roper (1987). The detailed property is shown in Şahin *et al.* (2014).

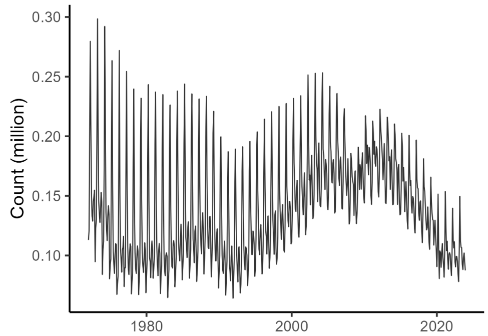
Table 3 presents the estimation results of the Cobb-Douglas matching function. Column (1) employs prefecture and month fixed effects, while Column (2) uses occupation and month fixed effects. The elasticity with respect to unemployment (log(U)) is estimated at 0.266 in Column (1) and 0.510 in Column (2), indicating greater sensitivity in the occupation-level specification. Similarly, the elasticity with respect to vacancies (log(V)) is 0.319 in Column (1) and 0.546 in Column (2), suggesting a stronger response at the occupation level.

Table 3: Cobb-Douglas matching function estimation results

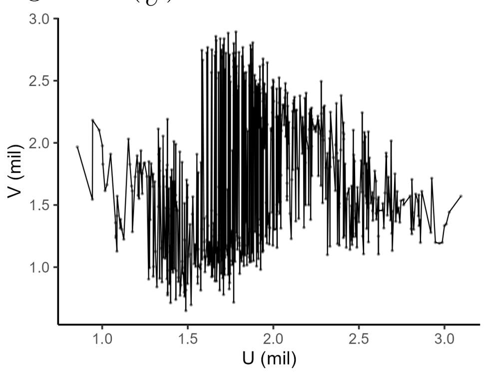
	(1)	(2)
Dependent var	log(H)	log(H)
log(U)	0.266	0.510
	(0.019)	(0.013)
log(V)	0.319	0.546
	(0.014)	(0.010)
FE	pref, month	occupation, month
Num.Obs.	6392	8908
R2	0.987	0.992
R2 Adj.	0.987	0.992



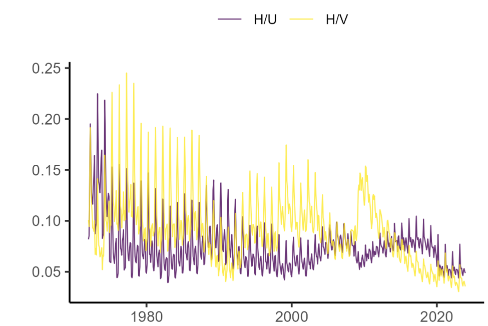
(a) Unemployed (U), Vacancy (V), and Tightness ($\frac{V}{U}$)



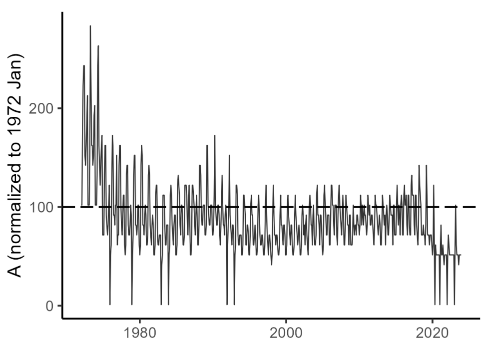
(b) Hire (H)



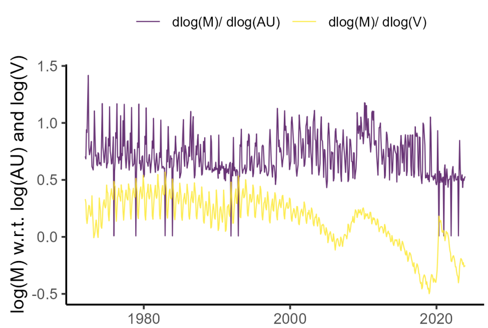
(c) (U,V) relationship



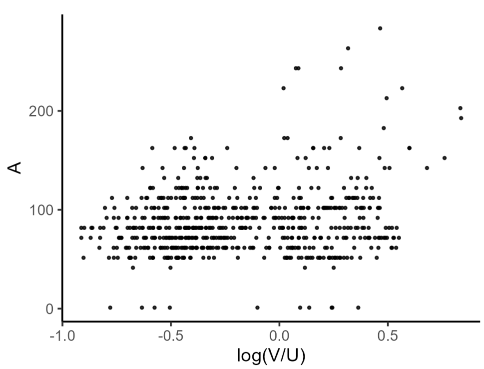
(d) Job Worker finding rate ($\frac{H}{U}$,$\frac{H}{V}$)



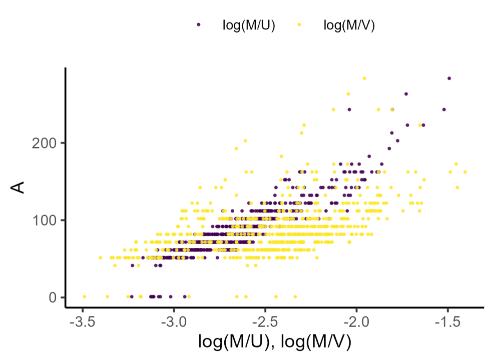
(e) Matching Efficiency (A)



(f) Matching Elasticity ($\frac{d\ln m}{d\ln AU}$, $\frac{d\ln m}{d\ln V}$)

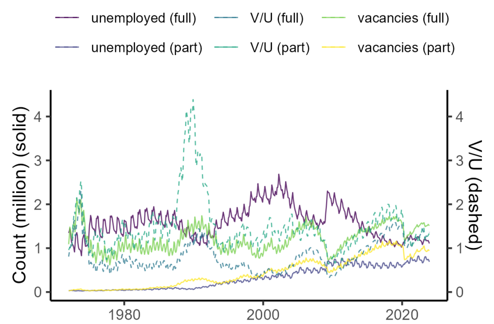


(g) Efficiency (A) and Tightness ($\ln\frac{V}{U}$)

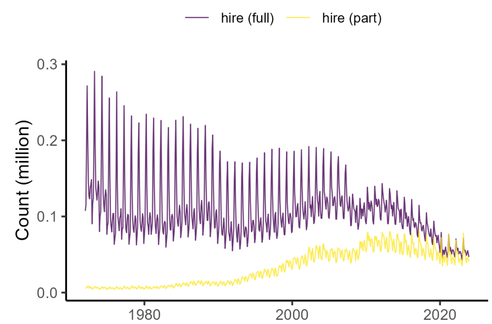


(h) Efficiency (A) and ($\ln\frac{H}{U}$, $\ln\frac{H}{V}$)

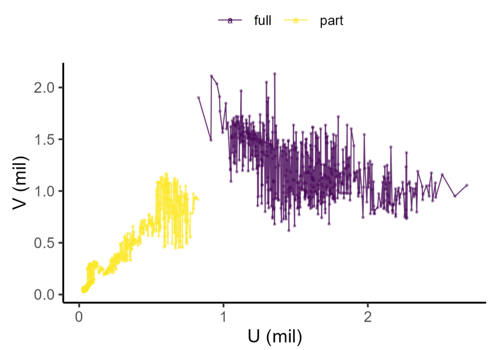
Figure 2: Month-level aggregate results 1972-2024



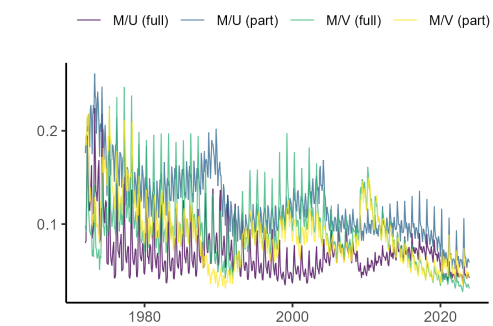
(a) Unemployed (U), Vacancy (V), and Tightness ($\frac{V}{U}$)



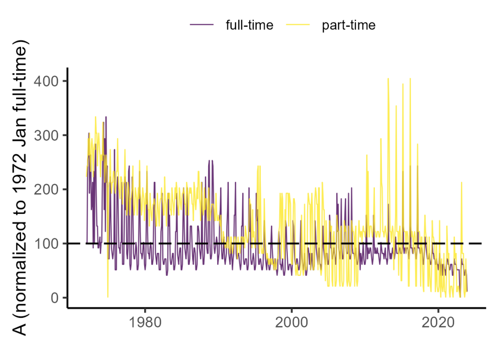
(b) Hire (H)



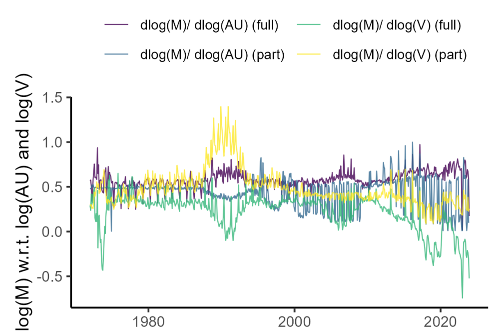
(c) (U,V) relationship



(d) Job Worker finding rate ($\frac{H}{U}$,$\frac{H}{V}$)

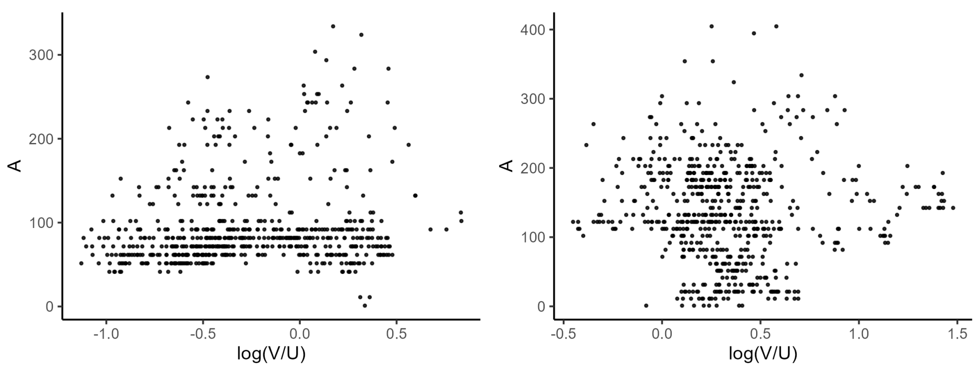


(e) Matching Efficiency (A)

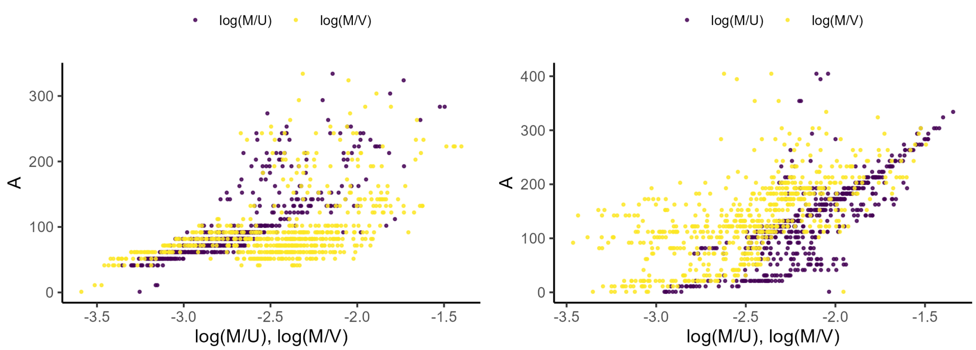


(f) Matching Elasticity ($\frac{d \ln m}{d \ln AU}$, $\frac{d \ln m}{d \ln V}$)

Figure 3: Month-level full-time and part-time results 1972-2024

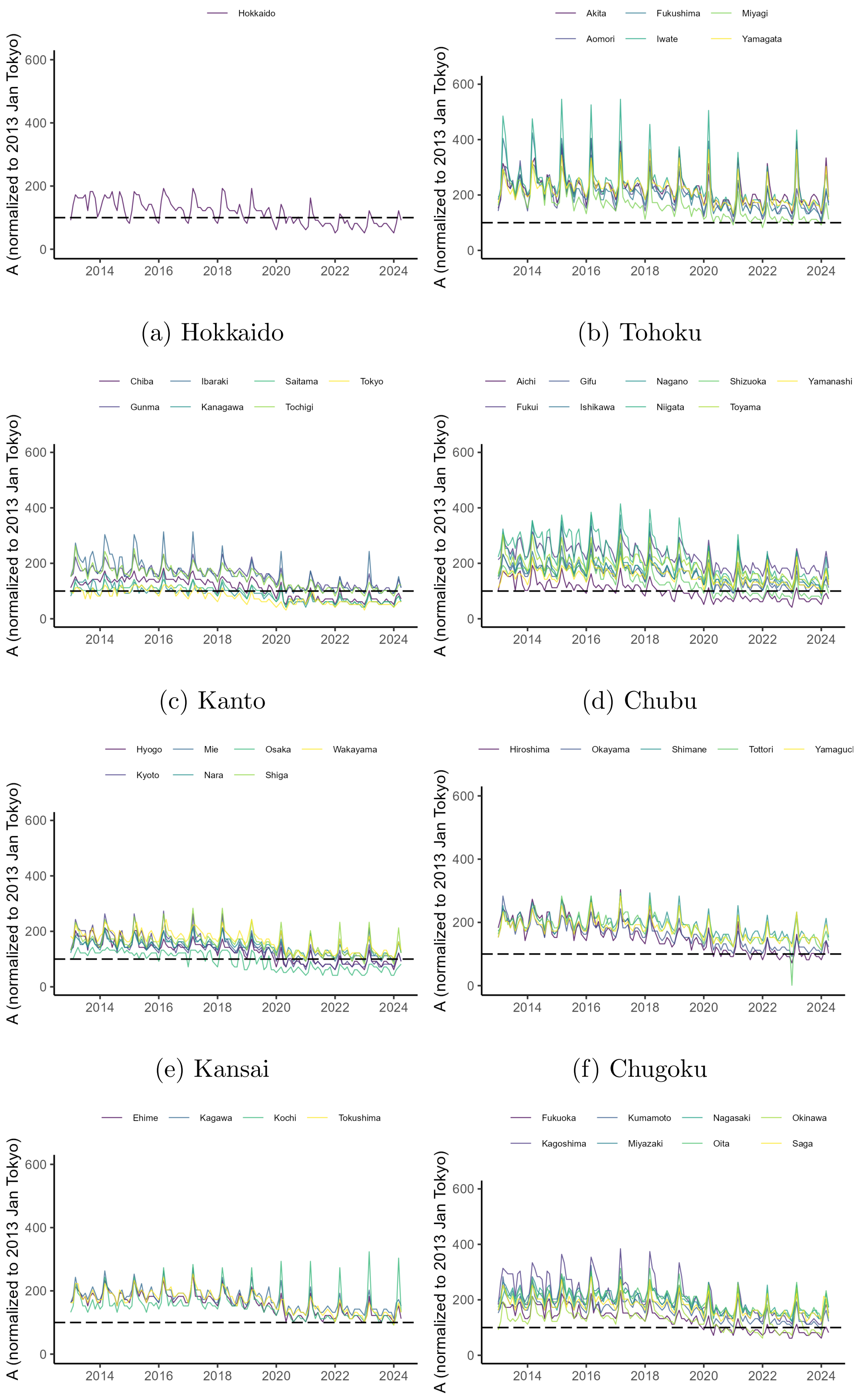


(a) Efficiency (A) and Tightness ($\ln \frac{V}{U}$), Full-time (left) and Part-time (right)



(b) Efficiency (A) and ($\ln \frac{H}{U}$, $\ln \frac{H}{V}$), Full-time (left) and Part-time (right)

Figure 4: Month-level full-time and part-time results 1972-2024 (Continued)



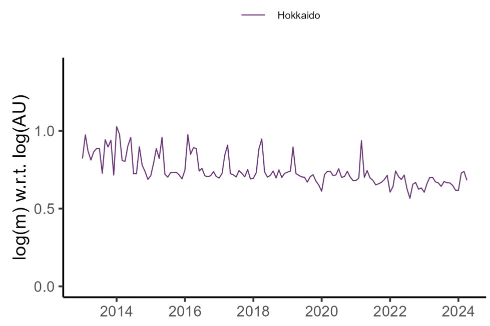
(a) Hokkaido (b) Tohoku

(c) Kanto (d) Chubu

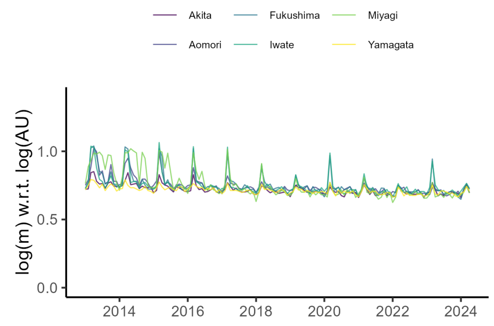
(e) Kansai (f) Chugoku

(g) Shikoku (h) Kyusyu, Okinawa

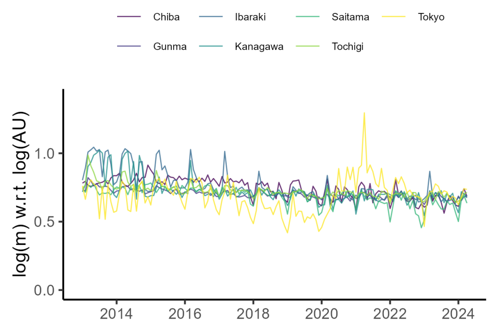
Figure 5: Month-prefecture level matching efficiency 2012-2024



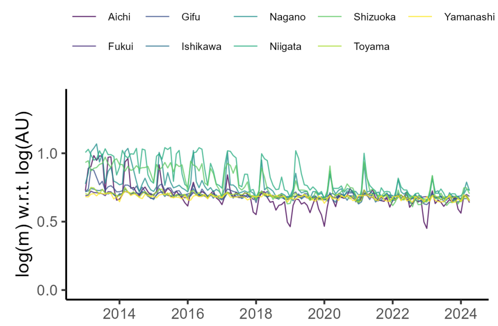
(a) Hokkaido



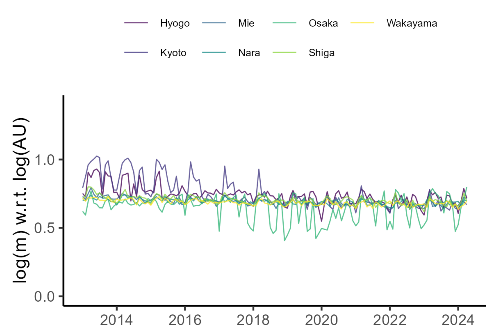
(b) Tohoku



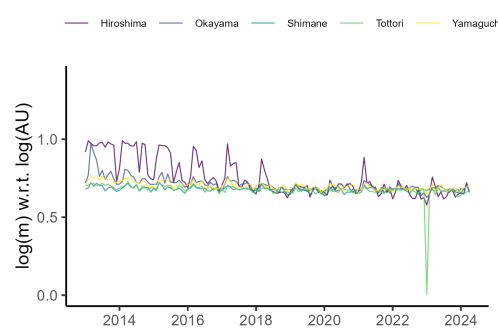
(c) Kanto



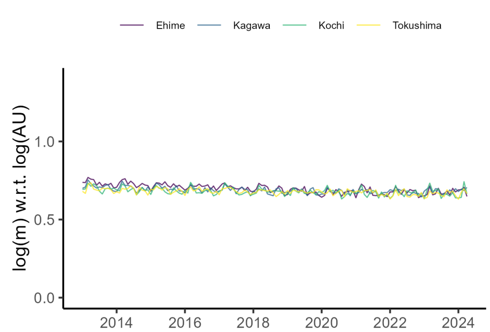
(d) Chubu



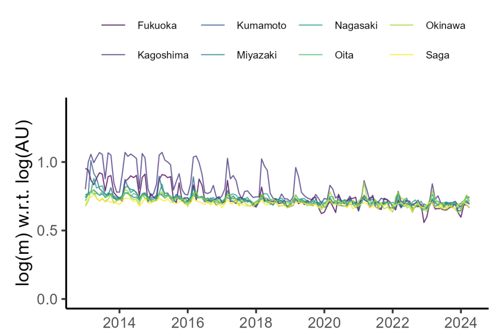
(e) Kansai



(f) Chugoku

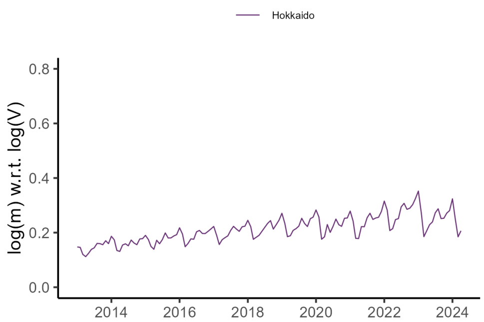


(g) Shikoku

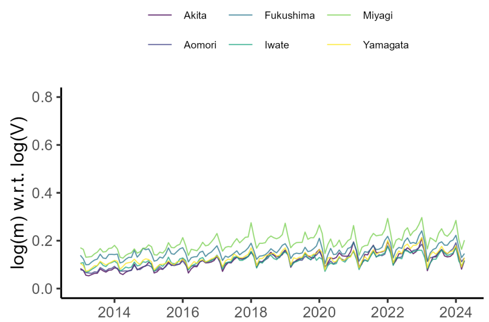


(h) Kyusyu, Okinawa

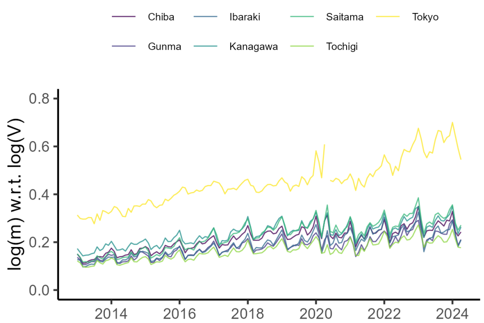
Figure 6: Month-prefecture level matching elasticity with respect to unemployed 2012-2024



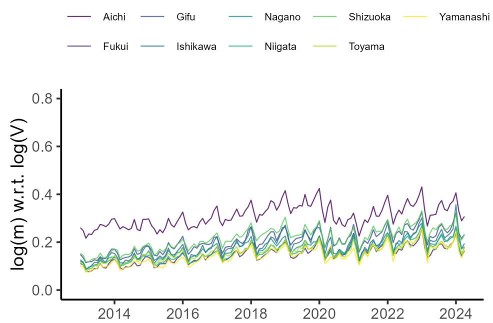
(a) Hokkaido



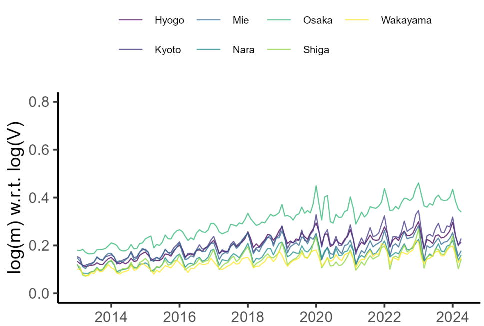
(b) Tohoku



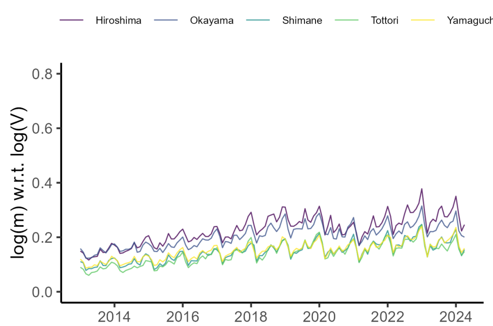
(c) Kanto



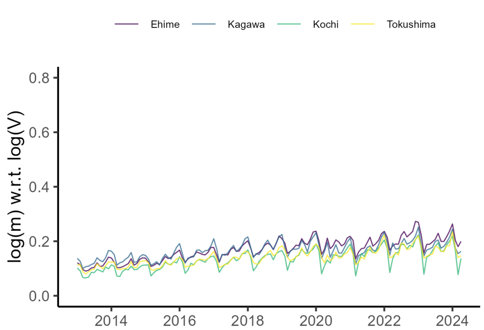
(d) Chubu



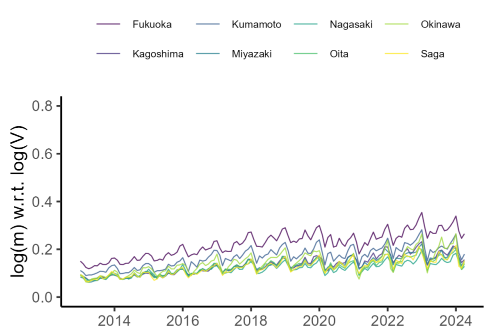
(e) Kansai



(f) Chugoku

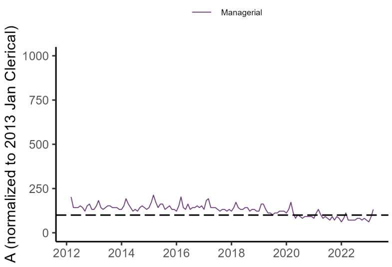


(g) Shikoku

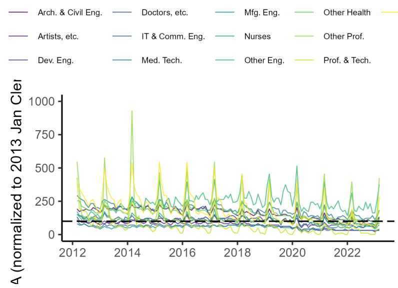


(h) Kyusyu, Okinawa

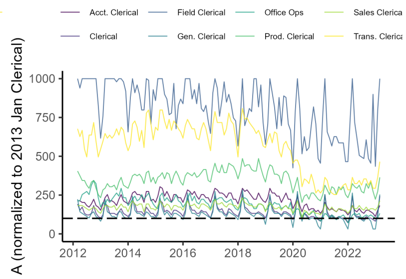
Figure 7: Month-prefecture level matching elasticity with respect to vacancies 2012-2024



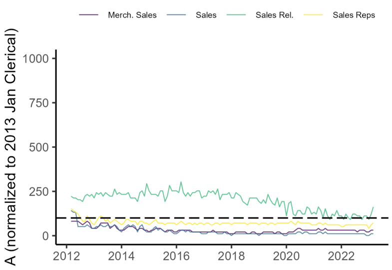
(a) managerial



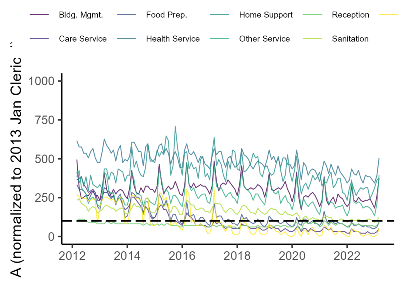
(b) professional and technical



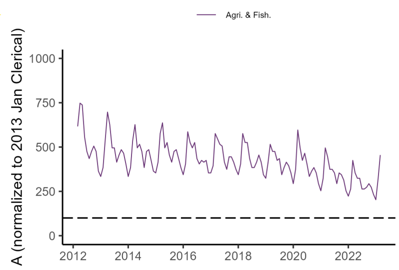
(c) clerical



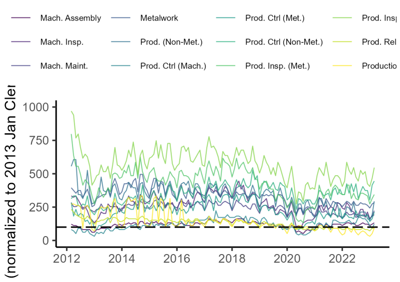
(d) sales



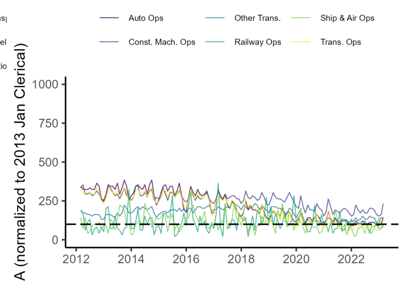
(e) service



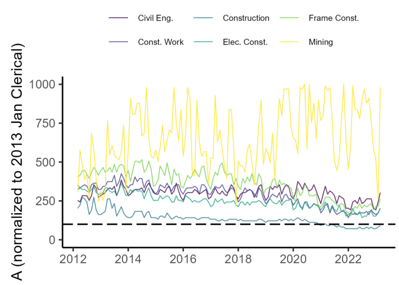
(f) agriculture forestry and fishing



(g) production

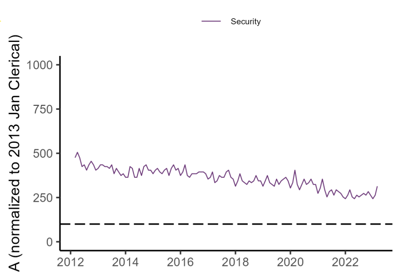


(h) transportation and machine operation



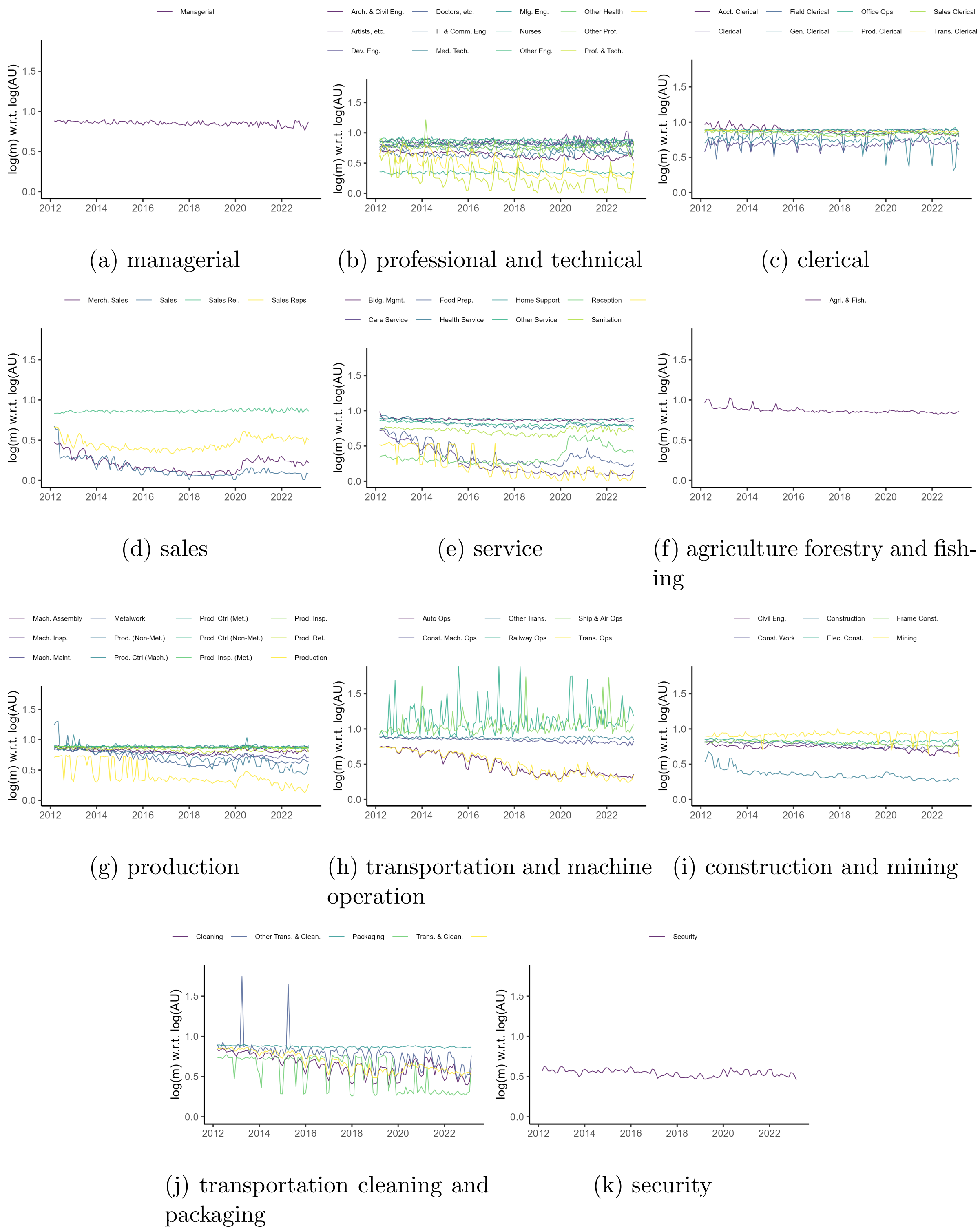
(i) construction and mining

(j) transportation cleaning and packaging



(k) security

Figure 8: Month-occupation level matching efficiency results 2012-2024



(a) managerial

(b) professional and technical

(c) clerical

(d) sales

(e) service

(f) agriculture forestry and fishing

(g) production

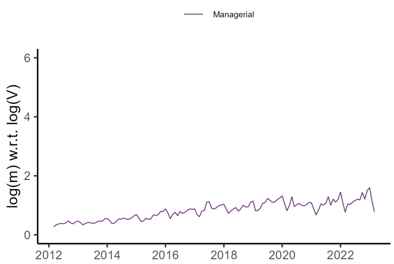
(h) transportation and machine operation

(i) construction and mining

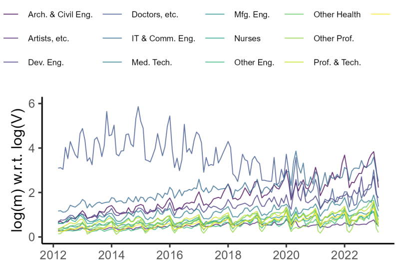
(j) transportation cleaning and packaging

(k) security

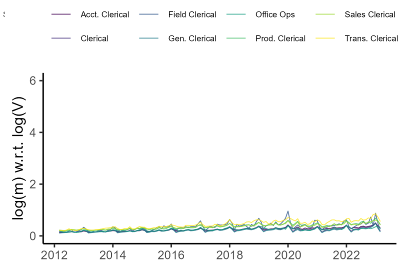
Figure 9: Month-occupation level matching elasticity with respect to unemployed 2012-2024



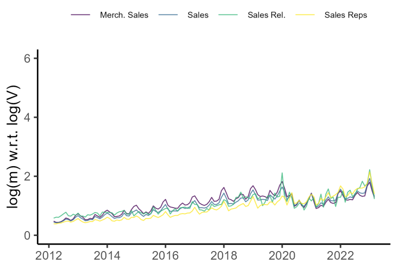
(a) managerial



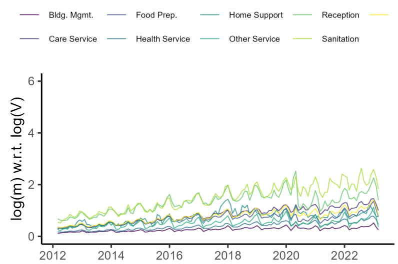
(b) professional and technical



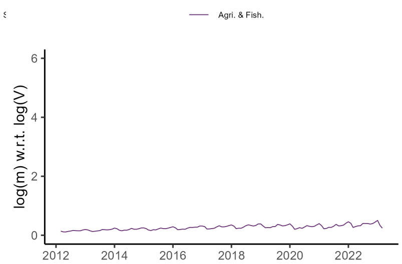
(c) clerical



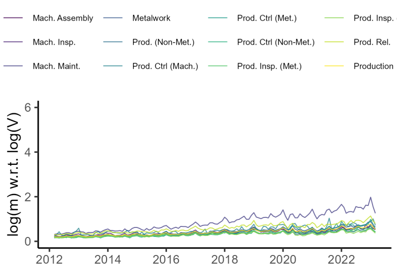
(d) sales



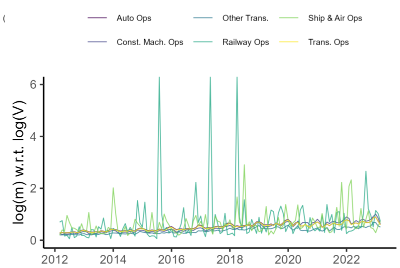
(e) service



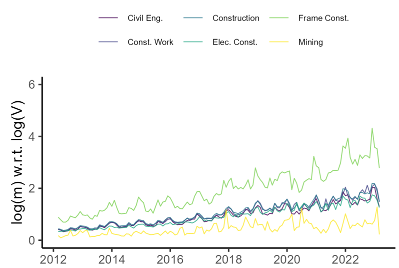
(f) agriculture forestry and fishing



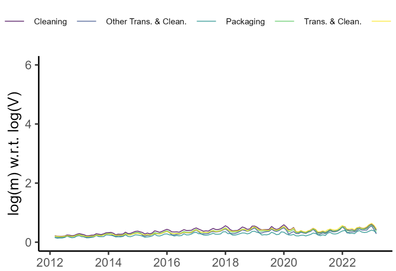
(g) production



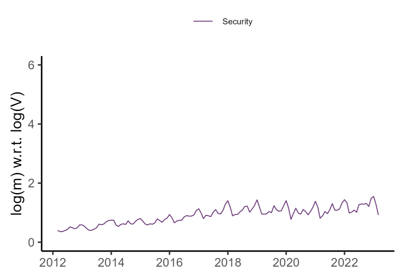
(h) transportation and machine operation



(i) construction and mining

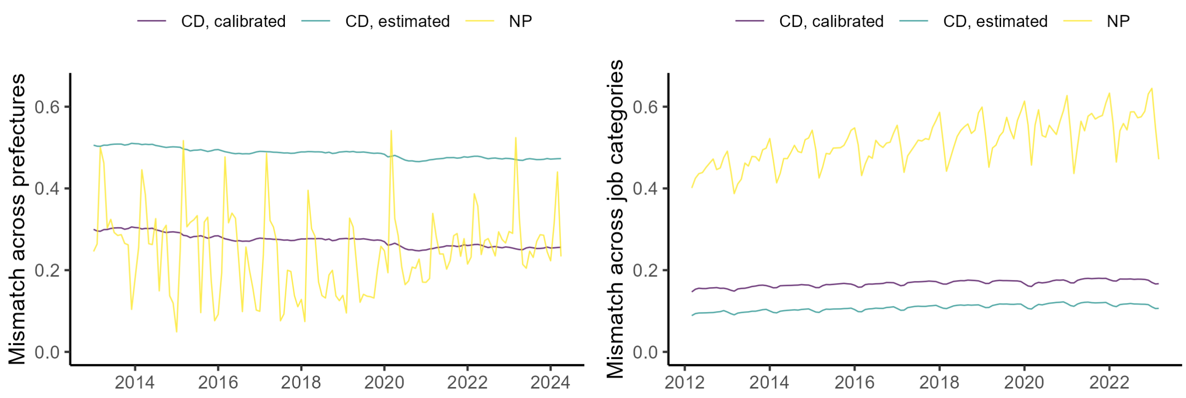


(j) transportation cleaning and packaging



(k) security

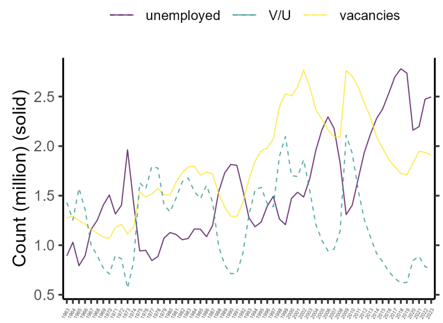
Figure 10: Month-occupation level matching elasticity with respect to vacancies 2012-2024



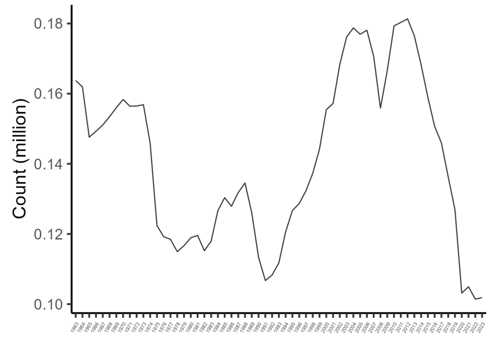
(a) Across prefectures (b) Across occupations

Figure 11: Estimated nonparametric mismatch index

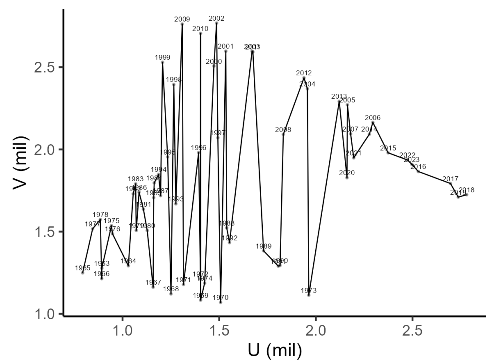
Note: CD, estimated = Cobb-Douglas mismatch index with estimated matching elasticity concerning vacancies, as in Şahin *et al.* (2014), Kawata (2019), and Shibata (2020). CD, calibrated = Cobb-Douglas mismatch index with calibrated matching elasticity set to 0.5 (Higashi and Sasaki 2023, Kawata 2019) across prefectures and 0.3 (Shibata 2020) across occupations for comparison with existing studies. NP = proposed nonparametric mismatch index.



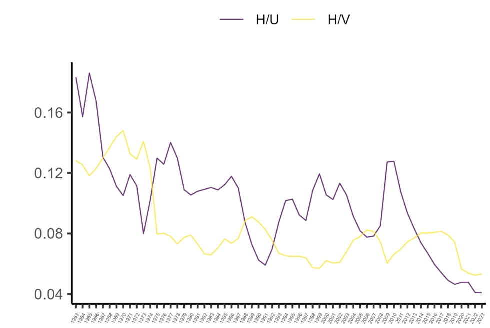
(a) Unemployed (U), Vacancy (V), and Tightness ($\frac{V}{U}$)



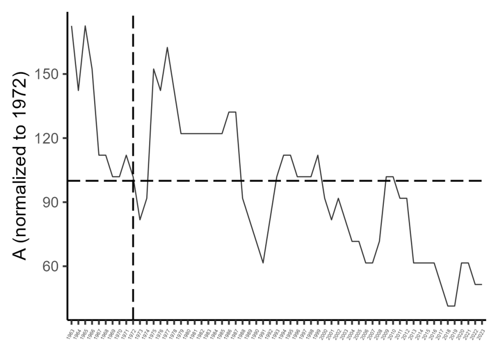
(b) Hire (H)



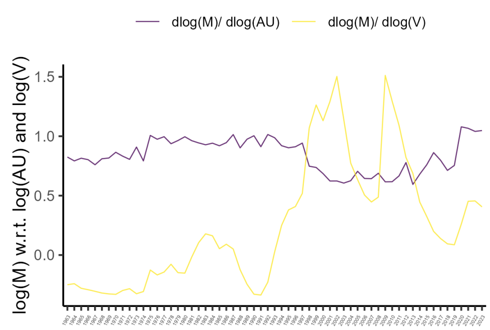
(c) (U,V) relationship



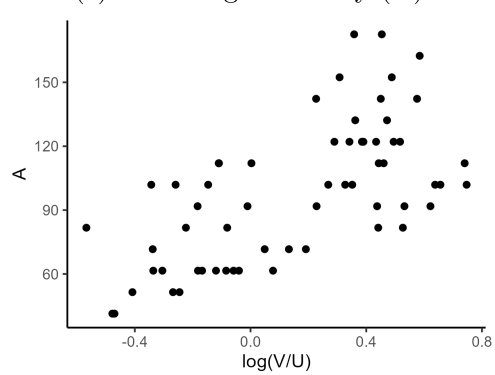
(d) Job Worker finding rate ($\frac{H}{U}$, $\frac{H}{V}$)



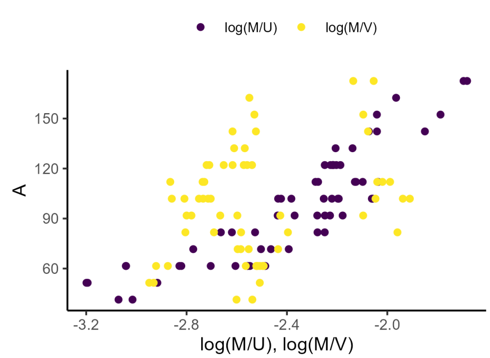
(e) Matching Efficiency (A)



(f) Matching Elasticity ($\frac{d\ln m}{d\ln U}$, $\frac{d\ln m}{d\ln V}$)



(g) Efficiency (A) and Tightness ($\ln \frac{V}{U}$)



(h) Efficiency (A) and ($\ln \frac{H}{U}$, $\ln \frac{H}{V}$)

Figure 12: Year-level results 1966-2024