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Equal Pay for *Similar* Work

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Abstract

Equal pay laws increasingly require that workers doing "similar" work are paid equal wages within firm. We study such "equal pay for similar work" (EPSW) policies theoretically and test our model's predictions empirically using evidence from a 2009 Chilean EPSW. When EPSW only binds across protected class (e.g., no woman can be paid less than any similar man, and vice versa), firms segregate their workforce by gender. When there are more men than women in a labor market, EPSW increases the gender wage gap. By contrast, EPSW that is not based on protected class can decrease the gender wage gap.

1 Introduction

> "No employee with status within one or more protected class or classes shall be paid a wage at a rate less than the rate at which an employee without status within the same protected class or classes in the same establishment is paid for... **similar work** [emphasis added]"
>
> -New York Labor Code, Section 194

Firms have some degree of wage-setting power in many labor markets (see, e.g., Card, 2022; Manning, 2005). Because of this, they may pay workers different relative salaries in ways that are repugnant to society at large. In particular, wage gaps between groups of workers, often men and women, are frequent rallying points for governmental action. A popular form of legislation seeks to prohibit firms from paying disparate wages to different workers, guided by the principle of "equal pay for equal work" (EPEW). In the United States, 49 states had EPEW laws in effect in 2015, requiring each firm to pay equal wages to all of its workers doing equal work.

However, EPEW may be difficult to enforce; "equal" pay is straightforward to define, but it is likely that no two workers are exactly "equal" within a firm. Firms can avoid the intent of these laws by pointing out differences between workers or making other maneuvers such as job title

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proliferation to marginally heterogenize their workforce (Baron and Bielby, 1986; Goldin, 1990).[1] To combat this enforceability issue, many EPEW laws have been updated to include a measure of coarseness–they require a firm to set "equal pay for similar work" (EPSW).[2] California was the first state in the US that moved from EPEW to EPSW in 2015, and as of January 2023, more of the US workforce is under the jurisdiction of a state EPSW law than a state EPEW law.[3] The equal pay provision in EPSW is frequently group based in that it binds only across groups of workers, prohibiting for example, that a man is paid more than a "similar" woman (and vice versa).

Despite the rapid growth of EPSW laws, little is known about their effects on labor market outcomes. Since EPSW is more constraining on firms than EPEW, EPSW may lead to a larger direct effect on wages. But how will firms adjust their employment policies to adapt in equilibrium? How will potential employment changes affect the goal of ensuring fairer pay?

We theoretically and empirically study the labor market effects of EPSW. Our findings suggest that the equilibrium effects of group-based EPSW overwhelm the direct effects, leading to increased occupational segregation and a shift in the wage gap in favor of the majority group of workers in a labor market. Therefore, these policies may counterintuitively exacerbate the problem they were intended to solve. We show that modifying EPSW to remove protected classes may have more positive effects.

We develop a theoretical framework to elucidate key economic forces at play. We begin by describing the simplest version of our model to derive key intuitions, and later extend our results to more general settings. In the basic model, there exist two homogeneous firms competing for a continuum of heterogeneous workers who all perform "similar" work in the eyes of the law. Each worker belongs to one of two groups, A or B (e.g., men or women), and is endowed with a "productivity" drawn from a distribution that potentially differs by group identity. Each firm has a constant-returns-to-scale production function and faces a wage monotonicity constraint to limit worker incentives to shirk: wages paid must be non-decreasing in productivity within group.

Several remarks are in order regarding our modeling decisions. First, we are agnostic about the underlying fundamentals of worker productivity, and therefore the differences in distributions between groups; we interpret productivity as a measure of firm willingness to pay for an individual worker net of any unmodeled, discriminatory factors. For example, in consumer-facing industries where consumers have discriminatory preferences in favor of a particular group of workers (Bar and Zussman, 2017; Holzer and Ihlanfeldt, 1998; Kelley et al., 2023; Kline et al., 2022), we may expect that group's productivity distribution to first-order stochastically dominate the other group's distribution. Similarly, taste-based bias by (managers of) firms can be incorporated. Second, even though our model assumes complete information in which each worker's productivity is common knowledge, a worker's productivity can instead be interpreted as the expected productivity at the time the hiring decision is made. Therefore, we can similarly include statistical discrimination into our framework. Third, our model assumes that all workers perform "similar" work in the eyes of the law.[4] Thus, our theoretical predictions should be viewed as applying within "job" in

[1] For example, a manufacturer told the *Washington Post* (1964) that his firm would "downgrade some job classifications for women and reassign higher-level, higher-paying duties to men" in response to EPEW. We are grateful to Martha Bailey for bringing this article to our attention.

[2] See Guppy and Vincent (2021) for a discussion of the transition in Canadian law—and the differences in allowable pay discrepancies—from EPEW to EPSW.

[3] The percent of the US workforce that is under the jurisdiction of EPSW and EPEW, respectively, are 45.9 and 45.6, respectively. These figures are calculated from 1) finding all states covered by each of these policies (see `https://www.dol.gov/agencies/wb/equal-pay-protections`), and 2) the share of the US workforce employed in each state (see `https://www.statista.com/statistics/223669/state-employment-in-the-us/`).

[4] We highlight that our modeling choices preclude an alternative where two workers are deemed "similar" if their productivities are within some distance. This is because (as we have discussed in this paragraph) we are agnostic about what "productivity" represents and, in particular, may include subjective factors not tied to an objective

a particular labor market, and should not be used to predict differential effects on, for example, custodians and lawyers working within the same firm.

Our model analysis reveals important effects of EPSW on worker sorting across firms. Without EPSW, each worker can be hired by either firm in equilibrium, regardless of group identity or productivity. Similarly to the classic Bertrand model, firms compete fiercely for each worker, and as a result, the average gap in pay across groups A and B is equal to the difference in average productivity between the groups. Thus, any discriminatory factors affecting firms' willingness to pay are exactly reflected in the wage gap. This result is obtained in the presence of rigidities imposed by the wage monotonicity constraint, suggesting that the "Bertrand" prediction may be more likely in unregulated labor markets than previously considered.

With EPSW, we show that firms segregate the workforce, with one firm hiring all $A-$group workers and the other hiring all $B-$group workers. To understand why this is the case, note that because each firm hires from only one group of workers, no firm is exposed to the constraint of equal pay in equilibrium. By contrast, EPSW makes poaching workers from its competitor costly: EPSW requires equal pay to any two workers from different groups and, by transitivity, this implies that equal wages must be paid to *all* workers it hires. Thus, EPSW serves as the enforcement mechanism for segregation, similarly to location choices in Hotelling's competition model.

The aforementioned analogy to the Bertrand and Hotelling models helps explain why EPSW leads to workforce segregation, but it leaves unanswered the question of the effect of EPSW on the wage gap. To address this question, we develop new analytical tools. First, we characterize the set of equilibrium outcomes under EPSW, which reveals novel economic forces and provides implications for policy. We show that (in any outcome involving worker segregation) three conditions on the wage schedules of the firms hold if and only if the wages are supported in an equilibrium outcome: (i) individual rationality—no firm pays a worker more than her productivity, (ii) equal profit across the firms, and (iii) a novel no desegregation condition—it is not profitable for a firm to pay any common wage and hire workers from both groups, possibly by poaching from the other firm. Second, given a wage function of a firm, say wage $w_2(\cdot)$ of firm 2, we define a wage function of the other firm, say function $\hat{w}_1(\cdot)$, that is lowest among all those that make it unprofitable to desegregate, i.e., $\hat{w}_1(\cdot)$ is the cheapest wage function that hires all $A-$group workers and satisfies the no desegregation condition. We establish that an equilibrium outcome with $w_2(\cdot)$ exists if and only if the profit of firm 1 under $\hat{w}_1(\cdot)$ is no smaller than the profit of firm 2 under $w_2(\cdot)$. This result facilitates our analysis by turning the hard problem of finding an equilibrium outcome into the simpler problem of evaluating an inequality involving firm profit under these two wage functions.

We use these tools to show that EPSW moves the wage gap in favor of the majority group of workers. Specifically, there exists a continuum of equilibria under EPSW: in one equilibrium, the wage gap is equal to that in the "Bertrand" outcome without EPSW, but in all other equilibria, the wage gap is strictly larger (i.e. more in favor of the majority group). This result follows from the equal profit condition between firms that must be satisfied in equilibrium. More specifically, if there are more $A-$group workers than $B-$group workers, the firm that hires these workers must receive smaller average profit from each worker than the other firm receives from the average $B-$group worker, so $A-$group workers' average wage is relatively higher. Notably, the wage gap widens under EPSW simply because the majority group has a larger population and, in particular, this conclusion holds regardless of the distributions of productivities of the two groups. Moreover, we also show that firm profit and the magnitude of increase in the wage gap co-move, implying that firms would benefit from selecting equilibria with larger wage gaps.

factor like output. Presumably, any EPSW uses objective measures to define similarity, and thus assuming that "similarity" is a function of the distance of worker "productivities" in our model is in conflict with our agnosticism about our interpretation of productivity.

One might claim that our model stacks the deck against EPSW because the outcome without EPSW is already "fair" in the sense that workers from both groups are paid their productivity. We disagree. Recall that our model does not make strong assumptions on discriminatory factors, so "productivity" could incorporate firms' bias. Given this observation, our model does *not* take a stance on whether or not the outcome without EPSW is fair. What we do show, by contrast, is that EPSW is relatively more advantageous to the majority group. And this conclusion is robust in that it does not depend on whether the outcome without EPSW is fair or not. Of course, if the majority group is favored without EPSW for discriminatory reasons, then our result implies that EPSW makes the labor market even less fair.

The effects of EPSW on the wage gap may be particularly bleak in some markets: with sufficient imbalance in the size of the groups, *any* individually rational wage schedule for workers in the minority group (including zero wages for all minority group workers) can be supported in equilibrium, while arbitrarily high wages *must* simultaneously obtain for workers in the majority group. This result casts doubt on folk wisdom that equal pay policies help minority workers enter fields dominated by majority workers.[5] While our model holds fixed relative group sizes and does not formally consider dynamic changes in responses to equilibrium conditions, Le Chatelier's Principle suggests that EPSW may cause or exacerbate "occupational tipping" patterns in which, for example, women select away from an industry if it becomes too male dominated (Pan, 2015).

We test our model by studying the effect of the enactment of EPSW in Chile in 2009. This EPSW was the first equal pay law in Chile, and it constrained how a firm could pay its workers across gender: no firm is permitted to pay a woman less than it pays a man for similar work, and vice versa. The law subjects firms in violation to substantial fines, and through a public-records request, we find direct evidence of policy enforcement. Importantly, EPSW binds only for firms above a particular size threshold. This allows for a straightforward event-study (difference-in-differences) design to estimate the causal effects of EPSW, wherein we compare differences in outcomes of firms above ("treated") and below ("control") the threshold following EPSW. Following Bennedsen et al. (2022); Böheim and Gust (2021); Duchini et al. (2022); Gulyas et al. (2023), we restrict our sample to firms just above and below this size threshold to limit size-based wage dynamics. That is, our identifying assumption is that parallel trends hold for similarly-sized firms.

Using matched employer-employee data from 2005–2013 we identify the following effects of EPSW consistent with our model predictions. First, EPSW increases gender segregation across firms. The share of firms with workers of only one gender increases by 4.6 percentage points, off a baseline of 31.2% of gender-segregated firms prior to EPSW enactment. While our theoretical results predict full gender segregation across all firms, there are clearly unmodeled factors that prevent such a stark empirical prediction. We show that EPSW leads to a "missing mass" of firms that are nearly-but-not-fully gender segregated, and moreover, that the share of "missing," nearly segregated firms is of a similar magnitude as the increased share of fully segregated firms. These findings suggest that firms on the margin of full segregation (e.g. those which can become fully segregated by firing the last worker of the "wrong" gender) are those most likely to fully segregate after EPSW.

Second, we show that EPSW moves the gender wage gap in favor of the majority group of workers in a local labor market. The average within-firm gender wage gap was 35.8% prior to EPSW. By including worker-firm fixed effects in our specifications, we are able to compare the impacts of EPSW on the same worker at the same firm. In local labor markets—defined by firm

[5] For example, the Obama administration claimed in 2013 that The Equal Pay Act of 1963 (an EPEW) "had a profound effect on job opportunities and earnings for women ... and laid the foundation for the movement of women into the paid labor force at unprecedented levels ... Since passage of the Equal Pay Act, ... [w]omen have integrated many previously exclusively male job fields." See `https://obamawhitehouse.archives.gov/sites/default/files/equalpay/equal_pay_task_force_progress_report_june_2013_new.pdf`.

industry and county—where a majority of workers are men, we find that EPSW *increases* (in favor of men) the gender wage gap by 3.8 percentage points, while in local labor markets where a majority of workers are women, we find that EPSW *decreases* (in favor of women) the gender wage gap by 5.2 percentage points. These findings exactly match our prediction that EPSW benefits the majority group of workers in a labor market. Because men in Chile dominate the overall labor market (5/6 of all workers are employed in majority male local labor markets), the overall effect of EPSW is to increase the gender wage gap (in favor of men) by 2.6 percentage points.

The results thus far paint a bleak picture of the impacts of EPSW. But, we theoretically analyze a simple design choice—removing protected classes—and show how this change can reverse the unintended equilibrium consequences of EPSW. On its face, requiring equal pay across classes makes a measure of sense; due to the "coarseness" of EPSW, if a policy maker is concerned only about gender-based discrimination by firms, it may seem reasonable to allow firms to pay different wages to different men (who are presumably heterogeneous). As we have shown, however, this asymmetry allows firms to segregate the workforce by gender to avoid the implied requirement that all similar workers are paid equally. By restricting that all similar workers be paid equal wages, regardless of group identity, such segregation is no longer an equilibrium feature. Instead, we show that under a non-group-based EPSW, firms segregate the workforce by productivity, with one firm hiring the most productive workers in the market and the other firm hiring less productive workers. We show that such a policy can decrease the gender wage gap in the market and reduce within-firm wage inequality.[6] However, we caution that such non-group-based EPSW can reduce overall employment.

1.1 Related Literature

While we are the first, we believe, to analyze the novel equilibrium effects of EPSW, there are rich theoretical and empirical literatures related to EPEW.

Theoretical studies of EPEW have typically focused on its unintended effects. This focus can be traced back to Milton Friedman who once famously said, "What you are doing, not intentionally, but by misunderstanding, when you try to get equal pay for equal work law... is reducing to zero the cost imposed on people who are discriminating for irrelevant reasons."[7] More recent work studies EPEW in Salop's classic location model; the first such paper is Bhaskar et al. (2002) and is succeeded by Berson (2016); Kaas (2009); Lagerlöf (2020); Lanning (2014). These papers must contend with the very motivation that led to EPSW: what is "equal work"? Doing so results in at least two difficulties. First, the authors interpret "equal work" literally and assume workers are equally productive, while in reality there may be very few workers whose productivities are exactly equal. Second, their analyses predict that EPEW can either increase or decrease differences in outcomes across groups of workers, often within the same paper. The lack of clear policy-relevant predictions is reflected in the empirical literature on EPEW, which we discuss shortly. By contrast, we find that EPSW has clear, if unintended, effects: our theoretical analysis unambiguously predicts both job segregation and widening wage gaps, and our empirical analysis of Chilean data confirms both predictions.

The empirical literature investigating equal-labor-rights legislation primarily considers US policies in the 1960s and 1970s. As with the theoretical literature we detail above, this empirical literature draws mixed conclusions about whether such legislation improves the employment rate or wages of protected classes of workers (see Altonji and Blank (1999); Bailey et al. (2022); Blau

[6] We note that complementary policies can be added to group-based EPSW to ensure the same equilibrium outcome as non-group-based EPSW. The key is that firms must be disincentivized from segregating occupations within job. For example, group quotas (Bertrand et al., 2019) or the proliferation of a sufficiently large number of protected classes prevent group-based segregation outcomes.

[7] See `https://www.aei.org/carpe-diem/milton-friedman-makes-the-case-against-equal-pay-for-equal-work-laws/`.

and Kahn (1992); Donohue III and Heckman (1991); Hyland et al. (2020); Neumark and Stock (2006) for detailed discussions).[8] Our paper adds to the equal-labor-rights literature in several ways. First, we solely observe the impact of an equal pay law. One difficulty in much of the literature is assessing the impacts of individual policies, as many related labor policies are often enacted in quick succession.[9] Donohue III and Heckman (1991) argue that it is difficult to attribute observed effects to any one of the contemporaneous policies, as there may be complementarities between them. Our empirical setting of Chile is notable as no existing equal pay laws were on the books at the time EPSW was enacted in 2009, and no significant related policies were enacted in quick succession.

Methodologically, our paper is more related to the theoretical literature on "best-price" guarantees, which commit firms to rebating past consumers if prices fall in the future. These policies have the direct effect of equalizing payments across heterogeneous buyers, but have the unintended equilibrium effect of raising firm market power (Butz, 1990; Cooper and Fries, 1991; Scott Morton, 1997a,b). In our paper, EPSW plays the role of a best-wage guarantee, but it binds only off the equilibrium path where firms fail to segregate. Nevertheless, this off-path restriction is key in driving the unintended wage effects of EPSW; as a result, firms in our model have an ex-ante identical willingness to pay for each particular worker, but the costs of hiring workers of the "wrong" type are differentiated in equilibrium. This force is similar to "artificial" costs that heterogenize ex-ante identical products in consumer markets, which can lead to local market power for firms (Klemperer, 1987).

Therefore, a key force in our model is firms' equilibrium behavior to segregate their workforce by group identity. Indeed, we show empirically that the Chilean EPSW leads to an increase in gender segregation across firms. One may suspect that such segregation is less likely to occur in other localities that enact EPSW.[10] Speaking to this point, however, group-based segregation across firms has been noted in the US (Blau, 1977; Goldin, 1990; Hellerstein and Neumark, 2008; Neumark et al., 1996), and recent research (Ferguson and Koning, 2018) argues that this segregation has increased over time. Therefore, it seems plausible that EPSW could further affect segregation in a wide variety of labor markets.

2 Model

There are two firms, 1 and 2, and a continuum of workers. Each worker is endowed with a type $e = (g,v) \in \{A,B\} \times [0,1]$, where $g \in \{A,B\}$ is the worker's group identity (say, men and women) and $v \in [0,1]$ is the worker's productivity. There is a $\beta \geq 1$ measure of $A-$group workers and 1 measure of $B-$group workers. F_A and F_B are cumulative distribution functions governing the productivities of workers in groups A and B, respectively. F_A and F_B are absolutely continuous and thus admit density functions f_A and f_B, respectively. We assume that $0 < \underline{f}_A \leq \bar{f}_A < +\infty$ and $0 < \underline{f}_B \leq \bar{f}_B < +\infty$, where $\underline{f}_A = \inf\{f_A(v)|v \in [0,1]\}$, $\bar{f}_A = \sup\{f_A(v)|v \in [0,1]\}$, $\underline{f}_B = \inf\{f_B(v)|v \in [0,1]\}$, and $\bar{f}_B = \sup\{f_B(v)|v \in [0,1]\}$. A *(labor)*

[8] The findings of Bailey et al. (2022) suggest that the conclusions from this literature may be sensitive to the econometric methods used.

[9] The Equal Pay Act of 1963 requires equal pay for men and women for equal work, while Title VII of the Civil Rights Act of 1964 prohibits discrimination in hiring, layoffs, and promotions. There were also other federal equal pay policies—Executive order 11246 in 1965 banning discrimination in hiring by federal contractors against minority candidates, and an extension to include women in 1967; the Equal Employment Opportunity Act in 1972 to increase enforcement; and many individual state policies.

[10] For example, recent research shows that gender-based occupational segregation may be especially likely when the local language has gendered nouns, as firms can target their hiring to workers of specific genders (Card et al., 2021; Kuhn and Shen, 2022; Kuhn et al., 2020). This may explain the high baseline level of gender segregation in Chile, where Spanish is the official language. Gendered nouns and targeted hiring may also play a role in the ability of Chilean firms to further segregate once EPSW is enacted.

market is a tuple (β, F_A, F_B). Note that the distribution of $A-$group workers may be different from that of $B-$group workers, allowing us to model situations in which firms discriminate against one of the groups of workers (i.e. the output of $B-$group workers are drawn from the same distribution as $A-$group workers, but the firms' willingness to pay for them is lower because firms have a taste-based preference for $A-$group workers). For each $g \in \{A,B\}$ we define $\mathbb{E}_g(v) := \int_0^1 v f_g(v) dv$.

For expositional ease, we study this environment via a cooperative game (all of our model predictions are unchanged if we instead consider a non-cooperative game, see Remark 4). Informally, we consider the following situation: An outcome specifies, for each worker, the firm she works for (or the outside option of staying unemployed) and the wage received (if employed). Each worker only cares about her wage and works for whichever firm offers her a higher wage (in case both firms offer the same wage to her, she may work for either of the firms), or stays unemployed if no firm makes a job offer (in case all wage offers made to the worker are zero, she may be employed or stay unemployed). A firm generates per-unit profit $v-w$ if it hires a worker of productivity v and pays her wage w. The firm does not have any capacity constraint (i.e., the firm can hire any measure of workers), and its payoff is the integral of profit generated from workers it employs.

Formally, an outcome for firm i is $O_i := \{(f_g^i(v), w_i^g(v))\}_{v \in [0,1], g=A,B}$, where:

1. $f_g^i(v) \in [0, f_g(v)]$ is the density of workers of type $e=(g,v)$ hired by firm i,

2. $w_i^g(v) \in [0,\infty)$ is the wage firm i pays to workers of type $e=(g,v)$ it hires. If $f_g^i(v)=0$, then we fix $w_i^g(v)=0$.

An outcome is a tuple $O := (O_1, O_2)$ where O_i is the outcome for firm i such that $f_g^1(v) + f_g^2(v) \leq f_g(v)$ for each v and g. That is, the (overall) outcome specifies the outcome for both firms such that the total hiring does not exceed the supply of workers (a feasibility requirement). We assume that f_g^i and w_i^g are measurable functions for each i and g. We also assume that wages must be monotone non-decreasing in worker productivity within each firm within each group. Formally, for each $i \in \{1,2\}$, $g \in \{A,B\}$, and any $v, v' \in [0,1]$, $w_i^g(v) \geq w_i^g(v')$ if $v \geq v'$ and $f_g^i(v) > 0$.

Remark **1.** Throughout, we maintain the assumption that the wage function is weakly monotone in the aforementioned sense. The motivation behind this assumption is the following. In many situations, although it may be difficult or impossible for a worker to convince the firm that they have a higher productivity than their true value, it is often easy for a worker to pretend to have a lower productivity than their true value. For example, a worker who is fluent in a foreign language can pretend to be otherwise simply by not speaking that language, while misrepresentation in the opposite direction may be impossible. Thus, if the wage function fails monotonicity, then such a misrepresentation may be both feasible and profitable for the worker. In other words, our monotone wage assumption within a firm and within a group is the condition we impose for robustness against a worker who considers destroying productivity or pretending to have lower productivity than their true value.

Under an outcome for i, $O_i := \{(f_g^i(v), w_i^g(v))\}_{v \in [0,1], g=A,B}$, firm i receives profit

$$\pi_i^{O_i} := \beta \int_0^1 [v - w_i^A(v)] f_A^i(v) dv + \int_0^1 [v - w_i^B(v)] f_B^i(v) dv.$$

Given an outcome $O = (O_1, O_2)$ and firm i, we denote $\pi_i^O := \pi_i^{O_i}$.

Given an outcome O, denote by AW_g^O the average wages for group $g \in \{A,B\}$, i.e.,[11]

$$AW_g^O := \int_0^1 w_1^g(v) f_g^1(v) dv + \int_0^1 w_2^g(v) f_g^2(v) dv.$$

We refer to $AW_A^O - AW_B^O$ as the *wage gap* in outcome O.

We view two outcomes for firm i, $O_i := \{(f_g^i(v), w_i^g(v))\}_{v \in [0,1], g=A,B}$ and $\tilde{O}_i := \{(\tilde{f}_g^i(v), \tilde{w}_i^g(v))\}_{v \in [0,1], g=A,B}$ as equivalent if, for each $g \in \{A,B\}$, $f_g^i(v) = \tilde{f}_g^i(v)$ and $w_i^g(v) = \tilde{w}_i^g(v)$ for almost all v. We view two outcomes O and $\tilde{O}$ as equivalent if either:

1. for every $i \in \{1,2\}$, O_i is equivalent to $\tilde{O}_i$, or
2. O_1 is equivalent to $\tilde{O}_2$, and O_2 is equivalent to $\tilde{O}_1$.

The first condition captures the usual notion that the outcomes are regarded as equivalent if both the employment patterns and wages are identical between them except for a measure-zero set. The second condition captures the case in which the employment patterns and wages are identical almost everywhere once the names of the firms are relabeled—recall that firms are homogeneous in the present model.

Remark **2.** We refer to the model presented so far as the model without EPSW. In the case with group-based EPSW we add a restriction that, for any outcome and any firm, no positive measures of workers from different groups receive different wages. The formal definition is given in Section 3.2. In the case with non-group-based EPSW, we add a restriction that, for any outcome and any firm, almost all workers at that firm receive the same wages. The formal definition is given in Section 3.3.

An outcome is said to be a *core outcome* if there is no firm and an alternative wage schedule for a subset of workers such that they are made better off being matched to each other, that is, both the firm and each worker in the hired subset obtain a higher payoff than the present outcome. Formally, we say that an outcome $O := \{(f_g^i(v), w_i^g(v))\}_{v \in [0,1], i=1,2, g=A,B}$ is *blocked* by firm j via an alternative outcome (for j) $\tilde{O}_j := \{(\tilde{f}_g^j(v), \tilde{w}_j^g(v))\}_{v \in [0,1], g=A,B}$ if $\pi_j^{\tilde{O}_j} > \pi_j^{O_j}$ and, for each $g \in \{A,B\}$ and almost all $v \in [0,1]$, one of the following conditions hold. Note that, because we define $\tilde{O}_j$ to be an outcome, it must satisfy all restrictions imposed on an outcome in addition to those listed below:

1. $\tilde{w}_j^g(v) \geq w_j^g(v)$ and $\tilde{w}_j^g(v) > w_{-j}^g(v)$,
2. $\tilde{w}_j^g(v) \geq w_j^g(v)$ and $\tilde{f}_g^j(v) + f_g^{-j}(v) \leq f_g(v)$,
3. $\tilde{w}_j^g(v) > w_{-j}^g(v)$ and $\tilde{f}_g^j(v) + f_g^j(v) \leq f_g(v)$, or
4. $\tilde{f}_g^j(v) + f_g^j(v) + f_g^{-j}(v) \leq f_g(v)$.

These cases enumerate all possibilities for the formation of a blocking coalition. Condition 1 states a "no wage cuts" requirement; if firm j weakly raises the wages of all workers involved, and strictly raises wages for workers employed by the other firm $-j$, then these workers are all willing to join the blocking coalition. Condition 2 considers the case in which firm j does not need to poach workers from firm $-j$ to construct the blocking outcome, so the only constraint on wages is

[11] Note that each unemployed worker from group g contributes a wage of 0 to the calculation of the average wage for group g.

that existing worker's wages are not reduced. Condition 3 considers the case in which firm j does not need to keep any existing workers to construct the blocking outcome, so the only restriction on wages is that the wage paid to poached workers is higher than those paid by $-j$ to the same workers. Condition 4 considers the case in which firm j can hire from unemployed workers to construct the blocking outcome, so there is no additional restriction on the wages of these workers.

We say that an outcome O is a *core outcome* if there exists no outcome that blocks it.

Remark **3.** The definition of block implies two restrictions that any core outcome must satisfy:

First, Condition 3 of the definition of block immediately implies the following **Equal Profit Condition**:

In any core outcome O, $\pi_1^O = \pi_2^O$.

This is because otherwise the firm earning strictly lower profit could fire all of its existing workers and hire all of the workers employed by the other firm with an arbitrarily small wage increase.

Second, the definition of the core implies the following **Individual Rationality Condition** for firms:

In any core outcome, there is no set $V \subset [0,1]$ of positive Lebesgue measure, a group $g \in \{A,B\}$, and a firm $i \in \{1,2\}$ such that $w_i^g(v) > v$ for all $v \in V$.

Intuitively this is because, if there were, firm i could simply fire all of the workers in question (i.e. set $\tilde{f}_g^i(v) = 0$ for all $v \in V$) and increase its profit. A formal argument for the case without EPSW is given in the proof of Proposition 1, and an essentially identical argument extends this observation to the case with (group-based or non-group-based) EPSW as well.

Remark **4.** Our setting and the solution concept of the core are of a cooperative nature. An alternative approach would be to set up a non-cooperative game and analyze its equilibria. In Appendix C, we present a non-cooperative game wherein the firms simultaneously make wage offers to workers, and each worker accepts at most one of the offers she receives (after observing all offers). The subgame perfect Nash equilibrium outcomes of this game exactly coincide with the set of core outcomes of the cooperative game we describe above. We choose to present the cooperative framework in the main text because its exposition is simpler, and the equivalence mentioned here provides a noncooperative foundation for doing so.

3 Results

In this section, we present theoretical results from our model. Throughout, we fix an arbitrary labor market (β, F_A, F_B) and present results within this labor market, except where explicitly stated otherwise.

3.1 Core without EPSW

We begin our analysis by studying core outcomes without EPSW. We establish that our model leads to very strong predictions both on employment patterns and wages.

Proposition 1. *Without EPSW, there exist a continuum of (non-equivalent) core outcomes. In any core outcome, almost every worker is employed and earns a wage equal to her productivity (formally, for all $i\in\{1,2\}$, all $g\in\{A,B\}$, and almost all $v\in[0,1]$: $f_g^1(v)+f_g^2(v)=f_g(v)$ and $w_i^g(v)=v$ if $f_g^i(v)>0$).*

Proposition 1 establishes that, while there are multiple core outcomes, they all feature full employment and result in wages to each worker equal to their productivity. We use this result as a benchmark and proceed to study how the employment patterns and wages are affected by EPSW in the following subsections.

Note that, by Proposition 1, in any core outcome O without EPSW, the wage gap is

$$\begin{aligned}AW_A^O-AW_B^O:=&\int_0^1 w_1^A(v)f_A^1(v)dv+\int_0^1 w_2^A(v)f_A^2(v)dv-\int_0^1 w_1^B(v)f_B^1(v)dv-\int_0^1 w_2^B(v)f_B^2(v)dv\\ =&\int_0^1 vf_A(v)dv-\int_0^1 vf_B(v)dv\\ :=&\mathbb{E}_A(v)-\mathbb{E}_B(v). \qquad (1)\end{aligned}$$

At first glance, Proposition 1 may seem quite intuitive and perhaps even straightforward: In the absence of EPSW, firms compete in a "Bertrand"-like manner, i.e., compete for each worker in isolation without any reference to wages paid to other workers, so the only core outcomes must feature wages equal to the worker's productivity. While this "Bertrand" intuition is reasonable, the actual proof is much more nontrivial and involved. The reason for this complexity is that throughout we assume that the wage function must be weakly non-decreasing within any given firm and each group of workers—recall that monotonicity is assumed to remove incentives for a worker from destroying productivity or pretending to have lower productivity (see Remark 1). Because of the monotonicity condition on wages, the wage for a particular worker cannot be freely chosen even without EPSW. The main content of the formal proof is that, even with this restriction, any outcome that does not satisfy the "Bertrand"-like features, i.e., that almost all workers are hired at their productivities, allows for a blocking outcome which itself satisfies the monotonicity of the wage. In that sense, one interpretation of our analysis is that the sharp prediction obtained by the Bertrand competition is in fact robust to a certain kind of wage rigidity, suggesting that the prediction may be more likely in applications than previously considered.

In the next subsections, we analyze whether EPSW affects wages and hiring in any substantive manner. As we will demonstrate, versions of EPSW in fact change the employment patterns such as the unemployment rate and job segregation, as well as wages paid to different groups of workers.

3.2 Core with Group-Based EPSW

Now we study core outcomes of our game under a group-based EPSW. Informally, this restriction requires that each firm pays the same wages to almost all workers it hires only if it hires a positive measure of workers from both groups. Formally, we modify the definition of outcome $O_i=\{(f_g^i(v),w_i^g(v))\}_{v\in[0,1],g=A,B}$ for all $i\in\{1,2\}$ to include the following restriction:

> There exist no sets $V_g\subset[0,1]$ and $V_{-g}\subset[0,1]$ with positive Lebesgue measure such that:
>
> 1. $f_g^i(v)>0$ for all $v\in V_g$,
> 2. $f_{-g}^i(v)>0$ for all $v\in V_{-g}$, and

3. $\inf_{v\in V_{-g}} w_i^{-g}(v) > \sup_{v\in V_g} w_i^g(v)$.

Informally, the preceding restriction prevents a firm from employing sets of workers from both groups with positive measure (points 1 and 2) such that all workers in one set receive strictly higher pay than all workers in the other (point 3).[12] Given the symmetry of the above definition across groups, group-based EPSW implies, by transitivity, that if a firm hires a positive measure of workers from both groups, it must pay almost all workers the same wages.

The next result shows that generically firms must fully segregate by group in any core outcome under group-based EPSW.[13]

Proposition **2.** *Generically, in any core outcome under group-based EPSW, firms completely segregate. Specifically, one firm hires almost all $A-$group workers, and the other hires almost all $B-$group workers (formally, for some $i\in\{1,2\}$, $f_A^i(v)=f_A(v)$ for almost all $v\in[0,1]$ and $f_B^{-i}(v)=f_B(v)$ for almost all $v\in[0,1]$).*

Remark **5.** The conclusion of this proposition holds only generically. An example of a non-generic case in which the conclusion fails features $\beta=1$ and $F_A(v)=F_B(v)=v$ for all $v\in[0,1]$. For this parameterization, it is straightforward to verify that there exists a core outcome where firm 1 hires all workers from both groups with $v\in[0,\frac{1}{2}]$ at wage zero while firm 2 hires all other workers at wage $\frac{1}{2}$.

Following the previous result, we assume throughout that any core outcome under group-based EPSW exhibits full segregation by group. Therefore, we assume in any core outcome O, without loss of generality, that firm 1 hires all $A-$group workers ($f_A^1(v)=f_A(v)$ for all v) and firm 2 hires all $B-$group workers ($f_B^2(v)=f_B(v)$ for all v). Consider a core outcome O where $w_1(\cdot)$ specifies firm 1's wages to $A-$group workers and $w_2(\cdot)$ specifies firm 2's wages to $B-$group workers. By Individual Rationality for the firms (see Remark 3), it suffices to consider $w_i(\cdot):[0,1]\to[0,1]$ for each $i\in\{1,2\}$ in any core outcome. Note that we can therefore represent the wage gap in a core outcome O under group-based EPSW as

$$AW_A^O - AW_B^O = \int_0^1 w_1(v)f_A(v)dv - \int_0^1 w_2(v)f_B(v)dv.$$

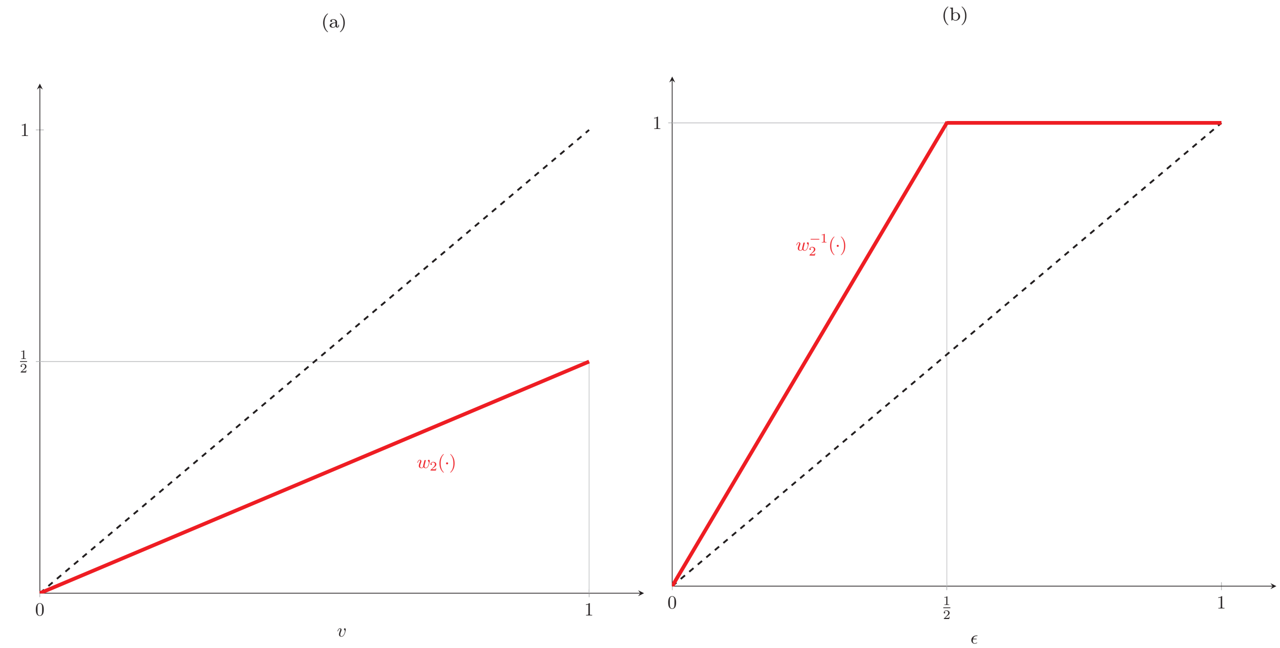
To understand the existence and properties of core outcomes under group-based EPSW, we introduce new machinery. Suppose firm 2 attempts to block an outcome O that involves segregation as detailed in the preceding paragraph. One potential blocking outcome that firm 2 could undertake is to desegregate and hire positive measures of workers from both groups. By the restrictions of group-based EPSW, this would require paying a common wage $\epsilon\in[0,1]$ to (almost) all workers it hires in the proposed blocking outcome. To denote the set of workers potentially available to be "poached," we define

$$w_i^{-1}(\epsilon) := \begin{cases} \sup\{v|w_i(v)\le\epsilon\} & \text{if } \{v|w_i(v)\le\epsilon\}\neq\emptyset, \\ 0 & \text{otherwise,} \end{cases}$$

[12]Note that because the above restriction must hold for every set V_g and V_{-g} of positive measure, we could equivalently state point 3 using the essential infemum and essential supremum of the wages, respectively.

[13]We consider the space of distributions F_A and F_B describing the distributions of productivities of $A-$ and $B-$group workers, respectively, that admit respective density functions f_A and f_B with respective lower bounds $\underline{f}_A,\underline{f}_B>0$ where $\underline{f}_A=\inf\{f_A(v)|v\in[0,1]\}$, $\underline{f}_B=\inf\{f_B(v)|v\in[0,1]\}$. We endow the space of distributions F_g, $g\in\{A,B\}$ with the weak-* topology and consider genericity with respect to the product topology over the product set of distributions, where we say that a property holds generically if the property holds in an open and dense subset.

Figure 1: Relationship between a wage function and its inverse



Notes: Panel (a) plots the wage function $w_2(v)=\frac{1}{2}v$. Panel (b) plots the inverse wage function $w_2^{-1}(\cdot)$.

for each $i\in\{1,2\}$ and $\epsilon\in[0,1]$, which characterizes the highest productivity worker hired by firm i who is paid no more than ϵ. $w_i^{-1}(\cdot)$ is a generalization of a traditional inverse function of $w_i(\cdot)$, in that we allow either or both of these functions to be weakly increasing instead of strictly increasing, which is necessary to study the class of wage functions that satisfy our monotonicity condition. Figure 1 plots a particular wage function $w_2(\cdot)$ and its inverse $w_2^{-1}(\cdot)$.

Consider the following inequalities which we refer to as the *No Desegregation Condition*:

$$\pi_2^O := \int_0^1 (v-w_2(v))f_B(v)dv \geq \beta \int_\epsilon^{w_1^{-1}(\epsilon)} (v-\epsilon)f_A(v)dv + \int_\epsilon^{w_2^{-1}(\epsilon)} (v-\epsilon)f_B(v)dv \qquad \text{for all } \epsilon\in[0,1] \tag{2}$$

It states that firm 2 does not wish to desegregate, pay a common wage of ϵ to all workers it employs, and hire only the workers with productivities above ϵ who are also paid less than ϵ according to $w_1(\cdot)$ and $w_2(\cdot)$. If the Equal Profit Condition also holds, then the No Desegregation Condition implies that firm 1 does not wish to block the outcome by desegregating either.

The following result finds that, in addition to Individual Rationality and the Equal Profit Condition which we have previously introduced, the No Desegregation Condition exactly characterizes the set of core outcomes.

Lemma **1.** *Consider an outcome O in which $f_A^1(v)=f_A(v)$ for all v and $f_B^2(v)=f_B(v)$ for all v. Then O is a core outcome if and only if:*

1. *$w_i(v)\leq v$ for all $v<1$ and all $i\in\{1,2\}$ (Individual Rationality),*

2. $\pi_1^O = \pi_2^O$ *(Equal Profit Condition), and*

3. (2) *is satisfied (No Desegregation Condition).*

The existence of a core outcome holds generally under group-based EPSW; consider the zero-profit outcome in which firm 1 hires all $A-$group workers and firm 2 hires all $B-$group workers, and all workers are paid wages equal to their productivities. Note that $w_i^{-1}(\epsilon) = \epsilon$ for any $i \in \{1,2\}$ and any $\epsilon \in [0,1]$, implying that the No Desegregation Condition is trivially satisfied, as the right-hand side is identically 0 for all ϵ. This outcome is similar to core outcomes without EPSW, except that the workforce is now necessarily segregated. However, there are also other core outcomes under group-based EPSW that result in positive firm profits and a gap of average pay between the two groups. Indeed, there always exist a continuum of (non-equivalent) core outcomes. Moreover, if the measure of $A-$group workers is strictly larger than the measure of $B-$group workers, all but one core outcome exhibits a larger wage gap than in the absence of group-based EPSW, and firm profits are higher in core outcomes with larger wage gaps.

Proposition **3.** *Suppose there is a group-based EPSW.*

1. *There exist a continuum of non-equivalent core outcomes.*

2. *Let $\beta > 1$. There exists one core outcome (and its equivalent outcomes) that yields the same wage gap as in the (essentially unique) core outcome without EPSW. In all other core outcomes under group-based EPSW, the wage gap is strictly larger.*

3. *Let $\beta > 1$. Consider any two core outcomes. The wage gap is larger in the first outcome if and only if firm profit is higher in the first outcome.*

As demonstrated by Part 2 of Proposition 3, group-based EPSW widens the wage gap between the two groups (and the widening of the wage gap is strict except for one core outcome among a continuum). Moreover, Part 3 shows that larger wage gaps under group-based EPSW are associated with higher firm profits. An implication of this last result is that firms prefer core outcomes that result in larger wage gaps, suggesting that a core outcome with a larger wage gap may be more likely to occur if firms can coordinate to select an outcome from the core.

When interpreting this result, we emphasize that we have made no assumptions about the relative productivities of the two groups of workers. Specifically, Proposition 3 predicts that if the wage gap is negative in the core outcomes without EPSW (which can occur, if for example, there are discriminatory factors against workers in the majority group), it will increase in a core outcome with group-based EPSW in the sense that it will either become less negative or change signs entirely. This consideration will become important in our empirical analysis of a Chilean EPSW in Section 5 where we find that the wage gap widened (i.e. in favor of men) after the introduction of EPSW in male-majority labor markets while the wage gap closed (i.e. in favor of women) in female-majority ones. Both findings are as predicted by Proposition 3.

3.2.1 How large can the wage gap be?

We have demonstrated that group-based EPSW can lead to an increase in the wage gap. In this section, we analyze how large this wage gap can become and how it is related to the relative size of the two groups of workers. We show that when the proportion of $A-$group workers in the market grows sufficiently large, nearly maximal wage inequality (subject to individual rationality) between $A-$ and $B-$group workers can be supported in a core outcome: all $A-$group workers are paid nearly their marginal products, and all $B-$group workers receive exactly zero pay, regardless of productivity. This core outcome in which $B-$group workers receive zero pay maximizes firm profits (by Proposition 2 and the Equal Profit Condition) and therefore also maximizes the wage gap (Proposition 3, Part 3). As discussed in the introduction, our results suggest that EPSW may cause or exacerbate "occupational tipping" patterns in which, for example, women select away from an industry if it becomes too male dominated (Pan, 2015).

We present two results that formalize those claims. First we show that for any sufficiently large β, all $A-$group workers receive a wage nearly equal to their productivity.

PROPOSITION **4.** *Suppose there is a group-based EPSW and fix F_A and F_B arbitrarily. For any $\delta > 0$ there exists $\beta^* \in [1,\infty)$ such that in the market (β, F_A, F_B) with any $\beta > \beta^*$, $w_1(v) > v - \delta$ for all v in any core outcome.*

The intuition for this result is simple and closely related to the rise in segregation under group-based EPSW (Proposition 2). First, because the two firms hire from different groups, the profit for each firm is bounded by the social surplus created by workers of the group from which the firm hires. Second, Proposition 2 also shows that almost all workers are employed, so the maximum social surplus is created by both groups. Third, the Equal Profit Condition implies that the firms must obtain the same profit. Therefore, when the proportion of $A-$group workers is large, most of the social surplus created by them must be paid to them as a wage. In other words, if some $A-$group workers receive wages bounded away from their productivity even for large β, then the firm hiring the $A-$group workers would receive higher profit than the firm hiring the $B-$group workers, a violation of the Equal Profit Condition.

Next, we proceed to explore the lowest wages we can sustain for $B-$group workers. To do so, we provide a necessary and sufficient condition for a wage schedule for $B-$group workers to be part of some core outcome. In light of Lemma 1, for any given $w_2(\cdot)$ we proceed by first characterizing the wage functions $w_1(\cdot)$ that satisfy the No Desegregation Condition stated in (2) (while addressing Individual Rationality and the Equal Profit Condition later). Consider the following construction. For each ϵ, define $\phi(\epsilon)$ implicitly as the supremum of $\tilde{v}$ such that

$$\int_0^1 (v - w_2(v)) f_B(v) dv \geq \beta \int_\epsilon^{\tilde{v}} (v - \epsilon) f_A(v) dv + \int_\epsilon^{w_2^{-1}(\epsilon)} (v - \epsilon) f_B(v) dv. \quad (3)$$

Then, we define

$$\hat{w}_1(v) := \begin{cases} \sup\{\epsilon | \phi(\epsilon) \leq v\} & \text{if } \{\epsilon | \phi(\epsilon) \leq v\} \neq \emptyset, \\ 0 & \text{otherwise.} \end{cases}$$

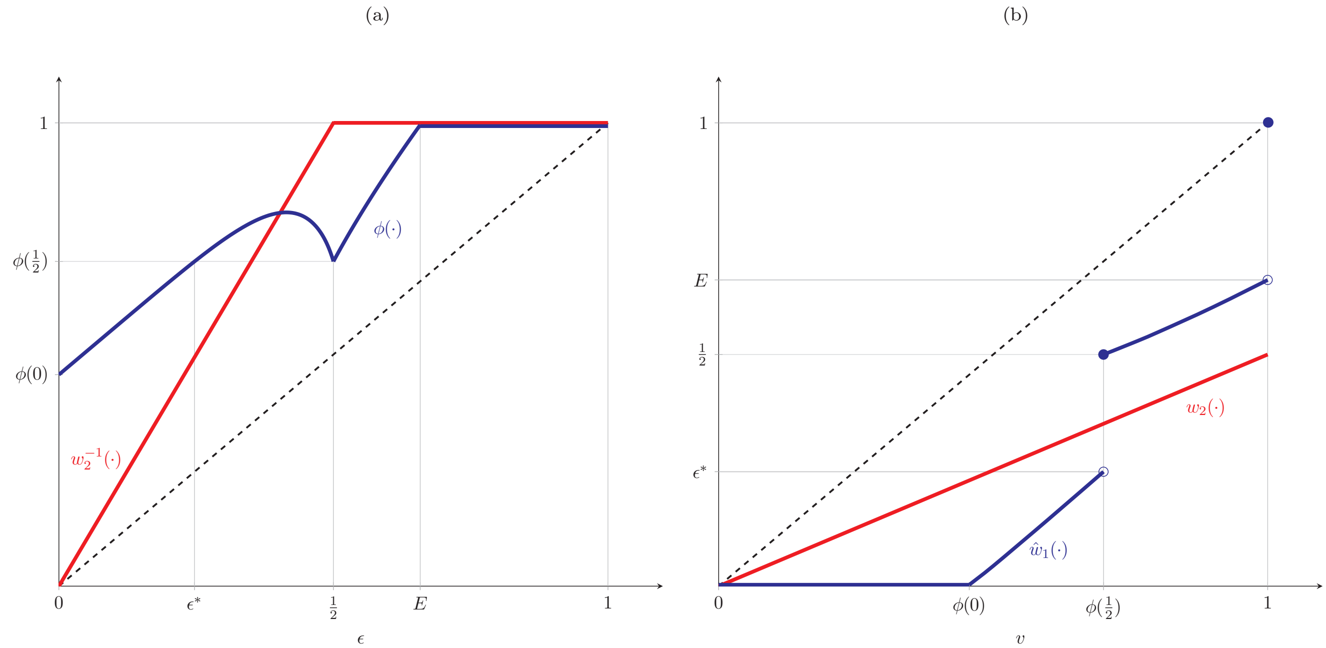
and

$$\hat{w}_1^{-1}(\epsilon) := \begin{cases} \sup\{v | \hat{w}_1(v) \leq \epsilon\} & \text{if } \{v | \hat{w}_1(v) \leq \epsilon\} \neq \emptyset, \\ 0 & \text{otherwise.} \end{cases}$$

Note that $\hat{w}_1(v)$ is weakly increasing in v. This is because the supremand $\{\epsilon | \phi(\epsilon) \leq v\}$ weakly expands (in the set inclusion sense) when we increase v. Figure 2 illustrates the construction of $\phi(\cdot)$ and $\hat{w}_1(\cdot)$ for a certain specification of the model primitives.

Figure 3 presents the relationship between $\phi(\cdot)$ and $\hat{w}_1^{-1}(\cdot)$. As can be seen—and as we show formally in Remark 8 in the appendix—$\hat{w}_1^{-1}(\cdot)$ is the largest monotone nondecreasing function that is pointwise no larger than $\phi(\cdot)$. Considering the differences between $\phi(\cdot)$ and $\hat{w}_1^{-1}(\cdot)$ is illuminating in

Figure 2: $\phi(\cdot)$ and $\hat{w}_1(\cdot)$



Notes: In this figure, F_A is the cumulative distribution function for the uniform distribution on [0,1] and $F_B(v)=v^5$ for all $v\in[0,1]$, which is the cumulative distribution function for the Kumaraswamy distribution on [0,1] with parameters $a=5$ and $b=1$. $w_2(v)=\frac{1}{2}v$, and $\beta=4$. Panel (a) plots $\phi(\cdot)$ and $w_2^{-1}(\cdot)$. Panel (b) plots the wage functions $\hat{w}_1(\cdot)$ and $w_2(\cdot)$. E is defined as the supremum value of ϵ for which the function $\phi(\cdot)<1$. ϵ^* is defined as the unique value $\epsilon<\frac{1}{2}$ such that $\phi(\epsilon^*)=\phi(\frac{1}{2})$. We note that F_B does not satisfy the regularity condition that there exists $\underline{f}_B>0$ such that $f_B(v)>\underline{f}_B$ for all $v\in[0,1]$, however, we have verified that the example is essentially unchanged by altering F_B slightly to satisfy this condition; details of this and all other calculations are available upon request.

order to understand the purpose of the additional machinery we have introduced in this section. $\phi(\cdot)$ and $\hat{w}_1^{-1}(\cdot)$ differ over interval $(\epsilon^*,\frac{1}{2})$. By construction, $\phi(\epsilon)$ is the highest productivity $A-$group worker that can be hired at a desegregation attempt ϵ while still deterring desegregation. But $\phi(\epsilon)>\phi(\epsilon^*)=\phi(\frac{1}{2})$ for any $\epsilon\in(\epsilon^*,\frac{1}{2})$, meaning that the highest productivity $A-$group worker that can be hired while still deterring desegregation is higher at ϵ than at ϵ^* and at $\frac{1}{2}$. We construct $\hat{w}_1(\cdot)$ to deter desegregation at any wage in [0,1], therefore, satisfying (3) at $\frac{1}{2}$ implies that the No Desegregation Condition is slack for $\epsilon\in(\epsilon^*,\frac{1}{2})$, which is reflected in $\hat{w}_1^{-1}(\cdot)$ being constant over $[\epsilon^*,\frac{1}{2}]$.

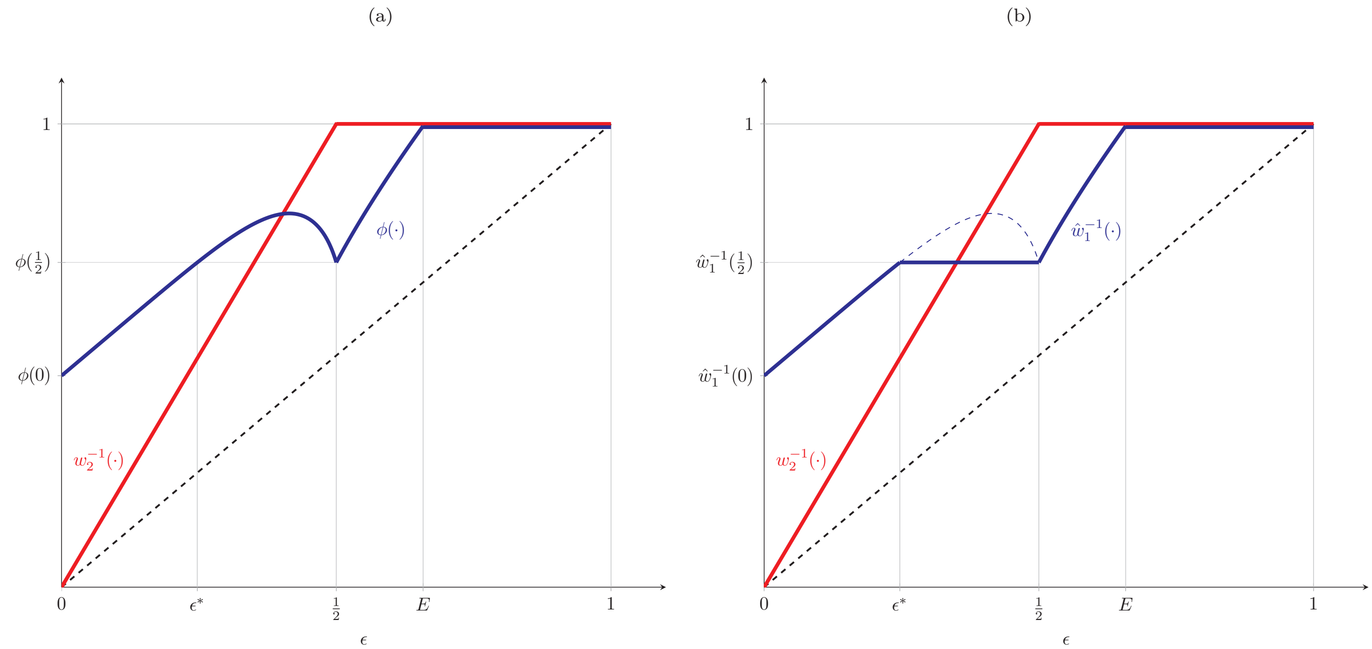
Not only is $\hat{w}_1(\cdot)$ constructed to deter desegregation, we show that it is the cheapest monotonic wage function for firm 1 that does so. Define the profit firm 1 receives if it were to hire all $A-$group workers and pay wage $\hat{w}_1(v)$ to each $A-$group worker of productivity v:

$$\hat{\pi}_1:=\beta\int_0^1(v-\hat{w}_1(v))f_A(v)dv.$$

The following lemma provides a necessary and sufficient condition under which $w_2(\cdot)$ can be supported in a core outcome.

Lemma **2.** *There exists a core outcome $O=(O_1,O_2)$ in which firm 2 pays wages $w_2(\cdot)$ to the workers it employs if and only if $\hat{\pi}_1\geq\pi_2^{O_2}$.*

Figure 3: $\phi(\cdot)$ and $\hat{w}_1^{-1}(\cdot)$



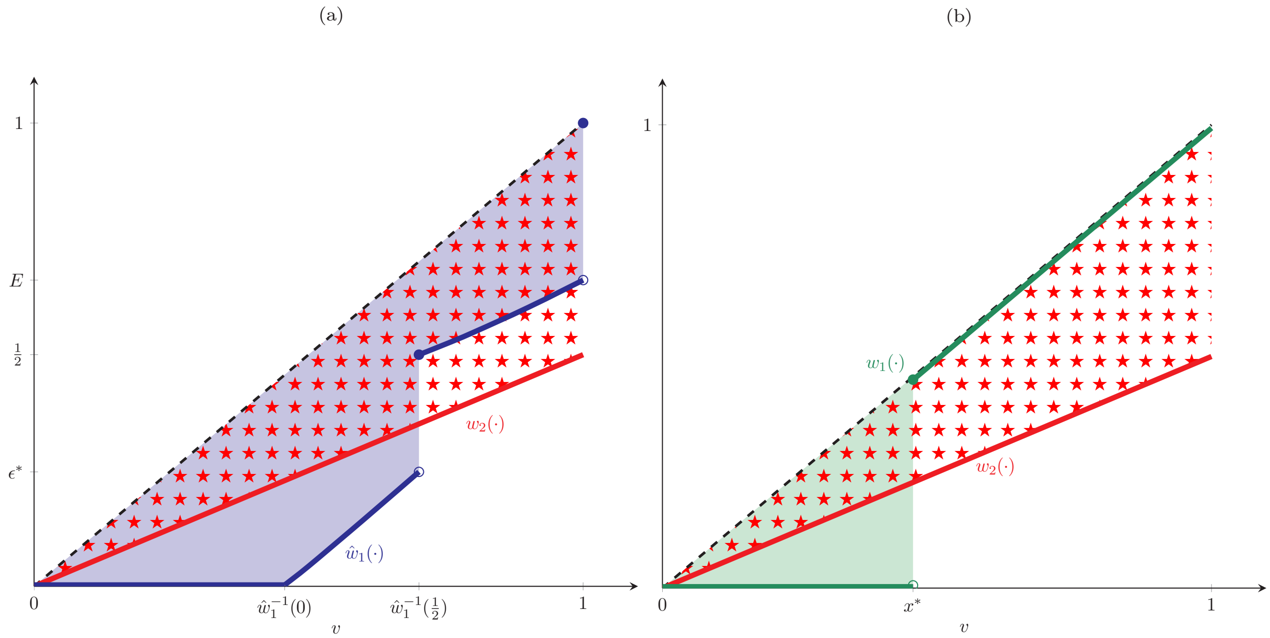
Notes: In this figure, F_A is the cumulative distribution function for the uniform distribution on [0,1] and $F_B(v)=v^5$ for all $v\in[0,1]$, which is the cumulative distribution function for the Kumaraswamy distribution on [0,1] with parameters $a=5$ and $b=1$. $w_2(v)=\frac{1}{2}v$, and $\beta=4$. Panel (a) plots $\phi(\cdot)$ and $w_2^{-1}(\cdot)$. Panel (b) plots inverse wage functions $\hat{w}_1^{-1}(\cdot)$ and $w_2^{-1}(\cdot)$. The dotted blue curve in panel (b) plots $\phi(\cdot)$ for values $\epsilon\in(\epsilon^*,\frac{1}{2})$, the interval over which two functions differ. E is defined as the supremum value of ϵ for which the function $\phi(\cdot)<1$. ϵ^* is defined as the unique value $\epsilon<\frac{1}{2}$ such that $\phi(\epsilon^*)=\phi(\frac{1}{2})$. We note that F_B does not satisfy the regularity condition that there exists $\underline{f}_B>0$ such that $f_B(v)>\underline{f}_B$ for all $v\in[0,1]$, however, we have verified that the example is essentially unchanged by altering F_B slightly to satisfy this condition; details of this and all other calculations are available upon request.

Figure 4 illustrates the "if" part of Lemma 2 for the environment described in Figure 2. In Panel (a), $\hat{\pi}_1$ corresponds to the blue region multiplied by $\beta=4$, which is larger than $\pi_2^{O_2}$ depicted by the starred area. Panel (b) illustrates an example of a wage function $w_1(\cdot)$ that supports a core outcome. To see that $w_1(\cdot)$ indeed supports a core outcome, observe that (i) clearly $\leq w_1(v)\leq v$ holds for all v, so Individual Rationality is satisfied; (ii) the green area (multiplied by β), which represents firm 1's profit under $w_1(\cdot)$, is equal to $\pi_2^{O_2}$, so that the Equal Profit Condition is satisfied; and (iii) by construction, $w_1(v)\geq\hat{w}_1(v)$ for all $v\in[0,1]$, and by inspection of (3) we can see that $w_1(\cdot)$ satisfies the No Desegregation Condition because $\hat{w}_1(\cdot)$ does. Thus, by Lemma 1, the wage profiles are part of a core outcome. Note that the property required in Lemma 2 that $\hat{\pi}_1\geq\pi_2^{O_2}$ is satisfied by this example, and this fact guarantees the existence of an appropriate $w_1(\cdot)$ satisfying the three conditions characterizing a core outcome.

With this machinery, we are now ready to state and prove the following result:

Proposition **5.** *Suppose there is a group-based EPSW and fix F_A and F_B arbitrarily. Let $w_2(\cdot)$ be an arbitrary wage function such that $w_2(v)\leq v$ for all $v\in[0,1)$. There exists $\beta^*\in[0,\infty)$ such that in the market (β,F_A,F_B) with any $\beta>\beta^*$, there exists a core outcome in which the wage schedule of $B-$group workers is given by $w_2(\cdot)$.*

Figure 4: Profit, and the existence of a core outcome with $w_2(\cdot)$



Notes: In this figure, F_A is the cumulative distribution function for the uniform distribution on [0,1] and $F_B(v) = v^5$ for all $v \in [0,1]$, which is the cumulative distribution function for the Kumaraswamy distribution on [0,1] with parameters $a=5$ and $b=1$. $w_2(v) = \frac{1}{2}v$, and $\beta = 4$. Panel (a) plots firm 1's profit given $\hat{w}_1(\cdot)$, $\hat{\pi}_1$, and firm 2's profit, $\pi_2^{O_2}$, given $w_2(\cdot)$. $\hat{\pi}_1$ is given by β times the shaded, blue area. $\pi_2^{O_2}$ is given by the starred area. Panel (b) is the same as panel (a) except that it plots a new wage function for firm 1, $w_1(v) := \begin{cases} \hat{w}_1(v) & \text{if } v < x^* \\ v & \text{otherwise} \end{cases}$, where x^* is the unique value such that $w_1(\cdot)$ and $w_2(\cdot)$ satisfy the Equal Profit Condition (see the proof of Lemma 2 for details). For this parameterization, $x^* = \hat{w}_1^{-1}(0)$. We note that F_B does not satisfy the regularity condition that there exists $\underline{f}_B > 0$ such that $f_B(v) > \underline{f}_B$ for all $v \in [0,1]$, however, we have verified that the example is essentially unchanged by altering F_B slightly to satisfy this condition; details of this and all other calculations are available upon request.

Remark **6.** For a subclass of wage functions, one can strengthen the conclusion of Proposition 5 to show that there exists a "threshold" value of β for which a given wage function $w_2(\cdot)$ can be supported in a core outcome. We state and prove this result in the appendix for a general class of functions $w_2(\cdot)$ while describing this result for a special case here, namely for the lowest-possible wage function, $w_2(v) = 0$ for all v:

> Suppose there is a group-based EPSW and fix F_A and F_B arbitrarily. There exists $\beta^* \in (0, \infty)$ such that in the market (β, F_A, F_B) with any $\beta > \beta^*$ there exists a core outcome in which $w_2(v) = 0$ for all v (i.e. the wage of each $B-$group worker is 0) while in the market (β, F_A, F_B) with any $\beta < \beta^*$ there exists no core outcome in which $w_2(v) = 0$ for all v.

Propositions 4 and 5, together with Remark 6, imply the wage gap between $A-$ and $B-$group workers can be large when there are many more $A-$group workers than $B-$group workers. In particular, if all $B-$group workers receive zero wages in a core outcome, then as β grows large, it follows from the Equal Profit Condition that there is nearly maximal inequality between $A-$ and $B-$groups: $A-$group workers are nearly paid their marginal products. While there exists a multiplicity of core outcomes, recall that our Proposition 3 implies that this core outcome with maximal inequality in pay between the groups is the firm-optimal core outcome.

3.3 Core with Non-Group-Based EPSW

Now we study core outcomes of our game under a non-group-based EPSW. Informally, this restriction requires that each firm pays the same wages to almost all workers it hires. Formally, we modify the definition of outcome $O_i = \{(f_g^i(v), w_i^g(v))\}_{v\in[0,1], g=A,B}$ for all $i \in \{1,2\}$ to include the following restriction:

> There exists $w_i \in [0,1]$ such that $w_i^g(v) = w_i$ for all $g \in \{A,B\}$ and almost all $v \in [0,1]$ such that $f_g^i(v) > 0$.

Observe that this restriction makes no reference to the group identity of a worker, so it is convenient for us to proceed with the following distribution of productivities of the entire population, with its cumulative distribution function F given by

$$F(v) := \frac{\beta}{1+\beta} F_A(v) + \frac{1}{1+\beta} F_B(v), \tag{4}$$

and denote the associated density function as $f(v)$. Denote $\bar{f} := \sup\{f(v)|v \in [0,1]\}$ and $\underline{f} := \inf\{f(v)|v \in [0,1]\}$ where $0 < \underline{f} < \bar{f} < +\infty$ by our previous assumptions on $\underline{f}_A, \underline{f}_B, \bar{f}_A$, and $\bar{f}_B$.

In any outcome we denote the density of workers hired by each firm $i \in \{1,2\}$ as $f^i(v) := \frac{\beta}{1+\beta} f_A^i(v) + \frac{1}{1+\beta} f_B^i(v)$. Given the non-group-based EPSW, it is without loss of generality to specify only $f^i(v)$ instead of both $f_A^i(v)$ and $f_B^i(v)$, $i \in \{1,2\}$. Furthermore, an outcome of the game specifies wages paid by firms 1 and 2 to (almost) all of its workers, $w_1 \in \mathbb{R}_+$ and $w_2 \in \mathbb{R}_+$, respectively. Without loss of generality, assume $w_1 \leq w_2$.

Proposition **6.** *Suppose there is a non-group-based EPSW.*

1. *There exist a continuum of non-equivalent core outcomes. In any core outcome, $w_1 < w_2$.*
2. *There exists one core outcome (and its equivalent outcomes) in which almost all workers are employed. In all other core outcomes, a strictly positive measure of workers are unemployed.*
3. *Consider any two core outcomes. The measure of unemployed workers is higher in the first outcome if and only if firm profit is lower in the first outcome.*

Several observations are in order under a non-group-based EPSW. First, segregation of workers across firms occurs by productivity. More specifically, one firm hires almost every worker (from both groups A and B) whose productivity is above a threshold, the other firm hires almost every worker with productivity below that threshold but above another threshold, and the lowest-productivity workers remain unemployed. Second, workers of the same productivity receive the same wage, irrespective of their group identity. Third, there is no wage gap between workers within the same firm.

Remark **7.** The effect of non-group-based EPSW on wage gaps between the groups is indeterminate. First we show that the difference in wages paid by the two firms can be arbitrarily large or arbitrarily small in different core outcomes *in the same market*. Second, we show that again in different core outcomes in the same market, a non-group-based EPSW can either increase or decrease the gap in average pay between $A-$ and $B-$group workers compared to the core outcome wage gap without EPSW (which is equal to $\mathbb{E}_A(v) - \mathbb{E}_B(v)$ by (1)). More formally, under non-group-based EPSW:

1. For any $\epsilon > 0$ there exists a market (β, F_A, F_B) such that there exist core outcomes O and O' (both with $w_2 > w_1$) such that: $w_2 - w_1 > 1 - \epsilon$ in O and $w_2 - w_1 < \epsilon$ in O'.

2. There exists a market (β, F_A, F_B) and two core outcomes O and O' (both with $w_2 > w_1$) such that: $AW_A^O - AW_B^O > \mathbb{E}_A(v) - \mathbb{E}_B(v)$ and $AW_A^{O'} - AW_B^{O'} < \mathbb{E}_A(v) - \mathbb{E}_B(v)$.

We provide a constructive proof of this remark in the appendix.

4 Discussion

Our analysis thus far has centered around a model with two homogeneous firms and two groups of workers, and moreover, EPSW is applied equally to both firms, so that the effect of EPSW can be meaningfully analyzed in the simplest setup possible. In this section, we discuss how relaxing these simplifying assumptions affects our theoretical predictions. Formal results are relegated to Appendix B.

4.1 Multiple Groups and/or Firms

In Appendix B.1 we analyze more general cases in which the number of firms or groups (or both) is larger than two. Most of our findings generalize to these cases, while we also find some subtleties under group-based EPSW. Without EPSW, there is a continuum of core outcomes and, in any core outcome, almost every worker is employed and receives a wage equal to her productivity (a generalization of Proposition 1). With a non-group-based EPSW, each firm "segregates by productivity" by setting a uniform wage in any core outcome (a generalization of Proposition 6). With a group-based EPSW, if the number of the firms is larger than the number of the groups, the outcome becomes similar to the case without EPSW: Specifically, firms completely segregate by group, each firm earns zero profit, and almost every worker is employed and receives a wage equal to her productivity. By contrast, if the number of the groups is sufficiently large, then the core outcomes under the group-based EPSW becomes equivalent to those under non-group-based EPSW.

Overall, we find that our analysis readily generalizes if there is no EPSW or the EPSW is not based on groups. By contrast, we find that the effect of group-based EPSW depends on some characteristics of the specific applications. More specifically, our analysis suggests that the key parameters to beware are the number of groups to be protected by the law, as well as the competitive environment in the sense of the firms competing for workers from the same segment of the labor market.

4.2 Heterogeneous Treatment EPSW

One form of homogeneity imposed in our base model is that both firms are constrained by EPSW. In Appendix B.2 we discuss the implications of our model in which a group-based EPSW applies to only one of the two firms, a situation we refer to as *heterogeneous treatment*. Cases of heterogeneous treatment have been documented and studied by economists; a U.S. federal EPEW policy restricted only federal contractors in the 1970s (see Donohue III and Heckman (1991)). As we discuss in our empirical application below, this model offers important predictions in the study of EPSW in Chile.

To fix ideas, consider again a model with two firms, but now let us assume that firm 1 is subject to the group-based EPSW while firm 2 is not. Proposition 11 demonstrates that the job segregation effect of homogeneous EPSW largely carries over to the case with heterogeneous EPSW, while the effect on the wage gap disappears completely. Specifically, in any core outcome, firm 1 is fully segregated in the sense that it hires from only (at most) one group, while firm 2 can hire workers from both groups; meanwhile, almost every worker receives a wage equal to her productivity. Therefore, introduction of heterogeneous-treatment EPSW leads to no changes in wages (and therefore, to the wage gap) compared to the case without EPSW.

4.3 Taste-Based Bias

Our base model does not explicitly incorporate (taste-based) bias against the minority group, nor does it consider differences in bias across firms. It is worth noting, however, that the base model allows for bias against the minority group, as long as there is no heterogeneity in the two firms in terms of their bias. Specifically, we allow the distributions of productivity of $A-$group workers to be different from those for $B-$group workers. By interpreting productivity of $B-$group workers as net of the disutility that (a manager of) the firm incurs when hiring a $B-$group worker, the model becomes one without any explicit disutility term associated with $B-$group workers, while their productivity distributions are shifted to reflect the effect of the disutility for the firms.

In Appendix B.3, we consider the case in which one firm (referred to as the biased firm) has biased preferences while the other firm is purely profit motivated.[14] Specifically, firm 1 incurs a constant per-worker disutility for hiring workers from $B-$group, while firm 2 does not incur any disutility from hiring a $B-$group worker. We show that the main predictions of our base models remain largely unchanged, though with some subtle changes. More specifically, without EPSW, firms completely segregate in any core outcome, just as in the case without a biased firm. With group-based EPSW, larger wage gaps arise in core outcomes compared to the case without EPSW.[15] Finally, the wage gap can be arbitrarily large if the share of $A-$group workers is sufficiently large (i.e. β is large). In these senses, the theoretical predictions for the case with a biased firm are largely unchanged from the case without a biased firm. We briefly discuss other forms of heterogeneity in Appendix B.4.

5 Empirical Analysis

In this section, we present an empirical test of our model findings by analyzing a 2009 EPSW in Chile.

5.1 Institutional Background

Chile is an OECD country with nearly 20 million inhabitants. The Chilean labor market is relatively concentrated in the formal employment sector; the informal labor market share around the time of the policy was 25%, the lowest in Latin America (Gasparini and Tornarolli, 2007). Only 10% of the (formal) workforce is unionized, and only union members are covered by collective bargaining agreements, implying that Chilean firms plausibly have a high degree of wage-setting power. Workers can be fired without cause and without notice at the cost of one month's wages.

The gender wage gap in Chile is similar to that in the United States. Chilean female workers earn 18–23% less than their male counterparts (Perticará and Bueno, 2009). Female labor force participation was roughly 30% in 2009, which is lower than in many OECD and other Latin American countries (Verick, 2014).

[14] Per the discussion of the last paragraph, one can also interpret the model as cases in which both firms have bias, but one of the firms incurs larger disutility from hiring a $B-$group worker than the other.

[15] Specifically, with group-based EPSW, the set of possible wage gaps across core outcomes expands in the set inclusion sense compared with the case without EPSW, and the "new" wage gaps that can be supported in a core outcome feature a larger wage gap than those without EPSW.

5.2 EPSW implementation

In June 2009, Law 20.348 was signed as an amendment to Chile's labor code with the General Secretary of the Chilean Senate declaring, "The main objective of the initiative is to establish the right to equal remuneration between men and women for the provision of services of similar value." We refer throughout to June 2009 as the time of announcement. The law took effect in November 2009, which we refer to as the time of enactment. An important part of the discussion and debate surrounding the law was providing a definition of "similar work." The law specifies that a firm cannot pay a man and a woman different wages for "arbitrary reasons"; pay differences across genders are allowable only if workers fall into different coarse categories based on skills, qualifications, suitability, responsibility, or productivity. Firms that do not comply are subject to sizable monetary fines per offense, as we discuss below. The law also establishes a 10% discount for any other labor fines a firm is subject to if it pays men and women the same wages for "similar jobs and responsibilities." We therefore classify the law for our purposes as (a gender-based) EPSW.[16]

The law has different consequences for firms of different sizes, based on the number of a firm's workers with long-term employment contracts.[17] Firms with 10 or more long-term workers are required to explicitly have a grievance procedure for gender-based pay discrimination. Workers in firms above this threshold who allege the firm has violated Law 20.348 must receive a sufficient response from the firm within 30 days. If no such response is received, the worker can file a complaint at the Labor Inspection Office or can directly raise the issue with a labor court. Financial penalties also differ per infraction by firm size. Firms with 10–49 long-term workers found to violate the law are subject to a fine of 69-1,384 USD per worker-month of violation, while firms with fewer than 10 long-term workers are not subject to a financial penalty.[18]

Initial evidence suggests that the law was both widely known to workers, and enforced. In a 2013 governmental survey,[19] 11% of respondents stated that they know someone that has complained using the law. Through a public-records request, we found that 9,577 complaints were filed by workers alleging violations of Law 20.348, 9,723 inspections were carried out by the government, and that 489 individual firms were punished. The average fine amount was 1,167 USD per violation (each worker-month of unfair pay is a separate violation of the law).

5.3 Data

We study the effects of EPSW using matched worker-firm administrative data from the Chilean unemployment insurance system from January 2005 to December 2013. We observe a random sample of firms, stratified by size, totaling roughly 4% of all firms. In our data, an observation is a worker-

[16]Guideline 1187/018 published on April 2010 by the Directorate of Labor clarifies that 1) the law does not bind within gender group, and 2) that a firm paying even a single man more than a single woman (or vice versa) despite both performing similar work is in violation of the law.

[17]The vast majority of workers in Chile have either long-term contracts (no end date is specified ex ante) or fixed-term contracts (an end date is specified ex ante, although such contracts are automatically transitioned into long-term contracts if the worker continues to be employed beyond the contract end date). EPSW protections for workers of different contract types within the same firm are identical. Nearly half of firm-months in our dataset contain no workers with fixed-term contracts.

[18]Cruz and Rau (2022) further discuss how the law imposed mandatory transparency guidelines on worker roles in the firm, and additional fines for violations, for firms with at least 200 long-term workers. They show that the disclosure policy reduced the gender wage gap through a bargaining channel. Our analysis avoids firms treated by this additional policy, due to the potential confounding equilibrium effects of transparency policies (for further discussion on potential equilibrium effects, see Cullen and Pakzad-Hurson, 2023).

[19]See `https://www.evaluaciondelaley.cl/wp-content/uploads/2019/07/ley_20348_igualdad_remuneraciones.pdf`.

firm-month. For each observation, the data include worker pay[20] and demographic information including gender, education level, contract status, age, and marital status; additionally, we observe the firm's geographic location and industry code. We observe the entire employment history of each worker ever employed at a sampled firm. We discuss further details of our dataset in Appendix D.5.

Firms in our sample are typically small, with a median of 9 concurrent workers. However, there are outliers. Following Bennedsen et al. (2022); Böheim and Gust (2021); Duchini et al. (2022); Gulyas et al. (2023) we only consider firms of similar sizes at the time of policy announcement in order to limit size-based wage dynamics. In our main specifications, we consider firms with at least 6 and no more than 13 total workers at announcement, which includes roughly 40% of firms in our data. In order to limit the impacts of firms that close after EPSW announcement (who potentially fail due to the policy) or those that are founded after announcement (who came into being in a labor environment with wage constraints), we restrict our analysis to a balanced sample of firms that are in operation during the entire window of our analysis. As we discuss in the following pages and in Appendix D.3, our findings are qualitatively similar under alternative size and balance restrictions.

In Table 1, we present descriptive statistics for (column I) the set of firms prior to the size restriction, (column II) the set of firms following the size restriction, and (column III) the set of firms following balancing which constitute our baseline sample.

5.4 Empirical Strategy

Based on our theoretical findings, we investigate the effect of EPSW on gender segregation within firms and on the differential pay of men and women. To obtain the causal effect of EPSW on our outcomes of interest, we consider an event-study analysis wherein firms are considered "treated" if they were subject to EPSW at announcement (i.e. the firm in question employed at least 10 long-term workers in June 2009) and "control" otherwise.[21] We present and discuss supportive evidence for our designation of treatment status in Appendix D.1. Appendix D.1 also presents "placebo" tests which do not find statistically or economically meaningful effects at alternative firm size thresholds, supporting that the observed effects of EPSW around the size threshold specified in the law are plausibly causal.

Our theoretical analysis, and extensions in Section 4, predict an increase in segregation for all firms that are treated by EPSW. One conceptual hurdle is that we do not observe which workers perform "similar" work within a firm. Therefore, we consider complete firm-level gender segregation, implying the gender segregation of every set of similar workers within the firm. We also predict a shift in the wage gap in favor of the majority gender group of workers in the local labor market in which a firm treated by EPSW operates in. An important question is, therefore, how we define local labor markets. There are 21 industry codes and 321 geographic counties in our data. We define a local labor market a firm operates in by the firm's geographic county and industry code pair.

[20]Worker monthly pay at each firm is top coded in our data. The treshold for top coding varies over time; in June 2009, the top-code threshold was roughly $3,550. The share of observations in our data that are top coded is 1.7%.

[21]Following the discussion in Section 4, the magnitude of the effects of EPSW on a firm are predicted to depend on the number and treatment status of its competitors. We do not observe all firms in a local labor market, nor do we separately observe any firm's direct competitors. Our event-study analysis assumes that only the treatment status of the own firm is relevant for determining EPSW's directional effect on the outcomes we study, and the outcome variables we analyze comport with this assumptions. By contrast, a discontinuity-based identification strategy around the size threshold of 10 would implicitly require additional assumptions about the share of a firm's competitors that are treated by EPSW. As we have little support in making such assumptions, we do not proceed with this alternative approach.

Table 1: Descriptive Statistics

	(I) All firms	(II) 6 to 13 workers at announcement	(III) Balanced 6 to 13 workers at announcement
Panel (a): Workers			
Average age at announcement	34.464	34.170	34.172
Share with tertiary education	0.129	0.130	0.134
Share male	0.700	0.736	0.733
Share married	0.378	0.373	0.374
Share with residence in Santiago Region	0.370	0.393	0.395
Share in female majority industry-county at announcement	0.180	0.161	0.165
Number of workers	828,906	414,344	269,170
Panel (b): Firms			
Average number of workers at announcement	12.104	8.656	8.716
Share in Santiago Region	0.487	0.553	0.568
Share in Agriculture	0.104	0.090	0.097
Share in Manufacturing	0.087	0.130	0.159
Share in female majority industry-county at announcement	0.141	0.154	0.166
Number of firms	23,449	6,124	3,201

Notes: This table displays summary statistics for the different samples used in the paper. The unit of our panels is the worker-firm-month. In Panel (a), we display figures about the workers present in our data. In Panel (b), we display figures about the firms in our data. In column I, we display figures for the data without firm size restrictions or balancing. In column II, we display figures for the data in column I after restricting for firms that employed between 6 and 13 workers in June 2009. In column III, we display figures for the data in column II after further restricting for firms that are in our data in every month between January 2005 and December 2013. Column III is the sample we use in the majority of our analysis. The rows within each panel label the displayed variables.

5.5 Effect of EPSW on Segregation

To study the effect of EPSW on gender segregation within the firm, we consider a panel in which an observation is a firm-month. We let j index a firm and let t index a month. We construct a full segregation indicator, $full_{jt}$, that equals 1 in time t if all workers employed by firm j are of the same gender at time t, and 0 otherwise. We also construct an indicator for whether the firm employs at least 10 long-term workers in June 2009 (policy announcement date), which we call $above10_j$. Finally, we construct a post-treatment indicator $post_t$ that is 1 if the time t of the observation is from June 2009 or later, and zero otherwise. We estimate difference-in-difference models of the following form:

$$full_{jt} = \alpha_j + \alpha_{k(j)t} + \beta^{seg}(above10_j \times post_t) + X_{jt}\Lambda + \epsilon_{jt} \tag{5}$$

where X_{jt} is a vector of controls indicating the share of workers (strictly) younger than the median age in the industry-region, the share of workers with tertiary education, and the share of workers that have long-term contracts. α_j is a fixed effect for firm j, and $\alpha_{k(j)t}$ are time-varying fixed effects at the level of firm j's industry-county (we provide results from additional empirical specifications which consider time trends at more aggregated levels in Appendix D.2). Our coefficient of interest is β^{seg}, and we interpret it as the effect of the policy on the share of gender-segregated firms.

To understand more about the dynamic effects of EPSW, we consider the following difference-in-differences model year by year, where we omit the year before policy announcement as the reference period, so that the set of years included is $\mathcal{T} = \{2005,2006,2007,2009,...,2013\}$. By construction, each year (indexed by τ) corresponds to twelve time periods (indexed by t).

$$full_{jt} = \alpha_j + \alpha_{k(j)t} + \sum_{\tau \in \mathcal{T}} \beta_{\tau}^{seg} D_{jt} + X_{jt}\Lambda + \epsilon_{jt} \quad (6)$$

where D_{jt} is an indicator that equals 1 in time period t if firm j employs at least 10 long-term workers at policy announcement, and zero otherwise. β_{τ}^{seg} is the average difference in segregation between treated and control firms in year τ (relative to 2008).

Our identifying assumption is that parallel trends hold between treated and control firms. That is, $\mathbb{E}[\epsilon_{jt} \cdot D_{jt}] = 0$ for all t. This strategy builds in a partial falsification test, in that we expect coefficient estimates of β_{τ}^{seg} to be zero for all $\tau < 2009$.

Table 2 presents estimates on the effect of EPSW on segregation. Column I presents our baseline results from (5). We find a 4.6 percentage point increase in segregation following EPSW, from a baseline of 31% of firms that were fully segregated at EPSW announcement. Columns II-VII present results on segregation from alternative empirical specifications and alternative sample selections. We discuss these specifications further in Appendix D.3. Across all specifications, the increase in segregation due to EPSW is statistically significant at conventional levels. Additionally, we present estimates from (6) to support our parallel trends assumption. We show that the mean pre-treatment estimates are small and statistically insignificant, and we cannot reject an F-test that all pre-treatment coefficients are jointly zero.

Figure 5 displays the estimated coefficients of interest from (6). Prior to EPSW, the coefficient of interest is statistically indistinguishable from 0 in all years $\tau < 2009$. In the first year of EPSW, segregation rises by 1.99 percentage points in the treated group compared to the control group (p-value = 0.135) and rises to 5.11 percentage points (p-value= 0.038) by year five of EPSW.

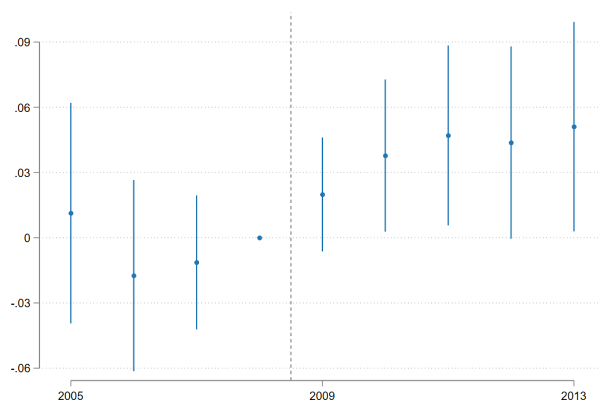
To better understand how EPSW affects firm incentives to segregate, we consider its effect on the share of firms that are mostly-but-not-fully segregated. Our model makes strong predictions that firms bound by EPSW will have incentives to fully segregate in equilibrium, but not that there are particular incentives to partially segregate. Of course, our model does not capture every complexity present in the labor market, and there may be other barriers preventing some firms from fully segregating. Nevertheless, even with such complexities, the equilibrium channels we study in this paper suggest that treated firms are likely to fully segregate if doing so would present a small change to their workforce. That is, firms that would have otherwise had only a small number of workers of the "wrong" gender that prevent full segregation may be particularly likely to end their relationship with these workers to achieve full segregation. Therefore, the economic forces present in our model likely indicate a decrease in the share of treated firms that are almost-but-not-fully segregated after EPSW.

Table 2: Effect of EPSW on Segregation

	(I) Baseline	(II) No firm FEs	(III) No controls	(IV) Unbalanced sample	(V) Doughnut hole	(VI) Narrower band	(VII) Wider band
($\hat{\beta}^{seg}$) Post × Treated	0.0455***	0.0420**	0.0368**	0.0471***	0.0431**	0.0432**	0.0651***
	(0.0171)	(0.0168)	(0.0171)	(0.0144)	(0.0199)	(0.0203)	(0.0157)
Mean Pre-Treatment	-0.0044	-0.0033	-0.0004	0.0032	-0.0002	-0.0066	-0.0051
	(0.0140)	(0.0134)	(0.0141)	(0.0125)	(0.0189)	(0.0162)	(0.0129)
No Pre-Trend p-value	0.3237	0.2556	0.6262	0.2009	0.7872	0.1485	0.3475
Number of Firms	2,638	2,638	2,638	4,275	2,244	1,920	3,325
Number of Observations	284,904	284,904	284,904	418,049	242,352	207,360	359,100
Fixed effects							
Firm	Yes	No	Yes	Yes	Yes	Yes	Yes
Month × industry × county	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm-month level controls	Yes	Yes	No	Yes	Yes	Yes	Yes

Notes: The unit of analysis is the firm-month and the dependent variable is a binary variable that indicates whether all workers at the firm in question are of a single gender in a given month. Column I presents estimated coefficient $\hat{\beta}^{seg}$ for our baseline difference-in-differences regression presented in (5). Firm-month controls included are: the share of workers younger than the median age in the industry-region, the share of workers with tertiary education, and the share of workers that have long-term contracts. The mean pre-treatment effect is the mean of $\hat{\beta}_{\tau}^{seg}$ for $\tau \in \{2005,2006,2007\}$ calculated from (6), and the no pre-trend p-value is derived from a joint F-test that $\beta_{\tau}^{seg}=0$ for all $\tau \in \{2005,2006,2007\}$. Columns II-VII present the analogous information for alternative sample selections and empirical specifications as described in Appendix D.3: column II removes firm fixed effects, column III removes the vector of controls, column IV considers the sample of all firms that exist at announcement, column V drops firms with 9 or 10 workers at announcement from our baseline sample, column VI drops firms with 6 or 13 workers at announcement from our baseline sample, and column VII adds firms with 5 or 14 workers at announcement to our baseline sample. Throughout, standard errors in parentheses are two-way clustered at the firm and month levels. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Figure 5: Dynamic Path of EPSW's Effect on Segregation



Notes: This figure displays estimated coefficients $\hat{\beta}_{\tau}^{seg}$ for the difference-in-differences regression described in (6). The specification corresponds to that in column I of Table 2. Bars depict 95% confidence intervals.

We reanalyze our firm-based analysis with a different dependent variable: "near" segregation.

Specifically, we define a firm j to be nearly segregated at time t if the share of workers in the majority gender of its workforce is in the interval [0.8,1). Note that j is classified as "nearly" segregated at time t only if it is not fully segregated at time t. We select 0.8 as the lower end of the interval for the definition of this outcome variable due to our size restrictions; firms in our sample typically are nearly segregated only in time periods in which they employ either 1 or 2 workers of the non-majority gender; however, our findings are robust to other selections of the lower end of the range. We re-estimate (5) and (6) with near segregation as the outcome variable and present these results in Table 3 and Figure 6. We refer to the associated coefficients of interest as $\beta^{nearseg}$ and $\beta_\tau^{nearseg}$, respectively.

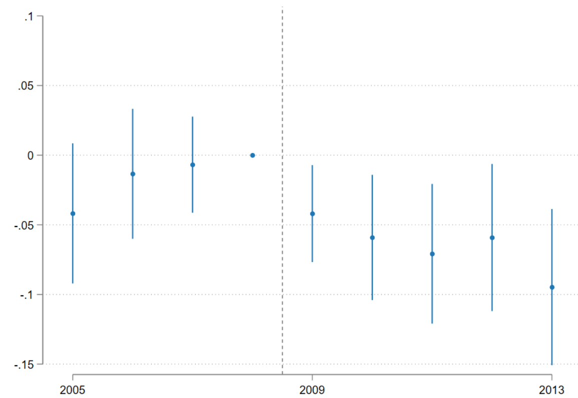
Table 3: Effect of EPSW on Near Segregation

	(I) Baseline	(II) No firm FEs	(III) No controls	(IV) Unbalanced sample	(V) Doughnut hole	(VI) Narrower band	(VII) Wider band
($\hat{\beta}^{nearseg}$) Post × Treated	-0.0505***	-0.0496***	-0.0418**	-0.0453***	-0.0414*	-0.0526**	-0.0796***
	(0.0179)	(0.0179)	(0.0180)	(0.0149)	(0.0230)	(0.0207)	(0.0169)
Mean Pre-Treatment	-0.0155	-0.0155	-0.0197	-0.0108	-0.0538***	-0.0052	-0.0105
	(0.0148)	(0.0143)	(0.0147)	(0.0123)	(0.0199)	(0.0168)	(0.0137)
No Pre-Trend p-value	0.2335	0.2641	0.2502	0.5456	0.0654	0.2477	0.3840
Number of Firms	2,638	2,638	2,638	4,275	2,244	1,920	3,325
Number of Observations	284,904	284,904	284,904	418,049	242,352	207,360	359,100
Fixed effects							
Firm	Yes	No	Yes	Yes	Yes	Yes	Yes
Month × industry × county	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm-month level controls	Yes	Yes	No	Yes	Yes	Yes	Yes

Notes: The unit of analysis is the firm-month and the dependent variable is "near" segregation, i.e. the dependent variable equals 1 if and only if the share of the majority gender of workers at firm j at time t is an element of [.8,1). Column I presents estimated coefficient $\hat{\beta}^{nearseg}$ for our baseline difference-in-differences regression presented in (5). Firm-month controls included are: the share of workers younger than the median age in the industry-region, the share of workers with tertiary education, and the share of workers that have long-term contracts. The mean pre-treatment effect is the mean of $\hat{\beta}_\tau^{nearseg}$ for $\tau \in \{2005,2006,2007\}$ calculated from (6), and the no pretrends p-value is derived from a joint F-test that $\beta_\tau^{nearseg} = 0$ for all $\tau \in \{2005,2006,2007\}$. Columns II-VII present the analogous information for alternative sample selections and empirical specifications as described in Appendix D.3: column II removes firm fixed effects, column III removes the vector of controls, column IV considers the sample of all firms that exist at announcement, column V drops firms with 9 or 10 workers at announcement from our baseline sample, column VI drops firms with 6 or 13 workers at announcement from our baseline sample, and column VII adds firms with 5 or 14 workers at announcement to our baseline sample. Throughout, standard errors in parentheses are two-way clustered at the firm and month levels. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 3 presents evidence to support our hypothesis on the effects of EPSW on near segregation. Each column presents a specification corresponding to the same column in Table 2. Our baseline specification in column I reveals that EPSW lowers the share of nearly segregated firms 5.1 percentage points following EPSW. Columns II-VII present results on near segregation from alternative empirical specifications and alternative sample selections. We discuss these specifications further in Appendix D.3. Across all specifications, our findings of a decrease in near segregation due to EPSW are statistically significant at conventional levels. Note that these estimates are similar in magnitude to the increase in the share of fully segregated, treated firms following EPSW (see Table 2). This "missing mass" of firms with near-but-not-full segregation suggests that firms that face relatively lower costs of fully segregating are those whose behavior most closely matches our theoretical predictions. Figure 6 shows the dynamic path of near segregation following EPSW. The pre-treatment coefficients are not statistically different from 0 in any year $\tau < 2009$; we note that pre-period point estimates are negative (the estimate in 2005, the first year of our panel, is -0.04 with a p-value of 0.11) which mitigates the magnitude of our coefficient estimate in Table 3.

Figure 6: Dynamic Path of EPSW's Effect on Near Segregation



Notes: This figure displays the estimated coefficients of interest for the difference-in-differences regression described in (6), where the outcome variable is "near" segregation, i.e. the dependent variable equals 1 if and only if the share of the majority gender of workers at firm j at time t is an element of [.8,1). The specification corresponds to that in column I of Table 3. Bars depict 95% confidence intervals.

5.6 Effect of EPSW on the Gender Wage Gap

Guided by our theoretical model, we are interested in studying the effect of EPSW on the wage gap between male and female workers. Our model predicts a relative benefit to men in male majority local labor markets and a relative benefit to women in female majority local labor markets. We define a local labor market as being female majority at time t if the share of female workers across all firms in a particular industry-county pair is at least 0.5, and otherwise, we define it as a male majority local labor market.

To estimate the effects of EPSW on the gender wage gap across local labor markets, we consider a panel in which an observation is a worker-firm-month. In order to remove potential confounds from workers who are simultaneously enrolled in formal schooling and those beyond Chile's official retirement age, we only consider workers aged 22-65. We let i index a worker, j index a firm, and t index a month. Let w_{ijt} be the earnings of worker i at firm j in month t. We construct an indicator, $male_i$, that equals 1 if the worker is a male, and 0 otherwise. We construct indicator $femalemaj_{jt}$ that equals 1 if firm j is in a female majority local labor market at time t, and 0 otherwise. We estimate difference models of the following form:

$$\begin{aligned}\ln w_{ijt} = {} & \alpha_i + \omega_{it} + \alpha_j + \alpha_{k(ij)t} + \gamma_1(above10_j \times post_t) + \psi_1(above10_j \times post_t \times femalemaj_{jt}) \\ & + \gamma_2(male_i \times post_t) + \psi_2(male_i \times post_t \times femalemaj_{jt}) \\ & + \gamma_3(above10_j \times male_i) + \psi_3(above10_j \times male_i \times femalemaj_{jt}) \\ & + \beta^{Mgap}(above10_j \times male_i \times post_t) + \beta^{Fgap}(above10_j \times male_i \times post_t \times femalemaj_{jt}) \\ & + X_{ijt}\Lambda + \epsilon_{ijt} \qquad (7)\end{aligned}$$

where α_i is a fixed effect for worker i, ω_{it} is worker i's age at time t, and X_{ijt} is a vector of firm-month level controls and worker-firm-month level controls. The firm-month level controls are the share of workers younger than the median age at the industry-region, share of workers with tertiary education, and the share of workers that have long-term contracts. The worker-firm-month level controls are the number of months at the firm and an indicator for whether the worker's earnings reach the top-coding threshold. $\alpha_{k(ij)t}$ are time fixed effects for workers in set $k(ij)$, where $k(ij)$ is a comparison group of workers to i employed at a comparison group of firms to j. Worker comparison groups are defined by equivalence across three binary dimensions at time t at firm j: an indicator for tertiary education, an indicator for long-term versus fixed-term contract, and an indicator for being above median age in the particular industry-region in which firm j operates. Firm comparison groups are defined by firms in the same industry-county in which firm j operates (we provide results from additional empirical specifications which consider time trends at more aggregated levels in Appendix D.2).

Our coefficients of interest are β^{Mgap} and β^{Fgap}. We interpret β^{Mgap} as the effect of the policy on the (percentage) wage gap between male and female workers in male majority labor markets. We interpret $\beta^{Fgap}+\beta^{Mgap}$ as the effect of the policy on the (percentage) wage gap between male and female workers in female labor markets.

To understand more about the dynamic effects of EPSW, we estimate the following triple difference model year by year, where we omit the year before policy announcement as the reference period, so that the set of years included is $\mathcal{T}=\{2005,2006,2007,2009,...,2013\}$. By construction, each year (indexed by τ) corresponds to twelve time periods (indexed by t). Let $year_\tau$ be an indicator that equals 1 in year τ, and zero otherwise.

$$\begin{aligned}\ln w_{ijt} = {} & \alpha_i+\omega_{it}+\alpha_j+\alpha_{k(ij)t}+\sum_{\tau\in\mathcal{T}}\gamma_{1\tau}(above10_j\times year_\tau)+\sum_{\tau\in\mathcal{T}}\psi_{1\tau}(above10_j\times femalemaj_{jt}\times year_\tau) \\ & +\sum_{\tau\in\mathcal{T}}\gamma_{2\tau}(male_i\times year_\tau)+\sum_{\tau\in\mathcal{T}}\psi_{2\tau}(male_i\times femalemaj_{jt}\times year_\tau) \\ & +\gamma_3(above10_j\times male_i)+\psi_3(above10_j\times male_i\times femalemaj_{jt}) \\ & +\sum_{\tau\in\mathcal{T}}\beta_\tau^{Mgap}D_{ijt}^M+\sum_{\tau\in\mathcal{T}}\beta_\tau^{Fgap}D_{ijt}^F \\ & +X_{ijt}\Lambda+\epsilon_{ijt} \qquad (8)\end{aligned}$$

where D_{ijt}^M is an indicator that equals 1 in time period t if firm j employs at least 10 long-term workers at the time of policy announcement, j's local labor market is coded as male majority in time t, and i is male, and zero otherwise. β_τ^{Mgap} is the average difference in log wages between men and women in treated versus control firms in year τ (relative to 2008) in male majority local labor markets. Similarly, D_{ijt}^F is an indicator that equals 1 in time period t if firm j employs at least 10 long-term workers at the time of policy announcement, j's local labor market is coded as female majority in time t, and i is male, and zero otherwise. β_τ^{Fgap} is the average difference in log wages between men and women in treated versus control firms in year τ (relative to 2008) in female majority labor markets *relative to male majority labor markets*. Therefore, $\beta_\tau^{Mgap}+\beta_\tau^{Mgap}$ is the average difference in log wages between men and women in treated versus control firms in year τ (relative to 2008) in female majority labor markets.

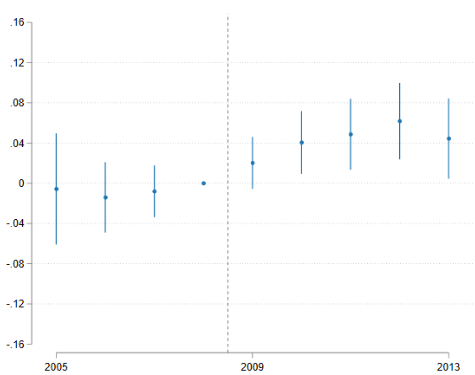
Our identifying assumption is that parallel trends hold between treated and control firms, that is, $\mathbb{E}[\epsilon_{ijt}\cdot D_{ijt}^M]=0$ and $\mathbb{E}[\epsilon_{ijt}\cdot(D_{ijt}^M+D_{ijt}^F)]=0$ for all t (Olden and Møen, 2022). This strategy builds in a partial falsification test, in that we expect coefficient estimates of β_τ^{Mgap}, and $\beta_\tau^{Mgap}+\beta_\tau^{Fgap}$ to be zero for $\tau<2009$.

Table 4 presents our estimates on the effect of EPSW on the gender wage gap. Column I presents our baseline results from (7). We find that EPSW increases the gender wage gap (in favor of men) by 3.8 percentage points in male majority labor markets, but decreases the gender wage gap (in favor of women) by 5.2 percentage points in female majority labor markets. For reference, the within-firm wage gap in favor of men, averaged across firms, is 35.8% at EPSW announcement among firms in our sample that employ both male and female workers. Columns II-VII present results on the gender wage gap from alternative empirical specifications and alternative sample selections. We discuss these specifications further in Appendix D.3. Across all specifications, our findings of an increase in the gender wage gap in male majority labor markets, and a decrease in the gender wage gap in female majority labor markets, are statistically significant at conventional levels. Additionally, we present estimates from (8) to support our parallel trends assumption. The mean pre-treatment estimates are small and statistically insignificant, and we cannot reject an F-test that all pre-treatment coefficients are jointly zero.

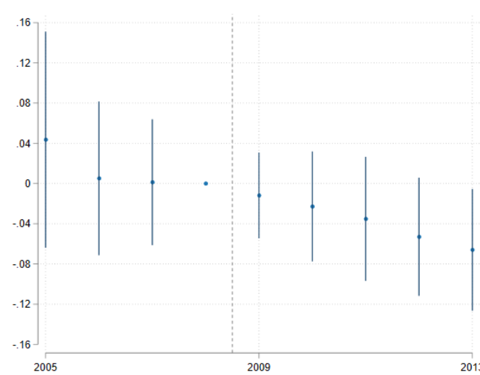
Figure 7 displays the estimated coefficients of interest from (8). Panel (a) presents estimates for male majority labor markets. The coefficient of interest is statistically indistinguishable from 0 in all years $\tau < 2009$. In the first year of EPSW, the wage gap rises (in favor of men) by 2.02 percentage points in the treated group compared to the control group (p-value = 0.123) and rises to 4.45 percentage points (p-value = 0.030) by year five of EPSW. Panel (b) presents estimates for female majority labor markets. The coefficient of interest is statistically indistinguishable from 0 in all years $\tau < 2009$. In the first year of EPSW, the wage gap falls (in favor of women) by 1.18 percentage points in the treated group compared to the control group (p-value= 0.583) and falls to 6.59 percentage points (p-value = 0.033) by year five of EPSW.

Figure 7: Dynamic Path of EPSW's Effect on Gender Wage Gap, by Majority Worker Group

(a): Male Majority Local Labor Markets



(b): Female Majority Local Labor Markets



Notes: Panel (a) displays estimated coefficients $\hat{\beta}_\tau^{Mgap}$ for the regression described in (8). Panel (b) displays estimated coefficients $\hat{\beta}_\tau^{Mgap} + \hat{\beta}_\tau^{Fgap}$ for the regression described in (8). In both, the specification corresponds to that in column I of Table 4. Bars depict 95% confidence intervals.

One potentially interesting policy-related question is the overall effect of EPSW on the gender wage gap across all industries. Given our findings that EPSW relatively benefits the majority group of workers in a local labor market, and the fact that the vast majority of workers are employed in male majority local labor markets, we expect EPSW to increase the wage gap in favor of men (although by a smaller magnitude that the increase in male majority labor markets). In Appendix D.4, we formally test this hypothesis using a triple difference model. We find that

Table 4: Effect of EPSW on Gender Wage Gap, by Majority Worker Group

	(I) Baseline	(II) No firm FEs	(III) No controls	(IV) Unbalanced sample	(V) Doughnut hole	(VI) Narrower band	(VII) Wider band
($\hat{\beta}^{Mgap}$) Treated × Male × Post	0.0378***	0.0340**	0.0387***	0.0273**	0.0494**	0.0266*	0.0171
	(0.0143)	(0.0138)	(0.0144)	(0.0113)	(0.0188)	(0.0152)	(0.0125)
($\hat{\beta}^{Fgap}$) Treated × Male × Post	-0.0902***	-0.0852***	-0.0934***	-0.0890***	-0.109***	-0.0750***	-0.0552**
× Female Majority Labor Market	(0.0233)	(0.0233)	(0.0234)	(0.0197)	(0.0298)	(0.0282)	(0.0226)
($\hat{\beta}^{Mgap}+\hat{\beta}^{Fgap}$) Effect in Female	-0.0524**	-0.0512**	-0.0547**	-0.0618***	-0.0600**	-0.0484*	-0.0381*
Majority Labor Market	(0.0217)	(0.0215)	(0.0216)	(0.0185)	(0.0286)	(0.0265)	(0.0210)
Mean Pre-Treatment	-0.0069	-0.0039	-0.0058	-0.0177*	-0.0017	-0.0253*	-0.0133
(Male Majority Labor Market)	(0.0126)	(0.0118)	(0.0129)	(0.0107)	(0.0170)	(0.0145)	(0.0122)
No Pre-Trend p-value	0.8285	0.8874	0.8361	0.1460	0.9369	0.2988	0.6646
(Male Majority Labor Market)							
Mean Pre-Treatment	0.0125	0.0189	0.0104	0.0129	-0.0065	-0.0023	0.0009
(Female Majority Labor Market)	(0.0253)	(0.0257)	(0.0254)	(0.0226)	(0.0276)	(0.0320)	(0.0215)
No Pre-Trend p-value	0.8427	0.6893	0.8480	0.8645	0.9228	0.3435	0.4014
(Female Majority Labor Market)							
Number of Firms	3,168	3,168	3,168	6,060	2,760	2,398	3,867
Number of Observations	3,333,272	3,333,272	3,333,272	5,321,733	2,839,244	2,634,618	3,982,352
Fixed effects							
Firm	Yes	No	Yes	Yes	Yes	Yes	Yes
Worker	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Worker age	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Month × industry × county × tertiary education × contract type × worker age	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm-month level controls	Yes	Yes	No	Yes	Yes	Yes	Yes
Worker-month level controls	Yes	Yes	No	Yes	Yes	Yes	Yes

Notes: The unit of analysis is the worker-firm-month and the dependent variable is the natural logarithm of the worker's wage at the firm in a given month. Column I presents estimated coefficients $\hat{\beta}^{Mgap}$ and $\hat{\beta}^{Fgap}$ for our baseline regression specification presented in (7). Time-varying fixed effects are defined as the intersection of: the firm's industry, the firm's county, an indicator for worker tertiary education, an indicator for worker contract type, an indicator for a worker being above median age in the industry-region. Firm-month controls included are: the share of workers younger than the median age in the industry-region, the share of workers with tertiary education, and the share of workers that have long-term contracts. Worker-firm-month levels controls included are the number of months at the firm and an indicator for reaching the earnings truncation threshold. The mean pre-treatment effects are the mean of $\hat{\beta}^{Mgap}_{\tau}$ and $\hat{\beta}^{Mgap}_{\tau}+\hat{\beta}^{Fgap}_{\tau}$, respectively, for $\tau \in \{2005{,}2006{,}2007\}$ calculated from (8), and the no pretrends p-values are derived from joint F-tests that $\beta^{Mgap}_{\tau}=0$ and $\beta^{Mgap}_{\tau}+\beta^{Fgap}_{\tau}=0$, respectively, for all $\tau \in \{2005{,}2006{,}2007\}$. Columns II-VII present the analogous information for alternative sample selections and empirical specifications as described in Appendix D.3: column II removes firm fixed effects, column III removes the vector of controls, column IV considers the sample of all firms that exist at announcement, column V drops firms with 9 or 10 workers at announcement from our baseline sample, column VI drops firms with 6 or 13 workers at announcement from our baseline sample, and column VII adds firms with 5 or 14 workers at announcement to our baseline sample. Throughout, standard errors in parentheses are two-way clustered at the firm and month levels. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

EPSW increases the overall gender wage gap (in favor of men) by 2.6 percentage points, which is statistically significant at conventional levels.

6 Conclusion

We find that the equilibrium effects of Equal Pay for Similar Work (EPSW) policies dominate their direct effects. Therefore, imposing these policies may lead to unintended outcomes. Our model demonstrates that EPSW targeted specifically to equalize pay across protected classes of workers leads to firms segregating their workforce in equilibrium to avoid the bite of the policy. Although discriminatory forces may lead to pay gaps across groups without EPSW, segregation caused by EPSW results in the minority group of workers in a labor market receiving even lower relative wages. Our empirical evaluation of Chile's 2009 EPSW—which prohibited unequal pay for similar work across genders—supports these predictions. The policy caused gender segregation within firm to rise, and the gender wage gap to rise. Importantly, the rise in the wage gap only occurred in male-majority local labor markets; in local labor markets with majority female workers, the wage gap closed. Both of these findings are as predicted by our theoretical analysis.

However, our model reveals how a change to EPSW can close wage gaps: removing clauses about protected classes of workers. Once they are removed, firms must pay all "similar" workers the same wage, regardless of group identity. Therefore, firms no longer have incentives to segregate their workforce by group identity in equilibrium. Such non-group-based EPSW can close wage gaps across groups. Additional design choices, such as imposing group employment quotas (Bertrand et al., 2019) or drastically *increasing* the number of protected classes can serve a similar purpose.[22]

Many important questions remain in understanding the equilibrium impacts of EPSW, and equity-related labor market policies in general. One difficulty is understanding the role of complementary policies; a benefit of studying the Chilean labor market is the relative dearth of alternative anti-discrimination policies prior to and contemporaneously with the enactment of EPSW. For this reason, further theoretical study of labor-market policies may be particularly fruitful. One particular avenue for further research is understanding firm incentives to heterogenize on-the-job responsibilities and duties to evade EPSW.

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[22] Many current US state EPSW policies define group identity as the intersection of: race, color, religion, sex, pregnancy, sexual orientation, gender identity, national origin, age, disability, or genetic information. Even taking a lower bound and assuming each of these is a binary characteristic leads to $2^{11} = 2048$ different protected classes, which removes the incentive for firms to specialize to one class, where the number of workers is likely quite small within any given local labor market.

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Appendix: For Online Publication

Appendix A presents proofs omitted from the main text. Appendix B presents extensions to our main model, as discussed in Section 4. Appendix C presents a non-cooperative formulation of the cooperative game studied in the main text. Appendix D presents additional empirical results and descriptions.

A Proofs

Proof of Proposition 1

Proof. Consider any outcome $O=\{(f_g^i(v),w_i^g(v))\}_{v\in[0,1],i=1,2,g=A,B}$ in which the following hold for almost all $v\in[0,1]$ and all $g\in\{A,B\}$:

1. $f_g^1(v)+f_g^2(v)=f_g(v)$, and
2. for all $i\in\{1,2\}$, $w_i^g(v)=v$ if $f_g^i(v)>0$.

We establish the desired result through three lemmas regarding these two enumerated conditions.

Lemma **3.** *Any outcome O satisfying the two enumerated conditions in the Proof of Proposition 1 is a core outcome.*

Proof of Lemma 3. Suppose not for the sake of contradiction. Then there are a firm j and a distinct outcome (for firm j) $\tilde{O}_j:=\{(\tilde{f}_g^j(v),\tilde{w}_j^g(v))\}_{v\in[0,1],g=A,B}$ that blocks O. In order for $\tilde{O}_j$ to block O it must be that $\pi_j^{\tilde{O}_j}>\pi_j^{O_j}$. However, it is also the case that

$$
\begin{aligned}
\pi_j^{O_j} &= \beta\int_0^1 [v-w_j^A(v)]f_A^j(v)dv+\int_0^1 [v-w_i^B(v)]f_B^j(v)dv \\
&= 0 \\
&\geq \beta\int_0^1 [v-\tilde{w}_j^A(v)]\tilde{f}_A^j(v)dv+\int_0^1 [v-\tilde{w}_i^B(v)]\tilde{f}_B^j(v)dv \\
&= \pi_j^{\tilde{O}_j}
\end{aligned}
$$

The second equality follows because, by the construction of O, either $f_g^j(v)=0$ or $w_j^g(v)=v$ for almost all v and all g, therefore, the integrand is almost always equal to zero. The inequality follows because of the following exhaustive cases for almost all v and all g, corresponding, respectively, to Conditions 1-4 of the definition of block:

- Suppose $\tilde{w}_j^g(v)\geq w_j^g(v)$ and $\tilde{w}_j^g(v)>w_{-j}^g(v)$, then it must be that $\tilde{w}_j^g(v)\geq v$ since $\max\{w_j^g(v),w_{-j}^g(v)\}=v$, which makes the integrand weakly negative,
- Suppose $\tilde{w}_j^g(v)\geq w_j^g(v)$ and $\tilde{f}_g^j(v)+f_g^{-j}(v)\leq f_g(v)$. If $\tilde{f}_g^j(v)=0$ then the integrand is weakly negative. If $\tilde{f}_g^j(v)>0$ then it must be that $f_g^{-j}(v)<f_g(v)$, and by the construction of O that $f_g^j(v)+f_g^{-j}(v)=f_g(v)$, it must be that $f_g^j(v)>0$. Therefore, it must be that $w_j^g(v)=v$, and the requirement that $\tilde{w}_j^g(v)\geq w_j^g(v)$ makes the integrand weakly negative.

- Suppose $\tilde{w}_j^g(v) > w_{-j}^g(v)$ and $\tilde{f}_g^j(v) + f_g^j(v) \leq f_g(v)$. If $\tilde{f}_g^j(v) = 0$ then the integrand is weakly negative. If $\tilde{f}_g^j(v) > 0$ then it must be that $f_g^j(v) < f_g(v)$, and by the construction of O that $f_g^j(v) + f_g^{-j}(v) = f_g(v)$, it must be that $f_g^{-j}(v) > 0$. Therefore, it must be that $w_{-j}^g(v) = v$, and the requirement that $\tilde{w}_j^g(v) > w_{-j}^g(v)$ makes the integrand strictly negative.

- Suppose $\tilde{f}_g^j(v) + f_g^j(v) + f_g^{-j}(v) \leq f_g(v)$ then it must be that $\tilde{f}_g^j(v) = 0$ since by the construction of O it is the case that $f_g^j(v) + f_g^{-j}(v) = f_g(v)$. Therefore, the integrand is weakly negative.

These cases reveal a contradiction with the premise that $\pi_j^{\tilde{O}_j} > \pi_j^{O_j}$. Therefore, O is a core outcome. □

LEMMA **4.** *There exist a continuum of (non-equivalent) core outcomes satisfying the two enumerated conditions in the Proof of Proposition 1.*

Proof. For any $v^* \in [0,1]$ there is an outcome O such that $f_g^1(v) = f_g(v)$ and $w_1^g(v) = v$ for all g and $v \leq v^*$, and $f_g^2(v) = f_g(v)$ and $w_2^g(v) = v$ for all g and $v > v^*$. □

It remains to show that any core outcome must be such that all workers are hired and all workers receive a wage equal to their productivity.

LEMMA **5.** *There exist no core outcomes which do not satisfy the two enumerated conditions in the Proof of Proposition 1.*

Proof. Suppose for contradiction that there is a core outcome $O = \{(f_g^i(v), w_i^g(v))\}_{v\in[0,1], i=1,2, g=A,B}$ such that there exists a set with positive measure V where the following fails for some $g \in \{A,B\}$ and all $v \in V$:

1. $f_g^1(v) + f_g^2(v) = f_g(v)$, and
2. for all $i \in \{1,2\}$, $w_i^g(v) = v$ if $f_g^i(v) > 0$.

Throughout, we assume that the above conditions are violated for $g = B$, and we construct a blocking outcome $\tilde{O}_j = \{(\tilde{f}_g^j(v), \tilde{w}_j^g(v))\}_{v\in[0,1], g=A,B}$, where $(\tilde{f}_A^j(v), \tilde{w}_j^A(v)) = (f_A^j(v), w_j^A(v))$ (or $\tilde{O}_{-j} = \{(\tilde{f}_g^{-j}(v), \tilde{w}_{-j}^g(v))\}_{v\in[0,1], g=A,B}$ where $(\tilde{f}_A^{-j}(v), \tilde{w}_{-j}^A(v)) = (f_A^{-j}(v), w_{-j}^A(v))$), i.e. we do not change the outcome for $A-$group workers for either firm. The argument in which the violations occur for $g = A$ is analogous, where terms related to the firm's profit must be multiplied by β. We show a contradiction by considering six exhaustive cases: By countable additivity of measure, the set of productivities that fails one of the above two enumerated points has positive measure if and only if at least one of the sets in the following six cases has a positive measure.

First, suppose there exist a firm j and a subset of productivities $V \subset [0,1]$ with positive measure such that $w_j^g(v) > v$ for all $v \in V$. Then $\tilde{O}_j$ where for all v:

$$\tilde{f}_g^j(v) := \begin{cases} f_g^j(v) & \text{if } v \notin V, \\ 0 & \text{if } v \in V. \end{cases} \qquad \tilde{w}_j^g(v) := \begin{cases} w_j^g(v) & \text{if } v \notin V, \\ 0 & \text{if } v \in V. \end{cases}$$

blocks O as j's profit increases and Condition 4 of the definition of block is satisfied for all $v \in V$ (i.e. the workers in V are fired) and for all $v \in [0,1] \setminus V$ Condition 2 of the definition of block is satisfied (i.e. there is no change in the hiring or wages of workers in $[0,1] \setminus V$). By construction, $\tilde{w}_j^g(v)$ satisfies our monotonicity condition.

Therefore, we proceed with the assumption that for each firm j, $w_j^g(v)\leq v$ for almost all v. Second, suppose there exist a firm j and a subset of productivities V with positive measure such that $f_g^1(v)+f_g^2(v)<f_g(v)$ and $w_j^g(v)<v$ for all $v\in V$. Then $\tilde{O}_j$ where for all v:

$$\tilde{f}_g^j(v):=f_g(v)-f_g^{-j}(v). \qquad \tilde{w}_j^g(v):=\begin{cases}0 & \text{if } \tilde{f}_g^j(v)=0,\\ \sup\limits_{v'\leq v} w_j^g(v') & \text{otherwise.}\end{cases}$$

blocks O as firm j's profit increases as some previously unemployed workers are hired at a wage strictly less than their productivity while all existing workers at j continue to be employed at the same wage as before, and Condition 2 of the definition of a block is satisfied for all $v\in[0,1]$ (i.e. no worker receives a wage cut and no workers are poached from firm $-j$). By construction, $\tilde{w}_j^g(v)$ satisfies our monotonicity condition.

Third, there exists a firm j and a subset of productivities V with positive measure such that $f_g^1(v)+f_g^2(v)<f_g(v)$ and $w_j^g(v)=v$ for all $v\in V$. Then, there exists $\varepsilon>0$ and V' with positive measure such that $f_g^1(v)+f_g^2(v)<f_g(v)-\varepsilon$ and $w_j^g(v)=v$ for all $v\in V'$.[23] Now, arbitrarily fix $p<1$ such that $\bar{f}_g(1-p)\leq p\varepsilon$ and $\frac{1}{2}p^2\varepsilon>\bar{f}_g(1-p)$ (note that those inequalities are satisfied by any sufficiently large $p<1$). By Halmos (1974, Theorem A, Page 68), there exists an interval $I:=[\underline{v},\bar{v}]\subseteq[0,1]$ such that $\mu(V'\cap I)>p\mu(I)$, where $\mu(\cdot)$ is the Lebesgue measure. Then $\tilde{O}_j$ where for all v:

$$\tilde{f}_g^j(v):=\begin{cases}f_g(v)-f_g^j(v)-f_g^{-j}(v) & \text{if } v\in I,\\ f_g^j(v) & \text{otherwise.}\end{cases} \qquad \tilde{w}_j^g(v):=\begin{cases}0 & \text{if } \tilde{f}_g^j(v)=0,\\ \sup\limits_{v'<\underline{v}} w_j^g(v') & \text{if } v\in I \text{ and } \tilde{f}_g^j(v)>0,\\ w_j^g(v) & \text{otherwise.}\end{cases}$$

blocks O. To see this, note Condition 4 of the definition of block is satisfied for all $v\in I$ (firm j fires all existing workers in set I and hires unemployed workers of the same productivity), and Condition 2 of the definition of block is satisfied for all $v\notin I$ (no worker receives a wage cut and are poached from firm $-j$). By construction, $\tilde{w}_j^g(v)$ satisfies our monotonicity condition.

To see that firm j's profit increases, let $\delta:=\underline{v}-\sup_{v\leq\underline{v}}w_j^g(v)$. Note that the firm makes an additional profit of at least

$$\delta p\mu(I)\varepsilon+\frac{1}{2}(p\mu(I))^2\varepsilon,$$

from hiring workers from set $V'\cap I$ while firing existing workers from $V'\cap I$ causes no loss (because those workers were hired at wages equal to their productivities), and the loss from losing workers from $I\setminus V'$ is bounded from above by $\bar{f}_g[(1-p)\mu(I)\times(\delta+\mu(I))]=\bar{f}_g(1-p)\delta\mu(I)+\bar{f}_g(1-p)\mu(I)^2$. Because p satisfies $\bar{f}_g(1-p)\leq p\varepsilon$ and $\frac{1}{2}p^2\varepsilon>\bar{f}_g(1-p)$ by assumption, the total change of firm j's payoff is strictly positive, as desired.

The previous two cases exhaust the possibility of a core outcome in which $f_g^1(v)+f_g^2(v)<f_g(v)$ for any g and a subset of productivities with positive measure. Therefore, we proceed with the assumption that $f_g^1(v)+f_g^2(v)=f_g(v)$ for almost all v.

[23]The proof is as follows: Suppose for contradiction that for each ε, any set of productivities such that $f_g^1(v)+f_g^2(v)<f_g(v)-\varepsilon$ and $w_j^g(v)=v$ has zero measure. Then, for each $n=1,2,...$ define the set $V_n:=\{v\in V|f_g(v)-f_g^1(v)-f_g^2(v)>\frac{1}{n}$ and $w_j^g(v)=v\}$. Then, by assumption, $V_1,V_2,...$ is an increasing sequence of sets and $\cup_n V_n=V^*:=\{v\in V|f_g(v)-f_g^1(v)+f_g^2(v)>0$ and $w_j^g(v)=v\}$. Therefore, by countable additivity of the Lebesgue measure, we have $\mu_g(V^*)=\lim_n\mu_g(V_n)=0$, which contradicts the assumption that V has positive measure and the fact that $V\subseteq V^*$.

Fourth, suppose that there exist j and a set V of productivities with positive measure such that $w_j^g(v) < v$ and $f_g^j(v) = f_g(v)$ for all $v \in V$. Then, there exists $\varepsilon > 0$ and V' with positive measure such that $w_j^g(v) < v - \varepsilon$ and $f_g^j(v) = f_g(v)$ for all $v \in V'$.[24] For any $p < 1$, by Halmos (1974, Theorem A, Page 68), there exists an interval $I^p := [\underline{v}^p, \bar{v}^p] \subseteq [0,1]$ such that $\mu(V' \cap I^p) > p\mu(I^p)$, where $\mu(\cdot)$ is the Lebesgue measure. Consider the following cases.

1. Suppose that there is no $V^p \subseteq [\bar{v}^p, 1]$ with positive measure such that $w_{-j}^g(v) < w_j^g(\bar{v}^p)$ and $f_g^{-j}(v) > 0$ for all $v \in V^p$ and for all p sufficiently close to 1. Let $w^p := \sup_{v < \underline{v}^p} w_{-j}^g(v)$. Then for a constant $\varepsilon' > 0$, $\tilde{O}_{-j}$ where for all v:

$$\tilde{f}_g^{-j}(v) := \begin{cases} f_g(v) - f_g^j(v) - f_g^{-j}(v) & \text{if } v \in I^p, \\ f_g^{-j}(v) & \text{otherwise.} \end{cases}$$

$$\tilde{w}_{-j}^g(v) := \begin{cases} 0 & \text{if } \tilde{f}_g^{-j}(v) = 0, \\ \max\{w^p, w_j^g(v) + \varepsilon'\} & \text{if } v \in I^p \text{ and } \tilde{f}_g^{-j}(v) > 0, \\ \max\{\sup_{v \leq \bar{v}^p} w_j^g(v) + \varepsilon', w_{-j}^g(v)\} & \text{if } v \in [\bar{v}^p, 1] \text{ and } \tilde{f}_g^{-j}(v) > 0, \\ w_{-j}^g(v) & \text{otherwise.} \end{cases}$$

blocks O for the following reasons: First, for all $v \in I^p$, Condition 4 of the definition of block is satisfied, and second, for all $v \notin I^p$, Condition 2 of the definition of block is satisfied. Note also that $\tilde{w}_j^g(v)$ satisfies our monotonicity condition by construction. It therefore remains only to show that $\tilde{O}_{-j}$ increases the profit of firm $-j$.

Firm $-j$ benefits from poaching workers in $I^p \cap V'$, which results in gains of at least

$$\underline{f}_g \int_{\underline{v}^p}^{\bar{v}^p - (1-p)(\bar{v}^p - \underline{v}^p)} \min\{v - w^p, \varepsilon - \varepsilon'\} dv.$$

This follows because, in the worst case, there are at most p fraction of workers in $I^p \cap V'$ who are lost by firm $-j$, and these p fraction of workers loaded into the right most part of I^p. The loss from losing existing (at most p fraction of) workers in I^p is upper bounded by

$$\bar{f}_g (1-p)(\bar{v}^p - \underline{v}^p)(\bar{v}^p - w^p).$$

Let $v^p := \max\{\min\{w^p + \varepsilon - \varepsilon', \bar{v}^p - (1-p)(\bar{v}^p - \underline{v}^p)\}, \underline{v}^p\}$. Then we can rewrite the lower bound on the gain as

$$\underline{f}_g \int_{\underline{v}^p}^{v^p} (v - w^p) dv + \underline{f}_g \int_{v^p}^{\bar{v}^p - (1-p)(\bar{v}^p - \underline{v}^p)} (\varepsilon - \varepsilon') dv. \tag{9}$$

We proceed by showing that the gain exceeds the loss over the interval I^p for sufficiently large p. We can rewrite (9) as:

$$(v^p - \underline{v}^p)(\underline{v}^p - w^p)\underline{f}_g + \frac{1}{2}(v^p - \underline{v}^p)^2 \underline{f}_g + (\varepsilon - \varepsilon')(\bar{v}^p - (1-p)(\bar{v}^p - \underline{v}^p) - v^p)\underline{f}_g,$$

[24] The proof is analogous to the one given in Footnote 23.

which is no smaller than

$$
\begin{aligned}
&(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g\left[(v^p-\underline{v}^p)(\underline{v}^p-w^p)+(v^p-\underline{v}^p)^2+(\bar{v}^p-(1-p)(\bar{v}^p-\underline{v}^p)-v^p)\right]\\
=&(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g[(v^p-\underline{v}^p)(v^p-w^p)+(\bar{v}^p-(1-p)(\bar{v}^p-\underline{v}^p)-v^p)]
\end{aligned}
$$

and therefore, a lower bound on the gain minus the loss is proportional to

$$
\frac{(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g}{(1-p)(\bar{v}^p-\underline{v}^p)}[(v^p-\underline{v}^p)(v^p-w^p)+(\bar{v}^p-(1-p)(\bar{v}^p-\underline{v}^p)-v^p)]-\bar{f}_g(\bar{v}^p-w^p) \qquad (10)
$$

We proceed to provide a lower bound of (10). We do so by considering three exhaustive alternatives.

First, suppose that $v^p=w^p+\varepsilon-\varepsilon'$. (10) is proportional to

$$
\frac{(v^p-\underline{v}^p)(v^p-w^p)+\bar{v}^p-v^p}{(1-p)(\bar{v}^p-\underline{v}^p)}-1-\frac{\bar{f}_g(\bar{v}^p-w^p)}{(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g},
$$

and

$$
\begin{aligned}
\frac{(v^p-\underline{v}^p)(v^p-w^p)+\bar{v}^p-v^p}{(1-p)(\bar{v}^p-\underline{v}^p)}-1-\frac{\bar{f}_g(\bar{v}^p-w^p)}{(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g}&=\frac{(v^p-\underline{v}^p)(v^p-w^p)+\underline{v}^p-v^p}{(1-p)(\bar{v}^p-\underline{v}^p)}+\frac{1}{1-p}-1-\frac{\bar{f}_g(\bar{v}^p-w^p)}{(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g}\\
&=\frac{(v^p-\underline{v}^p)[(v^p-w^p)-1]}{(1-p)(\bar{v}^p-\underline{v}^p)}+\frac{1}{1-p}-1-\frac{\bar{f}_g(\bar{v}^p-w^p)}{(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g}\\
&\geq\frac{(v^p-w^p)-1}{(1-p)}+\frac{1}{1-p}-1-\frac{\bar{f}_g(\bar{v}^p-w^p)}{(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g}\\
&=\frac{v^p-w^p}{1-p}-1-\frac{\bar{f}_g(\bar{v}^p-w^p)}{(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g}\\
&=\frac{\varepsilon-\varepsilon'}{1-p}-1-\frac{\bar{f}_g(\bar{v}^p-w^p)}{(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g},
\end{aligned}
$$

where the first inequality follows because $1\geq\bar{v}^p>\underline{v}^p$ and $\bar{v}^p\geq v^p\geq\underline{v}^p\geq w^p\geq 0$ for all p which implies that $\frac{v^p-\underline{v}^p}{\bar{v}^p-\underline{v}^p}\in[0,1]$ and $v^p-w^p\leq 1$, the final equality follows because $v^p=w^p+\varepsilon-\varepsilon'$. Renormalizing this term, a lower bound on the net profit over region $I^p\cap V'$ in this case is

$$
(1-p)(\bar{v}^p-\underline{v}^p)(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g\left[\frac{\varepsilon-\varepsilon'}{1-p}-1-\frac{\bar{f}_g(\bar{v}^p-w^p)}{(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g}\right].
$$

It is clear that this expression is positive for any sufficiently large $p<1$ since $\varepsilon-\varepsilon'>0$ by assumption.

Second, suppose that $v^p=\bar{v}^p-(1-p)(\bar{v}^p-\underline{v}^p)$. Then (10) can be rewritten as

$$\frac{(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g}{(1-p)}p(v^p-w^p)-\bar{f}_g(\bar{v}^p-w^p).$$

Noting that each of the terms in parentheses is non-negative by construction, then we have shown the desired result if we show that, for sufficiently large p,

$$\frac{(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g}{(1-p)}\frac{p(v^p-w^p)}{\bar{v}^p-w^p}-\bar{f}_g$$

is bounded away from zero. We can see that for all p

$$\begin{aligned}\frac{(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g}{(1-p)}\frac{p(v^p-w^p)}{\bar{v}^p-w^p}-\bar{f}_g &= \frac{(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g}{(1-p)}\left[p-p(1-p)\frac{\bar{v}^p-\underline{v}^p}{\bar{v}^p-w^p}\right]-\bar{f}_g\\ &\geq (\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g\left[\frac{p}{1-p}-p\right]-\bar{f}_g\\ &= (\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g\frac{p^2}{1-p}-\bar{f}_g\end{aligned}$$

where the first equality comes from substituting in $v^p=\bar{v}^p-(1-p)(\bar{v}^p-\underline{v}^p)$, and the inequality follows because $\frac{\bar{v}^p-\underline{v}^p}{\bar{v}^p-w^p}\leq 1$ for all p because $\bar{v}^p>\underline{v}^p$ and $\bar{v}^p\geq v^p\geq\underline{v}^p\geq w^p$. Renormalizing this term, a lower bound on the net profit over region $I^p\cap V'$ in this case is

$$(1-p)(\bar{v}^p-\underline{v}^p)(\bar{v}^p-w^p)\left[(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g\frac{p^2}{1-p}-\bar{f}_g\right].$$

It is clear that the final line is positive for any sufficiently large $p<1$ since $\varepsilon-\varepsilon'>0$ by assumption.

Third, suppose that $v^p=\underline{v}^p$. Then (10) is equal to

$$(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g\frac{p}{1-p}-\bar{f}_g(\bar{v}^p-w^p)$$

Noting that each of the terms in parentheses is non-negative by construction and $\bar{v}^p>\underline{v}^p\geq w^p$, then the above equation is bounded below by

$$(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g\frac{p}{1-p}-\bar{f}_g.$$

Renormalizing this term, a lower bound on the net profit over region $I^p\cap V'$ in this case is

$$(1-p)(\bar{v}^p-\underline{v}^p)\left[(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g\frac{p}{1-p}-\bar{f}_g\right].$$

This expression is positive for any sufficiently large $p<1$ since $\varepsilon-\varepsilon'>0$ by assumption.

Therefore, we have shown that firm $-j$'s change in profit from workers with $v \in I^p \cap V'$ is at least

$$(1-p)(\bar{v}^p-\underline{v}^p)\times\min\left\{(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g\left[\frac{\varepsilon-\varepsilon'}{1-p}-1-\frac{\bar{f}_g(\bar{v}^p-w^p)}{(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g}\right],(\bar{v}^p-w^p)\left[(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g\frac{p^2}{1-p}-\bar{f}_g\right],(\varepsilon-\varepsilon')\frac{1}{2}\underline{f}_g\frac{p}{1-p}-\bar{f}_g\right\},$$

and this expression is positive for every $\epsilon' \in (0,\epsilon)$ and sufficiently large $p<1$. Moreover, it can be observed by inspection that this expression is decreasing in ϵ'. Furthermore, the wage paid for workers in $[\bar{v}^p,1]$ may increase at most by ε', resulting in a loss of profit from the increased wage being bounded from above by $\varepsilon'\beta$. From these observations, for any sufficiently large $p<1$ and sufficiently small $\varepsilon'>0$, firm $-j$ strictly profits with the block, i.e. $\pi_{-j}^{\tilde{O}_{-j}} > \pi_{-j}^{O_{-j}}$ as desired.

2. Suppose that there exists a subset of $[0,1)$ whose supremum is 1 such that, for each p in that subset, there is a set $V^p \subseteq [\bar{v}^p,1]$ with positive measure such that $w_{-j}^g(v) < w_j^g(\bar{v}^p)$ and $f_g^{-j}(v)>0$ for all $v\in V^p$. Fix any such p and V^p. Consider $\tilde{O}_j$ where for all v:

$$\tilde{f}_g^j(v) := \begin{cases} f_g(v) & \text{if } v\in V^p, \\ f_g^j(v) & \text{otherwise.}\end{cases} \qquad \tilde{w}_j^g(v) := \begin{cases} 0 & \text{if } \tilde{f}_g^j(v)=0, \\ w_j^g(v) & \text{otherwise.}\end{cases}$$

$\tilde{O}_j$ blocks O_j for the following reasons: Condition 1 of the definition of block is satisfied for all $v \in V^p$ since $w_{-j}^g(v) < w_j^g(\bar{v}^p) \le \tilde{w}_j^g(v)$ for all $v\in V^p$ by construction, and Condition 2 of the definition of block is satisfied for all $v \notin V^p$. Moreover, firm j obtains a strictly higher profit under this outcome.[25]

Fifth, suppose that there exist j and $V\subset[0,1]$ with positive measure such that $0\le w_{-j}^g(v)\le w_j^g(v)<v$ and $f_g^j(v)\in(0,f_g(v))$ for all $v\in V$. Then, there exists $\varepsilon>0$ and $V'\subset V$ with positive measure such that $0\le w_{-j}^g(v)\le w_j^g(v)<v-\varepsilon$ and $f_g^j(v)\in(0,f_g(v)-\varepsilon)$ for all $v\in V'$. Then for a constant $\varepsilon'>0$, consider $\tilde{O}_j$ where for all v:

$$\tilde{f}_g^j(v) := \begin{cases} f_g(v) & \text{if } v\in V', \\ f_g^j(v) & \text{otherwise.}\end{cases} \qquad \tilde{w}_j^g(v) := \begin{cases} 0 & \text{if } \tilde{f}_g^j(v)=0, \\ w_j^g(v)+\varepsilon' & \text{otherwise.}\end{cases}$$

$\tilde{O}_j$ blocks O_j for the following reasons: Condition 1 of the definition of block is satisfied for all $v\in V'$ since $w_{-j}^g(v)\le w_j^g(\bar{v})<\tilde{w}_j^g(v)$ for all $v\in V'$ by construction, and Condition 2 of the definition of the block is satisfied for all $v\notin V'$. To see that firm j's profit increases, first note that firm j benefits from hiring workers from V', which results in an additional profit of at least $(\varepsilon-\varepsilon')\varepsilon\mu(V')$. Meanwhile, the firm may lose from paying more for existing workers, but the associated loss is bounded from above by $\varepsilon'\beta$. Therefore, for any sufficiently small ε', firm j's profit increases, as desired. Note also that monotonicity is satisfied by $\tilde{w}_j^g(\cdot)$ because $w_j^g(\cdot)$ is monotone and ε' is a constant.

Cases 4 and 5 exhaust the possibility of a core outcome in which there exists a set V' of positive Lebesgue measure such that $\max\{w_1^g(v),w_2^g(v)\}<v$ for almost all $v\in V'$. Therefore, we proceed with the assumption that for almost any $v\in[0,1]$ there exists a firm j such that $w_j^g(v)=v$.

Sixth, suppose there exist a set V'' of positive Lebesgue measure and a firm j such that $0\le w_{-j}^g(v)<w_j^g(v)=v$ and $f_g^{-j}(v)\in(0,f_g(v))$ for all $v\in V''$. Intuitively, we proceed by showing

[25] In the proposed block $\tilde{O}_j$, one could alternatively set $\tilde{w}_j^g(v):=w_j^g(v)$ for every $v\in[0,1]$, and the proof works without change.

that firm j can fire some subset of its workers who receive wages equal to productivity, and poach workers of the same productivity from firm $-j$. We proceed by constructing a set of workers with positive measure where such a maneuver is feasible.

Following earlier arguments, there exist $\delta > 0$ and a set V' with $\mu(V') > 0$ such that $0 \le w^g_{-j}(v) + \delta < w^g_j(v) = v$ and $f^{-j}_g(v) \in (0, f_g(v))$ for all $v \in V'$.

Let $\mathrm{cl}(V')$ be the closure of V'. $\mathrm{cl}(V')$ is compact because it is a closed and bounded subset of $[0,1]$. For any $v \in [0,1]$ and $\varepsilon > 0$, define $B_\varepsilon(v) := (v-\varepsilon, v+\varepsilon) \cap [0,1]$ to be the ε-ball around v. Consider a collection of sets $\{B_\varepsilon(v')\}_{v' \in \mathrm{cl}(V')}$ where $\varepsilon < \frac{\delta}{2}$. It is obvious that $\{B_\varepsilon(v')\}_{v' \in \mathrm{cl}(V')}$ covers $\mathrm{cl}(V')$ and, because $\mathrm{cl}(V')$ is compact, there exist $v_1, v_2, ..., v_n \in \mathrm{cl}(V')$ such that $\{B_\varepsilon(v_i)\}_{i=1}^n$ covers $\mathrm{cl}(V')$, that is,

$$\bigcup_{i=1}^n B_\varepsilon(v_i) \supseteq \mathrm{cl}(V').$$

Therefore, it follows that

$$\bigcup_{i=1}^n [B_\varepsilon(v_i) \cap V'] = V'.$$

Because $\mu(V') > 0$, this implies that

$$\mu\left(\bigcup_{i=1}^n [B_\varepsilon(v_i) \cap V']\right) > 0,$$

so there exists $i \in \{1, ..., n\}$ such that $\mu(B_\varepsilon(v_i) \cap V') > 0$.

Given the conclusion of the preceding paragraph, fix $i \in \{1, ...n\}$ such that $\mu(B_\varepsilon(v_i) \cap V') > 0$. We will show that there exists $v'_i \in B_\varepsilon(v_i) \cap V'$ such that $\mu([v_i - \varepsilon, v'_i] \cap V') > 0$. To see this, suppose not for contradiction. Let $\bar{v} := \sup B_\varepsilon(v_i) \cap V'$. Take a sequence $(v^k)_{k=1}^\infty$ such that $v^k \in B_\varepsilon(v_i) \cap V'$ for each k and $\lim_{k\to\infty} v^k = \bar{v}$ (such a sequence exists by definition of $\bar{v}$.) By the assumption made for the purpose of contradiction, we have that $\mu\big([v_i - \varepsilon, v^k] \cap V'\big) = 0$ for each $k = 1, 2, ...$ Since the sets $\big([v_i - \varepsilon, v^k] \cap V'\big)_{k=1}^\infty$ form an increasing sequence of measurable sets, we have $0 = \mu([v_i - \varepsilon, \bar{v}] \cap V') = \mu([v_i - \varepsilon, v_i + \varepsilon] \cap V') = \mu(B_\varepsilon(v_i) \cap V') > 0$, where the inequality is assumed at the beginning of the current paragraph. This is a contradiction.

Therefore, following the preceding paragraph, fix $v'_i \in B_\varepsilon(v_i) \cap V'$ with the property that $\mu([v_i - \varepsilon, v'_i] \cap V') > 0$. Because $v'_i < v_i + \varepsilon$ and $\varepsilon < \frac{\delta}{2}$, we have $[v'_i - \delta, v'_i] \supseteq [v_i - \varepsilon, v'_i]$. Hence, noting that $[v'_i - \delta, v'_i] \cap V'$ and $[v_i - \varepsilon, v'_i] \cap V'$ are measurable, $\mu([v'_i - \delta, v'_i] \cap V') \ge \mu([v_i - \varepsilon, v'_i] \cap V') > 0$.

We now show that firm j can block outcome O via workers whose productivities fall in $[v'_i - \delta, v'_i]$. To do so, we observe that $w^g_{-j}(v'_i) < v'_i - \delta$ because $v'_i \in V'$. Thus, by the monotonicity of w^g_{-j}, $w^g_{-j}(v) < v'_i - \delta$ for all $v \in [v'_i - \delta, v'_i]$. This implies that $w^g_{-j}(v) < v$ for all $v \in [v'_i - \delta, v'_i]$. Therefore, by the ongoing assumption (following the conclusion of Cases 4 and 5) that $\max\{w^g_1(v), w^g_2(v)\} = v$ for almost every $v \in [0,1]$, it follows that $w^g_j(v) = v$ for almost all $v \in [v'_i - \delta, v'_i]$.

Consider $\tilde{O}_j$ where

$$\tilde{f}^j_g(v) := \begin{cases} f^j_g(v) & \text{if } v \notin [v'_i - \delta, v'_i], \\ f^{-j}_g(v) & \text{if } v \in [v'_i - \delta, v'_i]. \end{cases} \qquad \tilde{w}^g_j(v) := \begin{cases} w^g_j(v) & \text{if } v \notin [v'_i - \delta, v'_i], \\ v'_i - \delta & \text{if } v \in [v'_i - \delta, v'_i] \text{ and } \tilde{f}^j_g(v) > 0, \\ 0 & \text{otherwise.} \end{cases}$$

$\tilde{O}_j$ blocks O_j for the following reasons: First, it is obvious from construction that $\tilde{w}^g_j$ satisfies monotonicity. Condition 3 of the definition of block is satisfied for all $v \in [v'_i - \delta, v'_i]$ (i.e. the workers previously employed by firm $-j$ are successfully poached and some workers are fired), and Condition 2 of the definition of block is satisfied for all $v \notin [v_i'-\delta, v_i']$ (i.e. workers in this set do not experience changes to hiring or wages). It is also the case that $\tilde{O}_j$ provides firm j with higher profit than O_j: newly poached workers from $[v_i'-\delta, v_i'] \cap V'$ (of whom there are a positive measure) are paid lower wages than their productivity in outcome $\tilde{O}_j$ while all newly-fired workers are from $[v_i'-\delta, v_i']$ and received wages equal to productivity from j in outcome O_j. Therefore, we have shown that O is not a core outcome as desired.

As these six cases are exhaustive and none of them admits a core outcome, we have completed the argument that any core outcome must be such that all workers are hired and all workers receive a wage equal to their productivity. □

□

Proof of Proposition 2

Proof. First, we show that if firm 1 does not hire any $B-$group workers, then it hires all $A-$group workers and firm 2 hires all $B-$group workers (the usual disclaimer about zero-measure sets applies). That is, in any core outcome O, if $f_B^1(v) = 0$ for almost all v, then it must be that $f_A^1(v) = f_A(v)$ for almost all v and $f_B^2(v) = f_B(v)$ for almost all v. We show this in several exhaustive cases. Throughout, it suffices to assume $w_i^g(v) \leq v$ for all $v \in [0,1]$, all $i \in \{1,2\}$, and all $g \in \{A,B\}$ by the Individual Rationality Condition.

1. We will show that if $f_B^1(v) = 0$ for almost all v, then it must be that there exists a set $V \subset [0,1]$ of positive Lebesgue measure such that $f_A^1(v) > 0$ for all $v \in V$. Suppose not toward a contradiction so that $f_B^1(v) = 0$ for almost all v and $f_A^1(v) = 0$ for almost all v. Then, $\pi_1^O = 0$. By the Equal Profit Condition it must be that $\pi_2^O = 0$. There are two possibilities to consider. First, it may be that there exist a group $g \in \{A,B\}$ and a set V with positive Lebesgue measure such that $f_g^2(v) < f_g(v)$ for all $v \in V$. Without loss of generality let $g = A$. Then firm 1 can block outcome O via $\tilde{O}_1$ where

$$\tilde{f}_A^1(v) := f_A(v) - f_A^2(v) - f_A^1(v) \text{ for all } v \qquad \tilde{w}_1^A(v) := 0 \text{ for all } v,$$
$$\tilde{f}_B^1(v) := 0 \text{ for all } v \qquad \tilde{w}_1^B(v) := 0 \text{ for all } v.$$

   which yields a positive profit, contradiction. Second, it may be that $f_A^2(v) = f_A(v)$ and $f_B^2(v) = f_B(v)$ for almost all v. By group-based EPSW, it must be that firm 2 pays a common wage to all workers: there exists $w_2 \geq 0$ such that $w_2 = w_2^A(v) = w_2^B(v)$ for almost all v. Moreover, by the ongoing assumption that $w_i^g(v) \leq v$ for all $v \in [0,1]$, all $i \in \{1,2\}$, and all $g \in \{A,B\}$, it must be the case that $w_2 = 0$. But then for any $w^* \in (0,1)$ firm 1 can block O via $\tilde{O}_1$ where for each $g \in \{A,B\}$:

$$\tilde{f}_g^1(v) := \begin{cases} f_g(v) - f_g^1(v) & \text{if } v \geq w^*, \\ 0 & \text{otherwise.} \end{cases} \qquad \tilde{w}_1^g(v) := \begin{cases} 0 & \text{if } \tilde{f}_g^1(v) = 0, \\ w^* & \text{otherwise.} \end{cases}$$

   which yields positive profit, contradiction.

2. We will prove that if $f_B^1(v) = 0$ for almost all v and there exists a positive Lebesgue measure set V such that $f_A^1(v) > 0$ for all $v \in V$, then there does not exist any pair of sets V_A and

V_B with positive Lebesgue measure such that $f_A^2(v)>0$ for all $v\in V_A$ and $f_B^2(v)>0$ for all $v\in V_B$. To show this, assume for contradiction that there exist sets V_A and V_B with positive Lebesgue measure such that $f_A^2(v)>0$ for all $v\in V_A$ and $f_B^2(v)>0$ for all $v\in V_B$. Then due to the group-based EPSW, it must be that firm 2 pays a common wage w_2 to almost all workers it hires, i.e. $w_2^A(v)=w_2^B(v)=w_2$ for all v such that $f_A^2(v)>0$ and $f_B^2(v)>0$, respectively. By our ongoing assumption that $w_2^g(v)\leq v$ for all $v\in[0,1]$ and all $g\in\{A,B\}$, it must be the case that $f_A^2(v)=0$ for all $v\leq w_2$. This implies that firm 2 is earning a positive profit from the $A-$group workers it hires as $w_2<v$ for almost all $v\in V_A$. By the proof of Case 4.2 of Lemma 5, firm 1 can block O by "poaching" some subset of these workers and increase its profits.

3. Next, we will establish that if $f_B^1(v)=0$ for almost all v and there exists a set V with positive Lebesgue measure such that $f_A^1(v)>0$ for all $v\in V$ then $f_A^2(v)=0$ for almost all v. To show this, by the previous case, it suffices to consider for contradiction that $f_B^2(v)=0$ for almost all v and that there exists a positive Lebesgue measure set V_A such that $f_A^2(v)>0$ for all $v\in V_A$. The conclusion of Proposition 1 applies, and the two firms "Bertrand" compete away profits from $A-$group workers such that they both earn zero profits, i.e. $w_1^A(v)=v$ for all v such that $f_A^1(v)>0$ and $w_2^A(v)=v$ for all v such that $f_A^2(v)>0$. But then an arbitrary firm j can block via outcome $\tilde{O}_j$:

$$\tilde{f}_A^j(v):=0 \text{ for all } v \qquad \tilde{w}_j^A(v):=0 \text{ for all } v$$
$$\tilde{f}_B^j(v):=f_B(v)-f_B^{-j}(v) \text{ for all } v \qquad \tilde{w}_j^B(v):=0 \text{ for all } v$$

The ongoing assumption that $f_B^1(v)=0$ for almost all v and $f_B^2(v)=0$ for almost all v demonstrates that $\tilde{O}_j$ indeed blocks O.

4. We will show that if $f_B^1(v)=0$ for almost all v, there exists a set V with positive Lebesgue measure such that $f_A^1(v)>0$ for all $v\in V$, and $f_A^2(v)=0$ for almost all v then it must be that $f_A^1(v)=f_A(v)$ for almost all v. This claim is shown via the second and third cases in Claim 3 of the proof of Proposition 1.

Therefore, the only remaining possible core outcomes in which firms do not completely segregate involves each firm hiring both groups of workers, i.e. there exist sets V_A^1,V_A^2,V_B^1,V_B^2 with positive Lebesgue measure such that $f_A^1(v)>0$ for all $v\in V_A^1$, $f_A^2(v)>0$ for all $v\in V_A^2$, $f_B^1(v)>0$ for all $v\in V_B^1$, and $f_B^2(v)>0$ for all $v\in V_B^2$.

Suppose for contradiction that there exists such a non-segregation core outcome O. We have argued that in any such core outcome, each firm $i\in\{1,2\}$ must pay a common wage w_i to almost every worker it hires. Without loss of generality, let $w_1\leq w_2$. We first claim that, if $w_1<w_2$:

1. For almost all $v<w_1$, all workers of productivity v are unemployed: for almost all $v<w_1$, $f_A^1(v)=f_B^1(v)=f_A^2(v)=f_B^2(v)=0$,
2. for almost all $v\in(w_2,1]$, all workers of productivity v are hired by firm 2, i.e., $f_A^2(v)=f_A(v)$ and $f_B^2(v)=f_B(v)$, and
3. for almost all $v\in[w_1,w_2]$, all workers of productivity v are hired by firm 1, i.e., $f_A^1(v)=f_A(v)$ and $f_B^1(v)=f_B(v)$.

Point 1 follows from our previous argument that no firm hires a positive measure of workers at wage higher than productivity in any core outcome. Point 2 is demonstrated with the following argument. As argued in the previous paragraph, firm 2 will hire almost no workers with productivity $v\in[w_1,w_2]$. If there exists a set $V\subset[w_2,1]$ with positive measure such that $f_A^2(v)<f_A(v)$ or $f_B^2(v)<f_B(v)$ for all $v\in V$, then firm 2 can block outcome O via $\tilde{O}_2$ such that for all $g\in\{A,B\}$:

$$\tilde{f}_g^2(v):=\begin{cases} f_g(v) & \text{if } v\in(w_2,1], \\ 0 & \text{otherwise.}\end{cases} \qquad \tilde{w}_2^g(v):=\begin{cases} 0 & \text{if } \tilde{f}_g^2(v)=0, \\ w_2 & \text{otherwise.}\end{cases}$$

which clearly blocks O as firm 2's profit increases (from hiring additional workers at a positive marginal profit) and Condition 1 of the definition of block is satisfied for all $v\in(w_2,1]$. Point 3 follows from a similar argument as the proof of point 2.

We now claim that in any core outcome O it must be that $w_1<w_2$ (recall that we have assumed $w_1\leq w_2$). To see this, suppose for contradiction that $w_1=w_2$. If $w_1=w_2<1$, then at least one firm $i\in\{1,2\}$ receives total profit $\pi_i^O<\int_{w_1}^1(v-w_1)[\beta f_A(v)+f_B(v)]dv$. But because F is atomless, for sufficiently small $\epsilon>0$, firm i would receive $\int_{w_1+\epsilon}^1(v-w_1-\epsilon)[\beta f_A(v)+f_B(v)]dv>\pi_i^O$ by instead setting wage $w'=w_1+\epsilon$ and hiring all workers with productivity strictly greater than $w_1+\epsilon$. This is a contradiction to the assumption that O is a core outcome. If $w_1=w_2=1$, then $\pi_1^O=\pi_2^O=0$ and so either firm could block O by setting any wage $w'\in(0,1)$ and hiring all workers with productivities $v\in[w',1)$, which yields profit $\int_{w'}^1(v-w')[\beta f_A(v)+f_B(v)]dv>0$. This contradicts the assumption that that O is a core outcome. As these two cases are exhaustive, it must be the case that $w_1<w_2$ in any core outcome in which $w_1\leq w_2$.

We now show that generically, any outcome O in which $f_A^1(v)=f_B^1(v)=f_A^2(v)=f_B^2(v)=0$ for almost all $v<w_1$, $f_A^1(v)=f_A(v)$ and $f_B^1(v)=f_B(v)$ for almost all $v\in[w_1,w_2]$, and $f_A^2(v)=f_A(v)$ and $f_B^2(v)=f_B(v)$ for almost all $v>w_2$ is not a core outcome. We consider two exhaustive cases.

1. $0<w_1<w_2$. Let $\pi_{i,g}$ represent the profit firm $i\in\{1,2\}$ receives from all workers in group $g\in\{A,B\}$ in outcome O, or more formally,

$$\pi_{i,A}:=\beta\int_0^1(v-w_i(v))f_A^i(v)dv, \qquad \pi_{i,B}:=\int_0^1(v-w_i(v))f_B^i(v)dv.$$

It must be the case that $\pi_{1,A}+\pi_{2,A}\geq\pi_{1,B}+\pi_{2,B}$, or vice versa. Without loss of generality, assume $\pi_{1,A}+\pi_{2,A}\geq\pi_{1,B}+\pi_{2,B}$. We claim that firm 1 can block outcome O via outcome $\tilde{O}_1$ where for some small $\epsilon'>0$:

$$\tilde{f}_A^1(v):=f_A(v) \quad \text{for all } v\in[0,1] \qquad \tilde{w}_1^A(v):=\begin{cases} 0 & \text{if } v<w_1, \\ w_1 & \text{if } v\in[w_1,w_2], \\ w_2+\epsilon' & \text{if } v>w_2.\end{cases}$$

$$\tilde{f}_B^1(v):=0 \quad \text{for all } v\in[0,1] \qquad \tilde{w}_1^B(v):=0 \quad \text{for all } v\in[0,1]$$

For sufficiently small ϵ', $\pi_1^{\tilde{O}_1}>\pi_{1,A}+\pi_{2,A}\geq\pi_1$, where the last inequality comes from the assumption that $\pi_{1,A}+\pi_{2,A}\geq\pi_{1,B}+\pi_{2,B}$ and the Equal Profit Condition $\pi_1=\pi_2=\frac{1}{2}(\pi_{1,A}+\pi_{2,A}+\pi_{1,B}+\pi_{2,B})$. For almost all $A-$group workers, Condition 1 of the definition of block is satisfied, and for almost all $B-$group workers, Condition 4 of the definition of block is satisfied. Therefore, $\tilde{O}_1$ blocks O.

2. $0=w_1<w_2$. The argument is analogous to case 1 if $\pi_{1,A}+\pi_{2,A}>\pi_{1,B}+\pi_{2,B}$ or $\pi_{1,A}+\pi_{2,A}<\pi_{1,B}+\pi_{2,B}$. Below, we argue that the condition $\pi_{1,A}+\pi_{2,A}=\pi_{1,B}+\pi_{2,B}$ is non-generic on the space of distributions:

   First, we will show the openness of the set of distributions F_A,F_B such that $\pi_{1,A}+\pi_{2,A}>\pi_{1,B}+\pi_{2,B}$ or $\pi_{1,A}+\pi_{2,A}<\pi_{1,B}+\pi_{2,B}$. To do so, assume that the former inequality holds at F_A,F_B (the other case is analogous), where the core outcome wages of firms 1 and 2 are 0 and w_2, respectively. Let $\epsilon>0$ be small enough that

$$\pi_{1,A}+\pi_{2,A}>\pi_{1,B}+\pi_{2,B}+\epsilon. \tag{11}$$

   By the Portmanteu Theorem,[26] for any $\tilde{w}_2\in[0,1]$ it follows that

$$\beta\int_0^{\tilde{w}_2} vf'_A(v)dv\to\beta\int_0^{\tilde{w}_2} vf_A(v)dv,$$

   as (F'_A,F'_B) with densities f'_A,f'_B converges in weak* topology to (F_A,F_B). Thus, there is a neighborhood of (F_A,F_B) such that, for any (F'_A,F'_B) in that neighborhood, we have

$$\beta\int_0^{\tilde{w}_2} vf'_A(v)dv\in\left(\beta\int_0^{\tilde{w}_2} vf_A(v)dv-\frac{\epsilon}{8},\beta\int_0^{\tilde{w}_2} vf_A(v)dv+\frac{\epsilon}{8}\right). \tag{12}$$

   Next, again by Portmanteu Theorem, it follows that for any $\delta<w_2$ (where w_2 is the wage paid by firm 2 when the distributions of worker productivities are given by (F_A,F_B)),

$$\beta\int_0^{w_2-\delta} vf'_A(v)dv+\int_0^{w_2-\delta} vf'_B(v)dv\to\beta\int_0^{w_2-\delta} vf_A(v)dv+\int_0^{w_2-\delta} vf_B(v)dv$$

   and

$$\beta\int_{w_2-\delta}^1(v-(w_2-\delta))f'_A(v)dv+\int_{w_2-\delta}^1(v-(w_2-\delta))f'_B(v)dv\to\beta\int_{w_2-\delta}^1(v-(w_2-\delta))f_A(v)dv+\int_{w_2-\delta}^1(v-(w_2-\delta))f_B(v)dv,$$

   as (F'_A,F'_B) converges in weak* topology to (F_A,F_B). Because we know that

$$\beta\int_0^{w_2-\delta} vf_A(v)dv+\int_0^{w_2-\delta} vf_B(v)dv<\beta\int_{w_2-\delta}^1(v-(w_2-\delta))f_A(v)dv+\int_{w_2-\delta}^1(v-(w_2-\delta))f_B(v)dv$$

   from the construction of outcome O at (F_A,F_B), it follows that there exists a neighborhood of (F_A,F_B) such that, for any pair of distributions (F'_A,F'_B) in that neighborhood, we have

$$\beta\int_0^{w_2-\delta} vf'_A(v)dv+\int_0^{w_2-\delta} vf'_B(v)dv<\beta\int_{w_2-\delta}^1(v-(w_2-\delta))f'_A(v)dv+\int_{w_2-\delta}^1(v-(w_2-\delta))f'_B(v)dv,$$

   and, with an analogous argument, that

$$\beta\int_0^{w_2+\delta} vf'_A(v)dv+\int_0^{w_2+\delta} vf'_B(v)dv>\beta\int_{w_2+\delta}^1(v-(w_2+\delta))f'_A(v)dv+\int_{w_2+\delta}^1(v-(w_2+\delta))f'_B(v)dv.$$

[26] See Theorem 2.8.1 (c) of Ash and Doléans-Dade (2000).

These inequalities imply that if firm 2 pays w_2' to all workers it employs and firm 1 pays $w'1=0$ to all workers it employs, both given distributions (F_A',F_B'), such that the Equal Profit Condition is satisfied, then $w_2'\in(w_2-\delta,w_2+\delta)$. Moreover, by monotonicity of the expression $\beta\int_0^{w_2'}vf_A'(v)dv$ in w_2', it follows that $\beta\int_0^{w_2'}vf_A'(v)dv\in\left(\beta\int_0^{w_2-\delta}vf_A'(v)dv,\beta\int_0^{w_2+\delta}vf_A'(v)dv\right)$ for any distribution (F_A',F_B') in the neighborhood. Take a sufficiently small $\delta>0$ such that

$$\beta\int_0^{w_2-\delta}vf_A(v)dv>\beta\int_0^{w_2}vf_A(v)dv-\frac{\epsilon}{8} \text{ and } \beta\int_0^{w_2+\delta}vf_A(v)dv<\beta\int_0^{w_2}vf_A(v)dv+\frac{\epsilon}{8}. \tag{13}$$

Moreover, we apply (12) to $\tilde{w}_2=w_2-\delta$ and $\tilde{w}_2=w_2+\delta$ to obtain

$$\beta\int_0^{w_2-\delta}vf_A'(v)dv\in\left(\beta\int_0^{w_2-\delta}vf_A(v)dv-\frac{\epsilon}{8},\beta\int_0^{w_2-\delta}vf_A(v)dv+\frac{\epsilon}{8}\right),$$
$$\beta\int_0^{w_2+\delta}vf_A'(v)dv\in\left(\beta\int_0^{w_2+\delta}vf_A(v)dv-\frac{\epsilon}{8},\beta\int_0^{w_2+\delta}vf_A(v)dv+\frac{\epsilon}{8}\right). \tag{14}$$

Thus, by (13) and (14) it follows that $\pi_{1,A}'\in(\pi_{1,A}-\frac{\epsilon}{4},\pi_{1,A}+\frac{\epsilon}{4})$. Similarly we can establish that $\pi_{i,g}'$ for $i\in\{1,2\},g\in\{A,B\}$, each satisfies $\pi_{i,g}'\in(\pi_{i,g}-\frac{\epsilon}{4},\pi_{i,g}+\frac{\epsilon}{4})$. This and (11) imply that

$$\pi_{1,A}'+\pi_{2,A}'>\pi_{1,B}'+\pi_{2,B}',$$

completing the proof of openness.

Second, we will show the denseness of the set of distributions F_A,F_B such that $\pi_{1,A}+\pi_{2,A}>\pi_{1,B}+\pi_{2,B}$ or $\pi_{1,A}+\pi_{2,A}<\pi_{1,B}+\pi_{2,B}$. To do so, suppose for a given F_A, F_B satisfying our regularity conditions there is a core outcome that does not feature segregation, i.e. $\pi_{1,A}+\pi_{2,A}=\pi_{1,B}+\pi_{2,B}$. We show that for any $\delta_1>0$ there exist distributions F_A', F_B' satisfying our regularity conditions such that $|F_A'(v)-F_A(v)|\leq\delta_1$ and $|F_B'(v)-F_B(v)|\leq\delta_1$ for all $v\in[0,1]$ and for which there does not exist a non-segregation core outcome.[27] We show this by modifying the distributions such that the same wages by firms 1 and 2, 0 and w_2, respectively, yield equal profit, but show that aggregate profit derived from $A-$group workers no longer equals that derived from $B-$group workers.

Take any $\delta\in(0,\min\{\underline{f}_A,\underline{f}_B,\delta_1\})$ and any $w^*\in(0,w_2)$ such that $w_2-w^*<1-w_2$. Define F_A' and F_B' via their respective densities:

$$f_A'(v):=\begin{cases} f_A(v) & \text{if } v<w^*,\\ f_A(v)-\frac{\delta}{\beta} & \text{if } v\in[w^*,w_2),\\ f_A(v)+\frac{\delta}{\beta} & \text{if } v\in[w_2,w_2+(w_2-w^*)),\\ f_A(v) & \text{otherwise}\end{cases},$$

$$f_B'(v):=\begin{cases} f_B(v) & \text{if } v<w^*,\\ f_B(v)+\delta & \text{if } v\in[w^*,w_2),\\ f_B(v)-\delta & \text{if } v\in[w_2,w_2+(w_2-w^*)),\\ f_B(v) & \text{otherwise}\end{cases}.$$

[27] Note that our pointwise notion of "closeness" corresponds to that of the weak-* topology on the set of distributions satisfying our regularity conditions.

Noting there exists a β measure of $A-$group workers, by construction of F'_A and F'_B it is the case that

$$\begin{aligned}\pi_{2,A}+\pi_{2,B} &= \beta\int_{w_2}^{1}[v-w_2]f_A(v)dv+\int_{w_2}^{1}[v-w_2]f_B(v)dv\\ &= \beta\int_{w_2}^{1}[v-w_2]f'_A(v)dv+\int_{w_2}^{1}[v-w_2]f'_B(v)dv\\ &= \pi'_{2,A}+\pi'_{2,B}\end{aligned}$$

and

$$\begin{aligned}\pi_{1,A}+\pi_{1,B} &= \beta\int_{0}^{w_2}vf_A(v)dv+\int_{0}^{w_2}vf_B(v)dv\\ &= \beta\int_{0}^{w_2}vf'_A(v)dv+\int_{0}^{w_2}vf'_B(v)dv\\ &= \pi'_{1,A}+\pi'_{1,B}\end{aligned}$$

where $\pi'_{i,g}$ represents firm i's profit from $g-$group workers it hires given distribution F'_g. The above two equations imply that

$$\pi'_i := \pi'_{i,A}+\pi'_{i,B} = \pi_i^{O_i} \text{ for } i\in\{1,2\}$$

It is also the case that

$$\begin{aligned}\pi_{1,A}+\pi_{2,A}-(\pi'_{1,A}+\pi'_{2,A}) &= \beta\int_{w_2}^{w_2+(w_2-w^*)}[v-w_2]\frac{\delta}{\beta}dv+\beta\int_{w^*}^{w_2}v\frac{\delta}{\beta}dv\\ &= \delta(w_2-w^*)w_2\\ &> 0\end{aligned}$$

where the final inequality comes from the assumptions that $w_2>w^*>0$. Similarly it can be shown that

$$\pi_{1,B}+\pi_{2,B}-(\pi'_{1,B}+\pi'_{2,B})<0.$$

The above two equations imply that

$$\pi'_{1,A}+\pi'_{2,A}<\pi_{1,A}+\pi_{2,A},$$

and

$$\pi_{1,B}+\pi_{2,B}<\pi'_{1,B}+\pi'_{2,B}.$$

The initial condition that $\pi_{1,A}+\pi_{2,A}=\pi_{1,B}+\pi_{2,B}$ further implies that

$$\pi'_{1,A}+\pi'_{2,A}<\pi'_{1,B}+\pi'_{2,B}$$

which completes the claim. $\square$

Proof of Proposition 3

Proof. **Proof of part 1.** We first constructively show that there exist a continuum of core outcomes $O(\delta,\delta')$ with group-based segregation, parameterized by the following class of wage functions for $\delta,\delta' \in [0,1]$:

$$w_1(v) := \begin{cases} v & \text{if } v < \delta, \\ \delta & \text{otherwise.} \end{cases} \qquad w_2(v) := \begin{cases} 0 & \text{if } v < \delta', \\ v & \text{otherwise.} \end{cases}$$

with firms 1 and 2 hiring all $A-$ and $B-$group workers, respectively. We show that these wage functions permit a core outcome if $\delta \geq \delta'$ and

$$\beta \int_{\delta}^{1} (v-\delta) f_A(v)dv = \int_{0}^{\delta'} v f_B(v)dv. \tag{15}$$

Note that the left-hand side of (15) is $\pi_1^{O(\delta,\delta')}$, while the right-hand side $\pi_2^{O(\delta,\delta')}$.

To see that the above wage functions permit a core outcome $O(\delta,\delta')$, first note that there is no firm i and outcome $\tilde{O}_i$ with group-based segregation that blocks $O(\delta,\delta')$. Suppose without loss of generality that in outcome $\tilde{O}_i$, firm i hires a positive measure of $A-$group workers only with associated wage function $\tilde{w}_i(\cdot)$. By the definition of block, for almost every v, $\tilde{f}_A^i(v) > 0$ only if $\tilde{w}_i(v) \geq w_1(v)$, implying that $\pi_i^{\tilde{O}_i} \leq \pi_1^{O(\delta,\delta')} = \pi_2^{O(\delta,\delta')}$. Therefore, $\tilde{O}_i$ is not a block.

Thus, consider a firm i and a potential blocking outcome $\tilde{O}_i$ in which i employs positive measures of both $A-$ and $B-$group workers. Under group-based EPSW, such a firm pays a common wage w to almost all workers. There are two exhaustive cases:

- Consider $w \leq \delta$. By the definition of block, firm i does not benefit from hiring $A-$group workers, as almost all $A-$group workers with $v \geq w$ are paid $w_1(v) = \min\{v,\delta\} \geq w$ in outcome $O(\delta,\delta')$. Therefore, $\pi_i^{\tilde{O}_i}$ is upper bounded by the case in which firm i hires only $B-$group workers. It has been shown in the preceding paragraph, however, that there exists no such block.
- Consider $w > \delta$. By the definition of block, firm i does not benefit from hiring $B-$group workers, as almost all $B-$group workers with $v \geq w$ are paid $w_2(v) = v$ in outcome $O(\delta,\delta')$. Therefore, $\pi_i^{\tilde{O}_i}$ is upper bounded by the case in which firm i hires only $A-$group workers. It has been shown in the preceding paragraph, however, that there exists no such block.

The preceding arguments demonstrate that any outcome $O(\delta,\delta')$ with $\delta \geq \delta'$ and $\pi_1^{O(\delta,\delta')} = \pi_2^{O(\delta,\delta')}$ is a core outcome. Next, we show that there is a continuum of pairs δ,δ' that satisfy these conditions. To see this, note that if $\delta = \delta' = 1$, then the right-hand side of (15) is strictly positive while the left-hand side is zero, and hence the former is strictly larger than the latter. Because of continuity of the left-hand side in δ, the left-hand side is strictly smaller than the right-hand side for any δ that is sufficiently close to 1. Now, noting continuity of the right-hand side in δ' and the fact that it is equal to zero for $\delta' = 0$, by intermediate value theorem, it follows that there exists δ' such that (15) holds with equality. This concludes the proof of part 1.

Proof of part 3. Recall that AW_A^O and AW_B^O denote the average wages for $A-$ and $B-$group workers, respectively. In addition, denote TS_A and TS_B to be the total surpluses created by $A-$

and $B-$group workers (together with the firms hiring them), respectively, i.e.,

$$TS_A := \beta \int_0^1 v f_A(v) dv = \beta \mathbb{E}_A(v),$$
$$TS_B := \int_0^1 v f_B(v) dv = \mathbb{E}_B(v).$$

Then, note that $\pi_1^O = TS_A - \beta AW_A^O$ and $\pi_2^O = TS_B - AW_B^O$. The Equal Profit Condition tells us that

$$\begin{aligned} & TS_A - \beta AW_A^O = TS_B - AW_B^O \\ \Leftrightarrow & TS_A - TS_B = \beta AW_A^O - AW_B^O \\ \Leftrightarrow & TS_A - TS_B + (1-\beta) AW_A^O = AW_A^O - AW_B^O. \end{aligned} \tag{16}$$

Proof of the "only if" part. Suppose that $AW_A^O - AW_B^O \leq AW_A^{O'} - AW_B^{O'}$ for core outcomes O and O'. Then (16) implies that $TS_A - TS_B + (1-\beta)AW_A^O \leq TS_A - TS_B + (1-\beta)AW_A^{O'}$. Note that $TS_A - TS_B$ is a constant. Since $\beta > 1$, this implies that $AW_A^O \geq AW_A^{O'}$. Since firm 1 hires almost all $A-$group workers in both O and O', this implies that $\pi_1^O \leq \pi_1^{O'}$. Then $\pi_2^O = \pi_1^O \leq \pi_1^{O'} = \pi_2^{O'}$, where the equalities follow from the Equal Profit Condition.

Proof of the "if" part. Suppose $\pi_1^O \leq \pi_1^{O'}$ for two core outcomes O and O' (which implies $\pi_2^O = \pi_1^O \leq \pi_1^{O'} = \pi_2^{O'}$ by the Equal Profit Condition). Since firm 1 hires almost all $A-$group workers in both O and O', this implies that $AW_A^O \geq AW_A^{O'}$. Since $\beta > 1$, this implies that $TS_A - TS_B + (1-\beta)AW_A^O \leq TS_A - TS_B + (1-\beta)AW_A^{O'}$. Thus, by (16), $AW_A^O - AW_B^O \leq AW_A^{O'} - AW_B^{O'}$.

Proof of Part 2. Consider the core outcome in which $w_1(v) = w_2(v) = v$ for all $v \in [0,1]$ and all $A-$ and $B-$group workers are hired by firms 1 and 2, respectively. The wage gap in this core outcome is the same as in any core outcome without EPSW, and both firms' profits are zero. Now, it is straightforward to see that in any non-equivalent core outcome firm profits are strictly positive. Therefore, applying the conclusion of part 3 completes the claim. □

Proof of Proposition 4

Proof. Following Proposition 2, without loss of generality, we focus on core outcomes such that $f_A^1(v) = f_A(v)$ and $f_B^2(v) = f_B(v)$ for all v. We show that there exists $\beta^* \in [1, \infty)$ such that for all $\beta > \beta^*$, any core outcome O must satisfy $w_1(v) > v - \delta$ for all v.

To show this, suppose not for the sake of contradiction. Then for any β^* there exists $\beta > \beta^*$, a core outcome O, and some v_β such that $w_1(v_\beta) \leq v_\beta - \delta$. Noting that $w_1(v)$ is non-decreasing in v, it must be the case that

$$w_1(v) \leq w_1(v_\beta) \leq v_\beta - \delta \text{ for all } v < v_\beta. \tag{17}$$

Therefore, we can bound the profit of firm 1 from below as follows:

$$\begin{aligned}\pi_1^O &= \beta\int_0^1 (v-w_1(v))f_A(v)dv \geq \beta\int_{v_\beta-\delta}^{v_\beta} (v-w_1(v))f_A(v)dv \\ &\geq \beta\int_{v_\beta-\delta}^{v_\beta} (v-(v_\beta-\delta))f_A(v)dv \geq \beta\int_{v_\beta-\delta}^{v_\beta} (v-(v_\beta-\delta))\underline{f}_A dv \\ &= \frac{\beta}{2}\delta^2\underline{f}_A,\end{aligned} \tag{18}$$

where the first inequality comes from the fact that $w_1(v) \leq v$ for all v, the second inequality comes from (17), and the third inequality comes from the fact that $f_A(v)$ is bounded below by $\underline{f}_A > 0$. But the right-most term in (18) which lower bounds π_1^O grows arbitrarily large as $\beta \to \infty$. Therefore, for any sufficiently large β, it follows that

$$\pi_1^O \geq \frac{\beta}{2}\delta^2\underline{f}_A > \bar{f}_B > \int_0^1 (v-w_2(v))\bar{f}_B dv \geq \int_0^1 (v-w_2(v))f_B(v)dv = \pi_2^O.$$

But this is a contradiction with the Equal Profit Condition, as we have shown $\pi_1^O > \pi_2^O$. □

Proof of Lemma 1

Proof. To show the "only if" part, we first note that it follows from previous arguments that any core outcome satisfies Individual Rationality and the Equal Profit Condition.

To show that the No Desegregation Condition is satisfied, suppose for contradiction that (2) is violated at some $\epsilon \in [0,1]$. Because the right-hand side of (2) is zero for $\epsilon = 1$, in the remainder we assume that $\epsilon < 1$. Define

$$\delta := \beta \int_{\epsilon}^{w_1^{-1}(\epsilon)} (v-\epsilon)f_A(v)dv + \int_{\epsilon}^{w_2^{-1}(\epsilon)} (v-\epsilon)f_B(v)dv - \int_0^1 (v-w_2(v))f_B(v)dv.$$

Let $\tilde{O}_1$ be an outcome such that,

$$\tilde{f}_A^1(v) := \begin{cases} f_A(v) & \text{if } v \in [\epsilon+\delta', \hat{w}_1^{-1}(\epsilon+\delta)], \\ 0 & \text{otherwise.}\end{cases} \qquad \tilde{w}_1^A(v) := \begin{cases} \epsilon+\delta' & \text{if } \tilde{f}_A^1(v) > 0, \\ 0 & \text{otherwise.}\end{cases}$$

and

$$\tilde{f}_B^1(v) := \begin{cases} f_B(v) & \text{if } v \in [\epsilon+\delta', \sup\{v' | w_2(v') < \epsilon+\delta'\}], \\ 0 & \text{otherwise.}\end{cases} \qquad \tilde{w}_1^B(v) := \begin{cases} \epsilon+\delta' & \text{if } \tilde{f}_B^1(v) > 0, \\ 0 & \text{otherwise.}\end{cases}$$

There exists $\delta' > 0$ such that,

$$\pi_1^{\tilde{O}_1} = \beta \int_{\epsilon+\delta'}^{w_1^{-1}(\epsilon+\delta')} (v-\epsilon-\delta') f_A(v) dv + \int_{\epsilon+\delta'}^{\sup\{v'|w_2(v')<\epsilon+\delta'\}} (v-\epsilon-\delta') f_B(v) dv$$
$$> \beta \int_{\epsilon}^{w_1^{-1}(\epsilon)} (v-\epsilon) f_A(v) dv + \int_{\epsilon}^{w_2^{-1}(\epsilon)} (v-\epsilon) f_B(v) dv - \delta$$

where the inequality holds due to the following arguments. The lower limit of the integrals are continuous in ϵ and the integrand $(v-\epsilon)f_g(v)$ for $g \in \{A,B\}$ are continuous by assumption, the latter because F_g, $g \in \{A,B\}$ has no mass points. Also, the upper limit of the integrals, respectively, are weakly higher in the first line than in the second line: $w_1^{-1}(\epsilon+\delta') \geq w_1^{-1}(\epsilon)$ by monotonicity of w_1^{-1} and $\sup\{v'|w_2(v')<\epsilon+\delta'\} \geq w_2^{-1}(\epsilon)$ by construction. Recalling that $\delta>0$ and that $\delta'>0$ is arbitrary, the desired conclusion follows.

$\tilde{O}_1$ blocks O because firm 1's profit increases by assumption, for all (v,A) such that $\tilde{f}_A^1(v)>0$, Condition 1 of the definition of block is satisfied, for all (v,B) such that $\tilde{f}_B^1(v)>0$, Condition 3 of the definition of block is satisfied, and for all other (g,v), $g \in \{A,B\}$, Condition 4 of the definition of block is satisfied. This contradicts the initial assumption that O is a core outcome.

To show the "if" direction, suppose the stated conditions hold for an outcome O, and suppose for contradiction that there is a firm i and a blocking outcome $\tilde{O}_i$. We take this firm i to be 1, and the argument for firm 2 is similar.

We consider the following three exhaustive cases:

1. $\tilde{f}_B^1(v)=0$ for almost all v. For any v, either $\tilde{f}_A^1(v)=0$ in which case firm 1 makes 0 infinitesimal profit from v, or $\tilde{f}_A^1(v)>0$. In the latter case, since $f_A^1(v)=f_A(v)$ and $f_B^1(v)=0$ for almost all v, it must be that for A and almost all v, Conditions 3 and 4 of the definition of block cannot be satisfied, while Conditions 1 and 2 of the definition of block become identical, implying $\tilde{w}_1^A(v) \geq w_1(v)$ for almost all v with $\tilde{f}_A^1(v)>0$. Given that $w_1(v) \leq v$ for almost all v by assumption, it must be that:

$$\pi_1^O = \beta \int_0^1 (v-w_1(v)) f_A(v) dv \geq \beta \int_0^1 (v-w_1(v)) \tilde{f}_A^1(v) dv \geq \beta \int_0^1 \left(v-\tilde{w}_1^A(v)\right) \tilde{f}_A^1(v) dv = \pi_1^{\tilde{O}_1},$$

   where the first equality comes from the fact that $f_A^1(v)=f_A(v)$ for almost all v by assumption, the first inequality follows since $w_1(v) \leq v$ for almost all v by assumption and since $\tilde{f}_A^1(v) \leq f_A(v)$ for all v, and the second inequality follows from the fact $\tilde{w}_1^A(v) \geq w_1(v)$ for all v such that $\tilde{f}_A^1(v)>0$ by Condition 2 of the definition of block. Therefore, the above inequalities contradict that $\pi_1^{\tilde{O}_1} > \pi_1^O$.

2. $\tilde{f}_A^1(v)=0$ for almost all v. For any v, either $\tilde{f}_B^1(v)=0$ in which case firm 1 makes 0 infinitesimal profit from v, or $\tilde{f}_B^1(v)>0$. In the latter case, since $f_B^2(v)=f_B(v)$ for almost all v, it must be that for B and almost all v, Conditions 2 and 4 of the definition of block cannot be satisfied, while Conditions 1 and 3 of the definition of block become identical, implying $\tilde{w}_1^B(v) > w_2(v)$ for almost all v with $\tilde{f}_B^1(v)>0$. Given that $w_2(v) \leq v$ for almost all v by assumption, it must be that:

$$\pi_2^O = \int_0^1 (v - w_2(v)) f_B(v) dv \geq \int_0^1 (v - w_2(v)) \tilde{f}_B^1(v) dv \geq \int_0^1 \left(v - \tilde{w}_1^B(v)\right) \tilde{f}_B^1(v) dv = \pi_1^{\tilde{O}_1},$$

where the first equality comes from the fact that $f_B^2(v) = f_B(v)$ for almost all v (Proposition 2), the first inequality follows since $w_2(v) \leq v$ for almost all v by assumption and since $\tilde{f}_B^1(v) \leq f_B(v)$ for all v, and the second inequality follows from the fact $\tilde{w}_1^B(v) > w_2(v)$ for all v such that $\tilde{f}_B^1(v) > 0$ by Condition 3 of the definition of block. The Equal Profit Condition guarantees that $\pi_2^O = \pi_1^O$, and therefore, the above inequalities contradict that $\pi_1^{\tilde{O}_1} > \pi_1^O$.

3. There exist positive Lebesgue measure sets V_A and V_B such that $\tilde{f}_A^1(v) > 0$ for all $v \in V_A$ and $\tilde{f}_B^1(v) > 0$ for all $v \in V_B$. Group-based EPSW implies that firm 1 must pay a common wage ϵ in $\tilde{O}_1$ to almost every (g,v) such that $\tilde{f}_g^1(v) > 0$. Therefore,

$$\begin{aligned}
\pi_1^{\tilde{O}_1} &\leq \beta \int_0^{w_1^{-1}(\epsilon)} (v-\epsilon) \tilde{f}_A^1(v) dv + \int_0^{w_2^{-1}(\epsilon)} (v-\epsilon) \tilde{f}_B^1(v) dv \\
&\leq \beta \int_\epsilon^{w_1^{-1}(\epsilon)} (v-\epsilon) \tilde{f}_A^1(v) dv + \int_\epsilon^{w_2^{-1}(\epsilon)} (v-\epsilon) \tilde{f}_B^1(v) dv \\
&\leq \beta \int_\epsilon^{w_1^{-1}(\epsilon)} (v-\epsilon) f_A(v) dv + \int_\epsilon^{w_2^{-1}(\epsilon)} (v-\epsilon) f_B(v) dv \\
&\leq \pi_2^O,
\end{aligned}$$

where the first inequality follows from two arguments:

- Consider the first integral. Since $f_A^1(v) = f_A(v)$ for almost all v by assumption, for A and almost all $v \in V_A$, Conditions 3 and 4 of the definition of block cannot be satisfied, while Conditions 1 and 2 of the definition of block become identical, implying $\tilde{w}_1^A(v) \geq w_1(v)$ for almost all $v \in V_A$. By definition of $w_1^{-1}(\cdot)$, it therefore must be the case that $\tilde{f}_A^1(v) = 0$ for almost all v such that $w_1^{-1}(\epsilon) < v$.
- Consider the second integral. Since $f_B^2(v) = f_B(v)$ for almost all v by assumption, for B and almost all $v \in V_B$, Conditions 2 and 4 of the definition of block cannot be satisfied, while Conditions 1 and 3 of the definition of block become identical, implying $\tilde{w}_1^B(v) > w_2(v)$ for almost all $v \in V_B$. By definition of $w_2^{-1}(\cdot)$, it therefore must be the case that $\tilde{f}_B^1(v) = 0$ for almost all v such that $w_2^{-1}(\epsilon) < v$.

The second inequality follows from the fact that $v - \epsilon < 0$ for all $v < \epsilon$ and the restriction that $\tilde{f}_A^1(v) \geq 0$ for all v and $\tilde{f}_B^1(v) \geq 0$ for all v. The third inequality follows from the fact that $v - \epsilon \geq 0$ over the range of integration, and that $\tilde{f}_A^1(v) \leq f_A(v)$ and $\tilde{f}_B^1(v) \leq f_B(v)$. The final inequality follows from the No Desegregation Condition.

The Equal Profit Condition guarantees that $\pi_2^O = \pi_1^O$, and therefore, the above inequalities contradict that $\pi_1^{\tilde{O}_1} > \pi_1^O$.

□

Proof of Lemma 2

Before beginning the proof, we define two terms.

Definition **1.** *A function* $g:[0,1]\to[0,1]$ *has an* upward jump *at* $x\in[0,1]$ *if either i)* $\exists\delta>0$ *such that* $\forall\eta>0$*, there exists* $x'\in(x-\eta,x)\cap[0,1]$ *for which* $g(x)>g(x')+\delta$*, or ii)* $\exists\delta>0$ *such that* $\forall\eta>0$*, there exists* $x'\in(x,x+\eta)\cap[0,1]$ *for which* $g(x')>g(x)+\delta$*.*

Definition **2.** *A function* $g:[0,1]\to[0,1]$ *is* upward continuous *at* $x\in[0,1]$ *if it does not have an upward jump at* x*.* g *is* upward continuous *if it is* upward continuous *at* x *for all* $x\in[0,1]$*.*

We now proceed with the proof of Lemma 2.

Proof. We begin the proof by showing several claims.

Claim **1.** *Take any* $\epsilon\in[0,1]$ *for which* $\phi(\epsilon)<1$*. Then* (3) *holds with equality at* ϵ *and* $\tilde{v}=\phi(\epsilon)$*.*

Proof. Because the right-hand side of (3) is continuous in $\tilde{v}$ while the left-hand side is constant in $\tilde{v}$, the definition of $\phi(\epsilon)$ implies that (3) holds, with either a strict inequality or equality, at ϵ and $\tilde{v}=\phi(\epsilon)$. Suppose for contradiction that (3) holds as a strict inequality at ϵ and $\tilde{v}=\phi(\epsilon)$. Then, because the right-hand side of the inequality is continuous in $\tilde{v}$ and $\phi(\epsilon)<1$ (while, again, the left-hand side is constant in $\tilde{v}$), there exists $v'\in(\phi(\epsilon),1]$ such that the inequality holds for $\tilde{v}=v'$. This is a contradiction to the definition of $\phi(\epsilon)$. □

Claim **2.** *Function* $\phi(\cdot)$ *is upward continuous.*

Proof. We consider for contradiction that $\phi(\cdot)$ has an upward jump of type (ii) at value $\epsilon<1$, and the case of a type (i) upward jump is similar and therefore omitted. Under this premise, there exists $\delta>0$ such that for all $\eta>0$ there exists $\epsilon'\in(\epsilon,\epsilon+\eta)\cap[0,1]$ for which $\phi(\epsilon')>\phi(\epsilon)+\delta$. Since $\phi(\cdot)$ is a function from $[0,1]$ to $[0,1]$ and $\phi(\epsilon')>\phi(\epsilon)+\delta$, it follows that $\phi(\epsilon)<1$. Claim 1 implies that (3) holds with equality at ϵ and $\tilde{v}=\phi(\epsilon)$.

First consider the case in which $\epsilon=\phi(\epsilon)$. Because the definition of an upward jump specifies the quantifier "for all $\eta>0$" it suffices to assume that $\delta>\eta$. Comparing (3) at ϵ and ϵ' respectively implies that

$$
\begin{aligned}
0 \leq & \int_{\epsilon}^{w_2^{-1}(\epsilon)} (v-\epsilon) f_B(v) dv - \beta \int_{\epsilon'}^{\phi(\epsilon')} (v-\epsilon') f_A(v) dv - \int_{\epsilon'}^{w_2^{-1}(\epsilon')} (v-\epsilon') f_B(v) dv \\
\leq & \int_{\epsilon}^{w_2^{-1}(\epsilon)} (v-\epsilon) f_B(v) dv - \beta \int_{\epsilon+\eta}^{\phi(\epsilon')} (v-\epsilon-\eta) f_A(v) dv - \int_{\epsilon+\eta}^{w_2^{-1}(\epsilon)} (v-\epsilon-\eta) f_B(v) dv \\
= & \int_{\epsilon}^{\epsilon+\eta} (v-\epsilon) f_B(v) dv + \int_{\epsilon+\eta}^{w_2^{-1}(\epsilon)} \eta f_B(v) dv - \beta \int_{\epsilon+\eta}^{\phi(\epsilon')} (v-\epsilon-\eta) f_A(v) dv \\
\leq & \eta \cdot \bar{f}_B + \eta \cdot \bar{f}_B - \beta \int_{\epsilon+\eta}^{\phi(\epsilon')} (v-\epsilon-\eta) \underline{f}_A dv \\
= & \eta \cdot \bar{f}_B + \eta \cdot \bar{f}_B - \frac{1}{2} \underline{f}_A \cdot \beta \cdot (\phi(\epsilon') - \epsilon - \eta)^2
\end{aligned}
$$

where the second inequality follows from the fact that $w_2^{-1}(\cdot)$ is non-decreasing and all integrands are weakly positive over their respective regions of integration. Rearranging terms implies that it must be that

$$
\frac{4}{\beta} \frac{\bar{f}_B}{\underline{f}_A} \geq \frac{(\phi(\epsilon') - \epsilon - \eta)^2}{\eta}
$$

Therefore, if $\phi(\epsilon') - \phi(\epsilon) = \phi(\epsilon') - \epsilon > \delta$ then the numerator on the right-hand side of the previous expression is positive, and the entire right-hand side grows unboundedly large as $\eta \to 0$, which is a contradiction with the inequality holding.

Second, consider the case in which $\epsilon < \phi(\epsilon)$. Because the definition of an upward jump specifies the quantifier "for all $\eta > 0$" it suffices to assume that $\delta > \eta$. Comparing (3) at ϵ and ϵ' respectively implies that

$$
\begin{aligned}
0 &\leq \beta \int_{\epsilon}^{\phi(\epsilon)} (v-\epsilon) f_A(v) dv + \int_{\epsilon}^{w_2^{-1}(\epsilon)} (v-\epsilon) f_B(v) dv - \beta \int_{\epsilon'}^{\phi(\epsilon')} (v-\epsilon') f_A(v) dv - \int_{\epsilon'}^{w_2^{-1}(\epsilon')} (v-\epsilon') f_B(v) dv \\
&\leq \beta \int_{\epsilon}^{\phi(\epsilon)} (v-\epsilon) f_A(v) dv + \int_{\epsilon}^{w_2^{-1}(\epsilon)} (v-\epsilon) f_B(v) dv - \beta \int_{\epsilon+\eta}^{\phi(\epsilon')} (v-\epsilon-\eta) f_A(v) dv - \int_{\epsilon+\eta}^{w_2^{-1}(\epsilon)} (v-\epsilon-\eta) f_B(v) dv \\
&= \beta \int_{\epsilon}^{\phi(\epsilon)} (v-\epsilon) f_A(v) dv + \int_{\epsilon}^{\epsilon+\eta} (v-\epsilon) f_B(v) dv + \int_{\epsilon+\eta}^{w_2^{-1}(\epsilon)} \eta f_B(v) dv - \beta \int_{\epsilon+\eta}^{\phi(\epsilon')} (v-\epsilon-\eta) f_A(v) dv \\
&= \beta \int_{\epsilon}^{\epsilon+\eta} (v-\epsilon) f_A(v) dv + \int_{\epsilon}^{\epsilon+\eta} (v-\epsilon) f_B(v) dv + \int_{\epsilon+\eta}^{w_2^{-1}(\epsilon)} \eta f_B(v) dv - \beta \int_{\phi(\epsilon)}^{\phi(\epsilon')} (v-\epsilon) f_A(v) dv + \beta \int_{\epsilon+\eta}^{\phi(\epsilon')} \eta f_A(v) dv \\
&\leq \beta \eta \bar{f}_A + \eta \cdot \bar{f}_B + \eta \cdot \bar{f}_B - \beta [\phi(\epsilon') - \phi(\epsilon)] \cdot [\phi(\epsilon) - \epsilon] \underline{f}_A + \beta \eta \bar{f}_A \\
&\leq \beta \eta \bar{f}_A + \eta \cdot \bar{f}_B + \eta \cdot \bar{f}_B - \beta \cdot \delta \cdot [\phi(\epsilon) - \epsilon] \underline{f}_A + \beta \eta \bar{f}_A,
\end{aligned}
$$

where the transition from the third to the forth line follows from the identity $\int_a^b g(x)dx - \int_c^d g(x)dx = \int_a^c g(x)dx - \int_b^d g(x)dx$, and that from the second-last to the last line follows from the assumption that $\phi(\epsilon') - \phi(\epsilon) > \delta$ (other manipulations are either straightforward or analogous to the first case, and hence the illustrations are omitted). However, for any sufficiently small η, the last expression is negative because all terms except for $-\beta \cdot \delta \cdot [\phi(\epsilon) - \epsilon] \underline{f}_A < 0$ converges to zero, a contradiction. □

Claim **3.** *$\phi(\epsilon) \geq \hat{w}_1^{-1}(\epsilon)$ for all ϵ.*

Proof. By the definition of $\hat{w}_1^{-1}(\cdot)$ it suffices to show that for any ϵ, if $\hat{w}_1(v) \leq \epsilon$ then $v \leq \phi(\epsilon)$. Suppose for contradiction that $\hat{w}_1(v) \leq \epsilon$ but that $v > \phi(\epsilon)$. Then by the upward continuity of $\phi(\cdot)$ (see Claim 2), there is some $\epsilon' > \epsilon$ such that $\phi(\epsilon') < v$. Then this implies, by the definition of $\hat{w}_1$, that $\hat{w}_1(v) \geq \epsilon' > \epsilon$. But this contradicts our ongoing assumption that $\hat{w}_1(v) \leq \epsilon$. □

Claim **4.** *$\hat{w}_1^{-1}(\epsilon) = \phi(\epsilon)$ for all $\epsilon > 0$ such that $\phi(\epsilon') \geq \phi(\epsilon)$ for all $\epsilon' > \epsilon$.*

Proof. Let $\epsilon > 0$ such that $\phi(\epsilon') \geq \phi(\epsilon)$ for all $\epsilon' > \epsilon$. Let $v := \phi(\epsilon)$. Note that $\hat{w}_1(v') > \epsilon$ for every $v' > v$ because $\phi(\cdot)$ is upward continuous (see Claim 2). Also note that $\hat{w}_1(v') \leq \epsilon$ for every $v' < v$ because $\phi(\epsilon') \geq \phi(\epsilon) = v > v'$ for every $\epsilon' > \epsilon$ by assumption. Now, consider

$$\hat{w}_1^{-1}(\epsilon) := \sup\{v' | \hat{w}_1(v') \leq \epsilon\}.$$

From the two previous inequalities, we conclude that $\hat{w}_1^{-1}(\epsilon) = \phi(\epsilon)$. □

Claim **5.** *Let $w_1 : [0,1] \to [0,1]$ be a non-decreasing function that satisfies (2) for every $\epsilon \in [0,1]$. Then $w_1(v) \geq \hat{w}_1(v)$ for almost all $v \in [0,1]$.*

Proof. We first show that for any v it is either the case that $w_1(v) \geq \hat{w}_1(v)$ or that $w_1(\cdot)$ is discontinuous from the right at v, i.e. $w_1(v) \neq \lim_{v' \to v+} w_1(v')$. Suppose the negation for contradiction: $w_1(v) < \hat{w}_1(v)$ for some $v \in [0,1]$ and $w_1(\cdot)$ is continuous from the right at v. Then by definition of $\hat{w}_1(\cdot)$ there exists ϵ^* such that $w_1(v) < \epsilon^*$ and $\phi(\epsilon^*) \leq v$. Since (2) holds for ϵ^* and

$w_1^{-1}(\epsilon^*)$ by assumption, it must be that $w_1^{-1}(\epsilon^*) \le \phi(\epsilon^*) \le v$. Since $w_1(v) < \epsilon^*$, the assumption that $w_1(v) = \lim_{v' \to v+} w_1(v')$ implies that there exists $\delta > 0$ such that $w_1(v+\delta) < \epsilon^*$. But then that implies that $w_1^{-1}(\epsilon^*) > v$ which contradicts $w_1^{-1}(\epsilon^*) \le v$.

Because w_1 is non-decreasing, there are at most countably many values v such that $w_1(v) \neq \lim_{v' \to v+} w_1(v')$.[28] By the previous paragraph, this implies that $w_1(v) \ge \hat{w}_1(v)$ for almost all $v \in [0,1]$.

□

Claim **6.** *For any $w_2(\cdot)$, $\hat{w}_1(v) \le v$ for all $v \in [0,1)$.*

Proof. Fix any $v \in [0,1]$. If the set $\{\epsilon | \phi(\epsilon) \le v\}$ is empty then $\hat{w}_1(v) = 0 \le v$. Therefore, we focus on the case in which the set $\{\epsilon | \phi(\epsilon) \le v\}$ is non-empty.

For each $\epsilon > 0$, it is the case that $\tilde{v} = \epsilon$ satisfies (3). To see this, note the first term of the right-hand side equals zero if $\tilde{v} = \epsilon$. Then comparing the left-hand side to the second term of the right-hand side, the inequality holds because: 1) the region of integration is larger for the left-hand side ($1 \ge w_2^{-1}(\epsilon)$ for all ϵ by definition of $w_2^{-1}(\cdot)$, and $0 < \epsilon$), and 2) the left-hand side integrand is weakly bigger than the right-hand side integrand since $\epsilon \ge w_2(v)$ for all $v < w_2^{-1}(\epsilon)$ by definition of $w_2(\cdot)$ and $w_2^{-1}(\epsilon)$.

Since $\phi(\epsilon)$ is defined as the supremum value of $\tilde{v}$ for which (3) holds for any ϵ, it must be that $\phi(\epsilon) \ge \epsilon$ for all $\epsilon \in [0,1]$. By the definition of $\hat{w}_1$, this implies that $\hat{w}_1(v) \le v$: if $\hat{w}_1(v) = \epsilon^* > v$ then $\epsilon^* := \sup\{\epsilon | \phi(\epsilon) \le v\}$ implies that $\phi(\epsilon^*) \le v < \epsilon^*$ which contradicts $\phi(\epsilon) \ge \epsilon$ for all $\epsilon \in [0,1]$.

□

We now return to the proof of the Lemma.

Proof of the "only if" part. Suppose $\hat{\pi}_1 < \pi_2^{O_2}$. By Claim 5, for any monotonic wage function $w_1(\cdot)$ satisfying the No Desegregation Condition, $\pi_1 := \beta \int_0^1 (v - w_1(v)) f_A(v) dv \le \hat{\pi}_1 < \pi_2^{O_2}$. Therefore, it is impossible to satisfy both the No Desegregation Condition and the Equal Profit Condition simultaneously. By Lemma 1, this implies there exists no core outcome in which firm 2 pays wage schedule $w_2(\cdot)$.

Proof of the "if" part. Suppose $\hat{\pi}_1 \ge \pi_2^{O_2}$. Consider a wage function $w_1^x(\cdot)$ for $x \in [0,1]$ where

$$w_1^x(v) := \begin{cases} \hat{w}_1(v) & \text{if } v < x \\ v & \text{otherwise.} \end{cases}$$

First, we claim that for any x, $w_1^x(\cdot)$ is monotonic. This follows from the monotonicity of $\hat{w}_1(\cdot)$, and the fact that $\hat{w}_1(v) < x$ for all $v < x$ (Claim 6).

Second, we claim that for any x, $w_1^x(\cdot)$ satisfies Individual Rationality. This follows from the facts that $\hat{w}_1(v) \le v$ for all $v < x$ (Claim 6), and that $w_1^x(v) = v$ for all $v \ge x$.

Third, we claim that for any x, $w_1^x(\cdot)$ satisfies the No Desegregation Condition. To see this, first note that $\hat{w}_1(\cdot)$ satisfies the No Desegregation Condition (Lemma 1 and Claim 3). This implies that any $w_1(\cdot)$ such that $w_1(v) \ge \hat{w}_1(v)$ for all v satisfies the No Desegregation Condition

[28] This claim can be shown as follows. Suppose that $w_1(v) \neq \lim_{v' \to v+} w_1(v')$. Then, because w_1 is non-decreasing, we have $w_1(v) < \lim_{v' \to v+} w_1(v')$. Then, because the set of rational numbers is dense in the set of real numbers, there exists at least one rational number in $(w_1(v), \lim_{v' \to v+} w_1(v'))$; denote one such rational number as ϵ_v. Moreover, because w_1 is non-decreasing, $v < \bar{v}$ implies $\lim_{v' \to v+} w_1(v') \le w_1(\bar{v})$, and hence $\epsilon_v \neq \epsilon_{\bar{v}}$. Therefore, we have established that there is an injective function from the sets of all points v such that $w_1(v) \neq \lim_{v' \to v+} w_1(v')$ to the set of rational numbers, which implies that the cardinality of the former set is at most as large as the cardinality of the latter.

as, $w_1^{-1}(\epsilon) \leq \hat{w}_1^{-1}(\epsilon)$ for all $\epsilon \in [0,1]$ and therefore the first integral on the right-hand side of the No Desegregation Condition is weakly smaller given wage schedule $w_1(\cdot)$.

Finally, we claim that for some $x^* \in [0,1]$, $\pi_1^{x^*} := \beta \int_0^1 \left(v - w_1^{x^*}(v)\right) f_A(v) dv = \pi_2^{O_2}$. To see this, note that $\pi_1^x = \beta \int_0^x (v - \hat{w}_1(v)) f_A(v) dv$. Then π_1^x can be viewed as a one-variable, weakly increasing function of x. By the assumption that F_A is atomless, π_1^x is continuous in x. Since $\hat{\pi}_1 = \pi_1^1 \geq \pi_2^{O_2}$ and $\pi_1^0 = 0 \leq \pi_2^{O_2}$, the intermediate value theorem applies, implying that there is some $x^* \in [0,1]$ such that $\pi_1^{x^*} = \pi_2^{O_2}$. We have shown all the conditions in Lemma 1, so we conclude that $O = (O_1, O_2)$, where O_1 is given by $f_A^1(v) = f_A(v), w_1^A(v) = w_1^{x^*}(v), f_B^1(v) = 0$, and $w_1^B(v) = 0$ for all v, is a core outcome.

□

Remark **8.** We observe that $\hat{w}_1^{-1}(\cdot)$ is the largest monotonic function that is everywhere below $\phi(\cdot)$. Formally,

Claim **7.** *If function $h:[0,1] \to [0,1]$ is monotone nondecreasing and $h(\epsilon) \leq \phi(\epsilon)$ for all $\epsilon \in [0,1]$, then $h(\epsilon) \leq \hat{w}_1^{-1}(\epsilon)$ for all $\epsilon \in [0,1]$.*

Proof. Suppose for contradiction that there exists ϵ such that $h(\epsilon) > \hat{w}_1^{-1}(\epsilon)$. Since $h(\epsilon) \leq \phi(\epsilon)$ by assumption, it follows that $\hat{w}_1^{-1}(\epsilon) < \phi(\epsilon)$.

First, suppose that the set $\{\tilde{\epsilon} \in [0,\epsilon] | \phi(\hat{\epsilon}) \geq \phi(\tilde{\epsilon}) \text{ for every } \hat{\epsilon} > \tilde{\epsilon}\}$ is nonempty. In that case, define $\epsilon^* := \sup\{\tilde{\epsilon} \in [0,\epsilon] | \phi(\hat{\epsilon}) \geq \phi(\tilde{\epsilon}) \text{ for every } \hat{\epsilon} > \tilde{\epsilon}\}$. Because $\phi(\cdot)$ is upward continuous, by Claim 4 it follows that $\phi(\epsilon^*) = \hat{w}_1^{-1}(\epsilon^*)$.[29] Because $\hat{w}_1^{-1}(\cdot)$ is nondecreasing, $\hat{w}_1^{-1}(\epsilon^*) \leq \hat{w}_1^{-1}(\epsilon)$. Combined with the assumption that $h(\epsilon) > \hat{w}_1^{-1}(\epsilon)$, it follows that $h(\epsilon) > \hat{w}_1^{-1}(\epsilon^*) = \phi(\epsilon^*)$. Since this is a contradiction to $h(\epsilon) \leq \phi(\epsilon)$ if $\epsilon^* = \epsilon$, we proceed with the assumption that $\epsilon^* < \epsilon$. By definition of ϵ^*, it follows that there exists $\bar{\epsilon} > \epsilon^*$ such that $\phi(\bar{\epsilon}) \leq \phi(\epsilon^*)$.[30] Moreover, we observe that $\epsilon \leq \bar{\epsilon}$.[31] Because $h(\cdot)$ is nondecreasing, it follows that $h(\bar{\epsilon}) \geq h(\epsilon) > \phi(\epsilon^*) \geq \phi(\bar{\epsilon})$, a contradiction.

Next, suppose that the set $\{\tilde{\epsilon} \in [0,\epsilon] | \phi(\hat{\epsilon}) \geq \phi(\tilde{\epsilon}) \text{ for every } \hat{\epsilon} > \tilde{\epsilon}\}$ is empty. Let $\underline{\epsilon}$ be a minimizer of $\phi(\cdot)$ over $[0,1]$.[32] Note that it cannot be that $\underline{\epsilon} \leq \epsilon$, because then $\underline{\epsilon} \in \{\tilde{\epsilon} \in [0,\epsilon] | \phi(\hat{\epsilon}) \geq \phi(\tilde{\epsilon}) \text{ for every } \hat{\epsilon} > \tilde{\epsilon}\}$, a contradiction to the assumption made in the present case. Thus, assume $\underline{\epsilon} > \epsilon$. From the constructions of $\hat{w}_1(\cdot)$ and $\hat{w}_1^{-1}(\cdot)$ on page 16, it follows that $\hat{w}_1^{-1}(\epsilon) = \phi(\underline{\epsilon})$. But this is a contradiction to monotonicity of $h(\cdot)$ because $h(\epsilon) > \hat{w}^{-1}(\epsilon) = \phi(\underline{\epsilon}) \geq h(\underline{\epsilon})$, where the last inequality follows from the hypothesis of the claim. This completes the proof.

□

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[29]The proof of this claim is as follows. By Claim 4, it suffices to show that ϵ^* is in the set $\{\tilde{\epsilon} \in [0,\epsilon] | \phi(\hat{\epsilon}) \geq \phi(\tilde{\epsilon}) \text{ for every } \hat{\epsilon} > \tilde{\epsilon}\}$. To show this, assume for contradiction that there exists $\hat{\epsilon} > \epsilon^*$ such that $\phi(\hat{\epsilon}) < \phi(\epsilon^*)$. Then, by upward continuity of $\phi(\cdot)$, there exists $\delta > 0$ such that $\phi(\epsilon^{**}) > \phi(\hat{\epsilon})$ for all $\epsilon^{**} \in (\epsilon^* - \delta, \epsilon^*)$. However, this is a contradiction to the definition of ϵ^*.

[30]The proof of this claim is as follows. Suppose for contradiction that $\phi(\bar{\epsilon}) > \phi(\epsilon^*)$ for all $\bar{\epsilon} > \epsilon^*$. Then, by upward continuity of $\phi(\cdot)$, ϵ^* would not be the supremum of the set $\{\tilde{\epsilon} \in [0,\epsilon] | \phi(\hat{\epsilon}) \geq \phi(\tilde{\epsilon}) \text{ for every } \hat{\epsilon} > \tilde{\epsilon}\}$, a contradiction.

[31]Suppose for contradiction that $\epsilon > \bar{\epsilon}$. Recall that $\phi(\bar{\epsilon}) \leq \phi(\epsilon^*)$. Since $\epsilon^* \leq \bar{\epsilon} < \epsilon$, then this is a contradiction with the fact that ϵ^* is the supreumum of the set $\{\tilde{\epsilon} \in [0,\epsilon] | \phi(\hat{\epsilon}) \geq \phi(\tilde{\epsilon}) \text{ for every } \hat{\epsilon} > \tilde{\epsilon}\}$.

[32]The claim that a minimizer of $\phi(\cdot)$ over $[0,1]$ exists is proven as follows. Let $\underline{v} := \inf_{\tilde{\epsilon} \in [0,1]} \phi(\tilde{\epsilon})$. By definition, there exists a sequence $(\epsilon_n)_{n=1}^{\infty}$ such that $\underline{\epsilon} := \lim_{n\to\infty} \epsilon_n$ exists and $\lim_{n\to\infty} \phi(\epsilon_n) = \underline{v}$. We will show that $\phi(\underline{\epsilon}) = \underline{v}$. To show this, suppose for contradiction that $\phi(\underline{\epsilon}) \neq \underline{v}$. Because $\underline{v}$ is the infimum of ϕ, this implies that $\phi(\underline{\epsilon}) > \underline{v}$. Suppose that there is a subsequence $(\epsilon_{n_k})_k$ of $(\epsilon_n)_n$ such that $\epsilon_{n_k} < \underline{\epsilon}$ for each k. Now, pick an arbitrary $\delta \in \left(0, \frac{\phi(\underline{\epsilon}) - \underline{v}}{2}\right)$: Note that $\phi(\underline{\epsilon}) - \underline{v} > 0$. Because $\underline{\epsilon} = \lim_{n\to\infty} \epsilon_n$ and $\lim_{n\to\infty} \phi(\epsilon_n) = \underline{v}$, for any $\eta > 0$ there exists k such that $\epsilon_{n_k} > \underline{\epsilon} - \eta$ and $\phi(\epsilon_{n_k}) + \delta < \underline{v} + 2\delta < \phi(\underline{\epsilon})$. But this is a contradiction to upward continuity of ϕ. Now, suppose that there is no subsequence $(\epsilon_{n_k})_k$ of $(\epsilon_n)_n$ such that $\epsilon_{n_k} < \underline{\epsilon}$ for each k. Then there is a subsequence $(\epsilon_{n_k})_k$ of $(\epsilon_n)_n$ such that $\epsilon_{n_k} \geq \underline{\epsilon}$ for each k. But this is a contradiction to the fact that, by inspection of the No Desegregation Condition, $\liminf_{\tilde{\epsilon} \to \underline{\epsilon}+0} \phi(\tilde{\epsilon}) \geq \phi(\underline{\epsilon})$.

Proof of Proposition 5

Proof. First, consider the case where $w_2(v)=v$ for almost all v. The conclusions of the proposition hold trivially with $\beta^*=0$ in this case because $w_1(v)=w_2(v)=v$ for all v can be supported in a core outcome for every β. Thus, in the following, we exclude the case in which $w_2(v)=v$ for almost all v.

From Lemma 2 note that, to tell whether there exists a core outcome in which firm 2 chooses payment schedule $w_2(\cdot)$ (for $B-$group workers), it suffices to check whether

$$\hat{\pi}_1 \geq \pi_2^{O_2}. \tag{19}$$

Lemma **6.** *There exists* $\beta^* \in [0,\infty)$ *such that* (19) *holds for all* $\beta > \beta^*$.

Proof. Let ϵ^* be the supremum of v such that $w_2(v') = v'$ for almost all $v' < v$, that is, $\epsilon^* := \{v \in [0,1] | w_2(v') = v' \text{ for almost all } v' \in [0,v)\}$. Note that we have already excluded the function $w_2(\cdot)$ such that $w_2(v)=v$ for almost all $v \in [0,1]$ from consideration, so it follows that $\epsilon^* < 1$. We proceed by proving the following three claims.

Claim **8.** $\phi(\cdot)$ *uniformly converges to the identity function as* $\beta \to \infty$.

Proof. By (3) and Claim 1, we have

$$\int_0^1 (v-w_2(v))f_B(v)dv \geq \beta \int_\epsilon^{\phi(\epsilon)} (v-\epsilon)f_A(v)dv.$$

The right-hand side of this inequality can be bounded from below as follows:

$$\begin{aligned}\beta \int_\epsilon^{\phi(\epsilon)} (v-\epsilon)f_A(v)dv &\geq \beta \int_\epsilon^{\phi(\epsilon)} (v-\epsilon)\underline{f}_A dv \\ &= \underline{f}_A \beta \int_\epsilon^{\phi(\epsilon)} (v-\epsilon)dv \\ &= \frac{\underline{f}_A \beta}{2} (\phi(\epsilon)-\epsilon)^2.\end{aligned}$$

This and the fact that $\phi(\epsilon) \geq \epsilon$ for all ϵ imply the following bound:

$$0 \leq \phi(\epsilon)-\epsilon \leq \left[\frac{2}{\underline{f}_A \beta} \int_0^1 (v-w_2(v))f_B(v)dv\right]^{1/2}. \tag{20}$$

Because the right-hand side of (20) does not depend on ϵ and converges to zero as $\beta \to \infty$, we have established uniform convergence of $\phi(\cdot)$ to the identity function as $\beta \to \infty$, as desired. □

Claim **9.** *For any* $n > 0$ *and any* $0 < \underline{\epsilon} < \overline{\epsilon} < 1$ *there exists* β^* *such that for all* $\beta > \beta^*$ *there exists a sequence* $\epsilon_1,...,\epsilon_n$ *satisfying:*

1. *Intervals* $[\epsilon_k, \hat{w}_1^{-1}(\epsilon_k))$ *and* $[\epsilon_\ell, \hat{w}_1^{-1}(\epsilon_\ell))$ *are disjoint for any* $k \neq \ell$,
2. $\phi(\epsilon_k) = \hat{w}_1^{-1}(\epsilon_k)$, *and*

3. $\underline{\epsilon} < \epsilon_k < \bar{\epsilon}$ *for all* k.

Proof. Define $v_1 = \underline{\epsilon}+2\gamma, v_2 = \underline{\epsilon}+4\gamma,...,v_n = \underline{\epsilon}+2n\gamma$, where $\gamma > 0$ is sufficiently small such that $v_n < \bar{\epsilon}$. Let β be large enough so that

$$\phi(\epsilon) \in [\epsilon, \epsilon+\gamma), \tag{21}$$

for every $\epsilon \in [0,1]$ (note that such β exists by Claim 8).

For each $k \in \{1,2,...,n\}$, let $\epsilon_k := \hat{w}_1(v_k)$. Then $\epsilon_k = \hat{w}_1(v_k) \in [v_k - \gamma, v_k] = [\underline{\epsilon}+(2k-1)\gamma, \underline{\epsilon}+2k\gamma]$, where the first relation follows from the definition of ϵ_k, the second relation follows from the definition of $\hat{w}_1(\cdot)$ because:

1. $\phi(\epsilon) \in [\epsilon, \epsilon+\gamma)$ for any ϵ by (21), so if $\epsilon < v_k - \gamma$ then $\phi(\epsilon) < v_k$. Because $\hat{w}_1(v_k)$ is defined as the supremum of all ϵ such that $\phi(\epsilon) \leq v_k$, we conclude that $\hat{w}_1(v_k) \geq v_k - \gamma$.

2. $\hat{w}_1(v_k) \leq v_k$ by Claim 6.

Moreover, $\hat{w}_1^{-1}(\epsilon_k) < \epsilon_k + \gamma \leq \underline{\epsilon}+(2k+1)\gamma$, where the first inequality follows from (21) and that $\phi(\epsilon) \geq \hat{w}_1^{-1}(\epsilon)$ by Claim 3, and the second inequality follows from our earlier assertion that $\epsilon_k \leq \underline{\epsilon}+2k\gamma$. We also know that $\epsilon_k \geq \underline{\epsilon}+(2k-1)\gamma$, so $\hat{w}_1^{-1}(\epsilon_k) \geq \underline{\epsilon}+(2k-1)\gamma$. Thus, it follows that $\hat{w}_1^{-1}(\epsilon_k) \in [\underline{\epsilon}+(2k-1)\gamma, \underline{\epsilon}+(2k+1)\gamma]$. Recalling that $\epsilon_k \in [\underline{\epsilon}+(2k-1)\gamma, \underline{\epsilon}+2k\gamma]$, the intervals $[\epsilon_k, \hat{w}^{-1}(\epsilon_k))$ are all disjoint, satisfying requirement 1 of the claim.

We can also see that $\phi(\epsilon') > \phi(\epsilon_k)$ for each k and all $\epsilon' > \epsilon_k$, because the definition $\epsilon_k = \hat{w}_1(v_k)$ and the upward continuity of $\phi(\cdot)$ (see Claim 2) imply that $\phi(\epsilon_k) = v_k$[33] and that there is no $\epsilon' > \epsilon_k$ such that $\phi(\epsilon') \leq v_k$. By Claim 4, this implies that $\phi(\epsilon_k) = \hat{w}^{-1}(\epsilon_k)$ for all k, satisfying requirement 2 of the claim.

Finally, the property that $\epsilon_k \in [v_k - \gamma, v_k]$ for every k and that $\underline{\epsilon}+2\gamma = v_1 < v_2 < \cdots < v_n < \bar{\epsilon}$ imply condition 3 of the claim holds.

□

For any ϵ at which the No Desegregation Condition (i.e. (2)) holds with equality, we can rearrange terms to obtain

$$\int_0^{\epsilon} (v - w_2(v)) f_B(v) dv + \int_{w_2^{-1}(\epsilon)}^{1} (v - w_2(v)) f_B(v) dv + \int_{\epsilon}^{w_2^{-1}(\epsilon)} (\epsilon - w_2(v)) f_B(v) dv = \beta \int_{\epsilon}^{\hat{w}_1^{-1}(\epsilon)} (v - \epsilon) f_A(v) dv. \tag{22}$$

Claim **10.** *For any* $\underline{\epsilon}$ *and any* $\bar{\epsilon}$ *such that* $\epsilon^* < \underline{\epsilon} < \bar{\epsilon} < 1$, *there exists* n *for which the left-hand side of* (22) *is strictly larger than* $\pi_2^{O_2}/n$ *for all* $\epsilon \in (\underline{\epsilon}, \bar{\epsilon})$.

Proof. By the definition of ϵ^*, $\int_0^{\underline{\epsilon}} (v - w_2(v)) f_B(v) dv := \delta > 0$. Moreover, because $v - w_2(v) \geq 0$ for every $v \in [0,1]$ by assumption, it follows that

$$\int_0^{\epsilon} (v - w_2(v)) f_B(v) dv \geq \int_0^{\underline{\epsilon}} (v - w_2(v)) f_B(v) dv = \delta,$$

[33]To see this, suppose for contradiction that $\phi(\epsilon_k) \neq v_k$. If $\phi(\epsilon_k) < v_k$, then since $\phi(\epsilon'_k) > v_k$ for all $\epsilon'_k > \epsilon_k$ by definition of $\epsilon_k = \hat{w}_1(v_k)$, $\phi(\cdot)$ has an upward jump of type (ii) at ϵ'_k. If $\phi(\epsilon_k) > v_k$, then since for any $\eta > 0$ there exists $x \in (\epsilon_k - \eta, \epsilon_k)$ such that $\phi(x) \leq v_k$, $\phi(\cdot)$ has an upward jump of type (i) at ϵ_k. In both cases, it is a contradiction to upward continuity of $\phi(\cdot)$.

for any $\epsilon \in (\underline{\epsilon},\overline{\epsilon})$. Moreover, each term of the left-hand side of (22) is nonnegative, so the left-hand side of (22) is at least $\delta > 0$ for all $\epsilon \in (\underline{\epsilon},\overline{\epsilon})$. Therefore, for any $n > \pi_2^{O_2}/\delta$, the left-hand side of (22) is strictly larger than $\pi_2^{O_2}/n$ for all $\epsilon \in (\underline{\epsilon},\overline{\epsilon})$, as desired. □

We now return to the proof of Lemma 6 to establish that $\hat{\pi}_1 > \pi_2^{O_2}$ for sufficiently large β. To do so, take any $\underline{\epsilon}$ and any $\overline{\epsilon}$ such that $\epsilon^* < \underline{\epsilon} < \overline{\epsilon} < 1$. By Claim 10 and (22), for any sufficiently large n and any $\epsilon \in (\underline{\epsilon},\overline{\epsilon})$ we have

$$\pi_2^{O_2} < \beta n \int_{\epsilon}^{\hat{w}_1^{-1}(\epsilon)} (v-\epsilon) f_A(v) dv.$$

Further noting that any $v \in [\epsilon, \hat{w}_1^{-1}(\epsilon)]$ satisfies $\epsilon \geq \hat{w}_1(v)$, we can see that

$$\pi_2^{O_2} < \beta n \int_{\epsilon}^{\hat{w}_1^{-1}(\epsilon)} (v-\hat{w}_1(v)) f_A(v) dv. \tag{23}$$

Consider any β^*, $\beta > \beta^*$ and a sequence $\epsilon_1, \epsilon_2, ..., \epsilon_n$ with $\underline{\epsilon} < \epsilon_k < \overline{\epsilon}$ for all $k \in \{1,...,n\}$ that satisfy the conditions in Claim 9. Let

$$k^* \in \arg \min_{k \in \{1,...,n\}} \int_{\epsilon_k}^{\hat{w}_1^{-1}(\epsilon_k)} (v-\hat{w}_1(v)) f_A(v) dv.$$

Then, by definition of k^*,

$$\int_{\epsilon_{k^*}}^{\hat{w}_1^{-1}(\epsilon_{k^*})} (v-\hat{w}_1(v)) f_A(v) dv \leq \int_{\epsilon_k}^{\hat{w}_1^{-1}(\epsilon_k)} (v-\hat{w}_1(v)) f_A(v) dv, \tag{24}$$

for every $k \in \{1,...,n\}$. Therefore,

$$\begin{aligned}
\beta n \int_{\epsilon_{k^*}}^{\hat{w}_1^{-1}(\epsilon_{k^*})} (v-\hat{w}_1(v)) f_A(v) dv &\leq \beta \sum_{k \in \{1,...,n\}} \int_{\epsilon_k}^{\hat{w}_1^{-1}(\epsilon_k)} (v-\hat{w}_1(v)) f_A(v) dv \\
&\leq \beta \int_0^1 (v-\hat{w}_1(v)) f_A(v) dv \\
&= \hat{\pi}_1,
\end{aligned}$$

where the first inequality follows from (24), the second inequality follows from the facts that $v - \hat{w}_1(v) \geq 0$ for every v and that all the intervals of the form $[\epsilon_k, \hat{w}_1^{-1}(\epsilon_k))$ are disjoint subsets of $[0,1]$, and the last equality follows from the definition of $\hat{\pi}_1$. This and (23) taken at $\epsilon = \epsilon_{k^*}$ imply $\hat{\pi}_1 > \pi_2^{O_2}$, as desired. □

Application of Lemma 6 completes the proof. □

Proof of Remark 6

In the main body, Remark 6 states

> Suppose there is a group-based EPSW and fix F_A and F_B arbitrarily. There exists $\beta^* \in (0,\infty)$ such that in the market (β, F_A, F_B) with any $\beta > \beta^*$ there exists a core outcome in which $w_2(v) = 0$ for all v (i.e. the wage of each $B-$group worker is 0) while in the market (β, F_A, F_B) with any $\beta < \beta^*$ there exists no core outcome in which $w_2(v) = 0$ for all v.

It also claims that this statement can be generalized. We formally state and prove the following result, which subsumes the case with $w_2(v) = 0$ for all v.

Remark **6** (Generalization). *Let $w_2(\cdot)$ be a continuous wage function such that $w_2(v) \leq v$ for all $v \in [0,1)$, $w_2^{-1}(\cdot)$ is absolutely continuous, and $\int_{\epsilon}^{w_2^{-1}(\epsilon)} (v-\epsilon) f_B(v) dv$ is non-increasing in ϵ. Also, let $f_A(\cdot)$ be continuous. Then there exists $\beta^* \in [0,\infty)$ such that in the market (β, F_A, F_B) with any $\beta > \beta^*$ there exists a core outcome in which the wage schedule of $B-$group workers is given by $w_2(\cdot)$ while in the market (β, F_A, F_B) with any $\beta < \beta^*$ there exists no core outcome in which the wage schedule of $B-$group workers is given by $w_2(\cdot)$.*

Proof. First, consider the case where $w_2(v) = v$ for almost all v. The conclusion of the proposition holds trivially with $\beta^* = 1$ in this case because $w_1(v) = w_2(v) = v$ for all v can be supported in a core outcome for every β. Thus, in the following, we exclude the case in which $w_2(v) = v$ for almost all v.

Fix w_2 satisfying the conditions in the proposition. From Proposition 5, there exists β' such that there exists a core outcome in which the wage schedule of $B-$group workers is given by w_2 for all $\beta > \beta'$. Let $\bar{\beta} > \beta'$, and consider the set $[1, \bar{\beta}]$. In the remainder of the proof, we will show that there exists $\beta^* \in [1, \bar{\beta})$ such that for all $\beta \in (\beta^*, \bar{\beta}]$ there exists a core outcome in which the wage schedule of $B-$group workers is given by $w_2(\cdot)$ while for all $\beta \in [1, \beta^*)$ there exists no core outcome in which the wage schedule of $B-$group workers is given by w_2 (which implies that $\beta^* \leq \beta'$): This will complete the proof because Proposition 5 implies that the same conclusion holds for all $\beta \in [\bar{\beta}, \infty)$.

With the additional assumptions made for this part, we first establish two technical properties: first, Lemma 7 establishes that $\hat{w}_1^{-1}(\epsilon) = \phi(\epsilon)$ for all ϵ, and second, Lemma 8 implies that $\hat{w}_1^{-1}(\epsilon)$ is differentiable in β and ϵ. Building on these results, the proof of the "threshold structure" is provided in Lemma 9.

Lemma **7.** *If $\int_{\epsilon}^{w_2^{-1}(\epsilon)} (v-\epsilon) f_B(v) dv$ is non-increasing in ϵ, then*

1. *There exists some $E \in [0,1]$ such that $\phi(\cdot)$ is strictly increasing in ϵ for all $\epsilon < E$ and $\phi(\epsilon) = 1$ for all $\epsilon \geq E$, and*
2. *$\hat{w}_1^{-1}(\epsilon) = \phi(\epsilon)$ for all $\epsilon \in [0,1]$.*

Proof of Part 1. Let $0 \leq \epsilon_1 < \epsilon_2 \leq 1$. First, suppose that $\tilde{v}_1 := \phi(\epsilon_1) = 1$. By inspection, we note the following properties from (3): the left-hand side does not depend on ϵ, the second integral on the right-hand side is non-increasing in ϵ by assumption, and the first term on the right-hand side is strictly decreasing in ϵ when fixing $\tilde{v}_1$ as the upper limit of integration, it becomes clear that the inequality is satisfied at ϵ_2 and $\tilde{v}_1 = 1$. Therefore, $\phi(\epsilon_2) = 1$, which demonstrates that there exists E such that $\phi(\epsilon) = 1$ for any $\epsilon > E$.

Second, suppose $\phi(\epsilon_1) < 1$. Then, $\tilde{v}_1 := \phi(\epsilon_1)$ satisfies (3) at ϵ_1 with equality. Define $\hat{v} := \max\{\tilde{v}_1, \epsilon_2\}$. We consider the following exhaustive cases, which jointly show that $\phi(\epsilon_1) < \phi(\epsilon_2)$, as desired.

1. Suppose that $\tilde{v}_1 \geq \epsilon_2$, which implies $\hat{v} = \tilde{v}_1$. By inspection, we note the following properties from (3): the left-hand side does not depend on ϵ, the second integral on the right-hand side is non-increasing in ϵ by assumption, and the first term on the right-hand side is strictly decreasing in ϵ when fixing $\tilde{v}_1$ as the upper limit of integration. Therefore, $\hat{v}$ satisfies (3) at ϵ_2 with a strict inequality. Noting that the right-hand side of (3) is continuous in the variable $\tilde{v}$ for all $\epsilon \in [0,1]$, there exists $\bar{v} > \hat{v}$ that satisfies (3) at ϵ_2. Hence, we obtain $\phi(\epsilon_2) > \phi(\epsilon_1)$, as desired.

2. Suppose that $\tilde{v}_1 < \epsilon_2$. The proof of Claim 6 reveals that $\phi(\epsilon) \geq \epsilon$ for all $\epsilon \in [0,1]$. Therefore, $\phi(\epsilon_2) \geq \epsilon_2 > \tilde{v}_1 = \phi(\epsilon_1)$, establishing that $\phi(\epsilon_2) > \phi(\epsilon_1)$ as desired.

□

Proof of Part 2. By Part 1, it follows that $\phi(\epsilon') \geq \phi(\epsilon)$ for any $\epsilon' > \epsilon$. By applying Claim 4, we complete the proof. □

Second, we establish relevant differentiability properties of $\phi(\cdot)$. Note that we have treated $\phi(\cdot)$ as a function of ϵ, but it also depends on β. Thus, in the following result, we regard $\phi(\cdot)$ as a function of ϵ and β.

LEMMA **8.** *Suppose that $w_2^{-1}(\cdot)$ is absolutely continuous and $\int_{\epsilon}^{w_2^{-1}(\epsilon)} (v-\epsilon) f_B(v) dv$ is non-increasing in ϵ.*

1. *Fix $\beta \geq 1$. For almost all $\epsilon \in (0,1)$, $\frac{\partial \phi(\cdot)}{\partial \epsilon}$ exists.*

2. *Fix $\epsilon \in (0,1)$. For almost all $\beta \geq 1$, $\frac{\partial \phi(\cdot)}{\partial \beta}$ exists.*

3. *Let $f_A(\cdot)$ be continuous. Then for almost all $\epsilon \in (0,E)$ and almost all $\beta \geq 1$, $\frac{\partial \hat{w}_1}{\partial \beta}(\hat{w}_1^{-1}(\epsilon))$ exists and is equal to $-\frac{\partial \hat{w}_1}{\partial v}(\hat{w}_1^{-1}(\epsilon)) \frac{\partial \hat{w}_1^{-1}}{\partial \beta}(\epsilon)$.*

4. *There exists constant $c > 0$ such that for every $\beta < \bar{\beta}$, $\hat{w}_1(\cdot)$ is Lipschitz continuous in $v \in [0,1)$ with constant $1/c$.*

Proof of Part 1.

First, consider the case in which $w_2(v) = v$ for all $v \in [0,1]$. Then, $\phi(\epsilon) = \epsilon$ for all $\epsilon \in [0,1]$, so the conclusion holds.

Second, suppose it is not the case that $w_2(v) = v$ for all v. Fix $\underline{\epsilon} > 0$. It suffices to show that for almost all $\epsilon \in [\underline{\epsilon},1]$, $\frac{\partial \phi(\cdot)}{\partial \epsilon}$ exists. To do so we establish the following claim:

CLAIM **11.** *Suppose it is not the case that $w_2(v) = v$ for all v. Then there exists $\delta > 0$ such that, for every $\epsilon \in [\underline{\epsilon},1]$ with $\phi(\epsilon) < 1$, we have $\phi(\epsilon) - \epsilon > \delta$.*

Proof. For any ϵ with $\phi(\epsilon) < 1$, (3) holds with equality at $\tilde{v} = \phi(\epsilon)$, so we have

$$\beta \int_{\epsilon}^{\phi(\epsilon)} (v-\epsilon) f_A(v) dv = \int_0^1 [v - w_2(v)] f_B(v) dv - \int_{\epsilon}^{w_2^{-1}(\epsilon)} (v-\epsilon) f_B(v) dv. \tag{25}$$

The left-hand side of (25) can be bounded from above by

$$\beta \int_0^1 (\phi(\epsilon) - \epsilon) f_A(v) dv = \beta(\phi(\epsilon) - \epsilon). \tag{26}$$

The assumption that $\int_{\epsilon}^{w_2^{-1}(\epsilon)}(v-\epsilon)f_B(v)dv$ is non-increasing in ϵ implies that the right-hand side of (25) is bounded from below by

$$\int_0^1[v-w_2(v)]f_B(v)dv-\int_0^{w_2^{-1}(0)}(v-0)f_B(v)dv=\int_{w_2^{-1}(0)}^1[v-w_2(v)]f_B(v)dv, \tag{27}$$

where the equality holds because $w_2(v)=0$ for all $v<w_2^{-1}(0)$. If the right-hand side of (27) is strictly positive, then we have established the desired conclusion, because (25), (26), and (27) imply that $\phi(\epsilon)-\epsilon\geq\frac{1}{\beta}\int_{w_2^{-1}(0)}^1[v-w_2(v)]f_B(v)dv>0$ for all ϵ.

So, we now assume that the right-hand side of (27) is zero. This implies that there exists $v^*\in[0,1]$ such that $w_2(\cdot)$ is equal (almost everywhere) to

$$w_2(v)=\begin{cases}0 & v<v^*\\ v & v\geq v^*.\end{cases}$$

Recall that we have assumed it is not the case that $w_2(v)=v$ for all v. Therefore, $v^*>0$. Now, assume $\underline{\epsilon}<v^*$ without loss of generality. For any $\epsilon\in[\underline{\epsilon},v^*)$, the right-hand side of (25) is equal to

$$\int_0^{v^*}[v-0]f_B(v)dv-\int_{\epsilon}^{v^*}(v-\epsilon)f_B(v)dv=\int_0^{\epsilon}vf_B(v)dv+\int_{\epsilon}^{v^*}\epsilon f_B(v)dv\geq\int_0^{\underline{\epsilon}}v\underline{f}_Bdv=\frac{\underline{\epsilon}^2}{2}\underline{f}_B>0, \tag{28}$$

where the first inequality holds because $\int_{\epsilon}^{v^*}\epsilon f_B(v)dv\geq0$, $\epsilon\geq\underline{\epsilon}$, and $f_B(v)\geq\underline{f}_B$ for all v. For any $\epsilon\in[v^*,1]$, the right-hand side of (25) is equal to

$$\int_0^{v^*}[v-0]f_B(v)dv\geq\frac{(v^*)^2}{2}\underline{f}_B>0, \tag{29}$$

where the first inequality follows because $f_B(v)\geq\underline{f}_B$ for all v. Let

$$\delta_\beta:=\frac{1}{2\beta}\min\left\{\frac{\underline{\epsilon}^2}{2}\underline{f}_B,\frac{(v^*)^2}{2}\underline{f}_B\right\}=\frac{\underline{\epsilon}^2\underline{f}_B}{4\beta}, \tag{30}$$

where the last equality follows from $v^*>\underline{\epsilon}$. Combining (26), (28), and (29) we have that for all $\epsilon>\underline{\epsilon}$, $\phi(\epsilon)-\epsilon>\delta_\beta$ as desired. □

We now return to the *Proof of Part 1* of Lemma 8. First, consider the case in which $\phi(\epsilon)=1$. Then for any $\Delta\epsilon>0$, $\phi(\epsilon+\Delta\epsilon)=1$, so $\phi(\epsilon+\Delta\epsilon)-\phi(\epsilon)=0$.

Next, consider the case in which $\phi(\epsilon)<1$. By Claim 11, let δ_β be defined as in (30). We reproduce (3) (No Desegregation Condition) holding with equality (note that $\phi(\epsilon)<1$ implies that No Desegregation Condition holds with equality) here:

$$\int_0^1[v-w_2(v)]f_B(v)dv=\beta\int_{\epsilon}^{\phi(\epsilon)}(v-\epsilon)f_A(v)dv+\int_{\epsilon}^{w_2^{-1}(\epsilon)}(v-\epsilon)f_B(v)dv. \tag{31}$$

For any $\Delta\epsilon \in (0,1-\epsilon)$, the No Desegregation Condition (with inequality) must also hold at $\epsilon+\Delta\epsilon$, so

$$\int_0^1 [v-w_2(v)]f_B(v)dv \geq \beta \int_{\epsilon+\Delta\epsilon}^{\phi(\epsilon+\Delta\epsilon)} (v-(\epsilon+\Delta\epsilon))f_A(v)dv + \int_{\epsilon+\Delta\epsilon}^{w_2^{-1}(\epsilon+\Delta\epsilon)} (v-(\epsilon+\Delta\epsilon))f_B(v)dv. \quad (32)$$

Suppose that $\Delta\epsilon < \delta_\beta/2$. We continue the proof by subtracting (31) from (32). Note that $\phi(\epsilon)-\epsilon > \delta_\beta$, as we have previously shown, implies that $\phi(\epsilon) > \epsilon+\delta_\beta > \epsilon+\Delta\epsilon$. Note that we do not know the sign of $w_2^{-1}(\epsilon)-(\epsilon+\Delta\epsilon)$ but our approach moving forward will not depend on that sign.

$$\begin{aligned}
0 \geq & -\beta \int_{\epsilon}^{\epsilon+\Delta\epsilon} (v-\epsilon)f_A(v)dv - \beta \int_{\epsilon+\Delta\epsilon}^{\phi(\epsilon)} \Delta\epsilon f_A(v)dv + \beta \int_{\phi(\epsilon)}^{\phi(\epsilon+\Delta\epsilon)} (v-(\epsilon+\Delta\epsilon))f_A(v)dv \\
& - \int_{\epsilon}^{\epsilon+\Delta\epsilon} (v-\epsilon)f_B(v)dv - \int_{\epsilon+\Delta\epsilon}^{w_2^{-1}(\epsilon)} \Delta\epsilon f_B(v)dv + \int_{w_2^{-1}(\epsilon)}^{w_2^{-1}(\epsilon+\Delta\epsilon)} (v-(\epsilon+\Delta\epsilon))f_B(v)dv \\
\Leftrightarrow \beta & \int_{\phi(\epsilon)}^{\phi(\epsilon+\Delta\epsilon)} (v-(\epsilon+\Delta\epsilon))f_A(v)dv \leq \\
& \beta \int_{\epsilon}^{\epsilon+\Delta\epsilon} (v-\epsilon)f_A(v)dv + \beta \int_{\epsilon+\Delta\epsilon}^{\phi(\epsilon)} \Delta\epsilon f_A(v)dv + \int_{\epsilon}^{\epsilon+\Delta\epsilon} (v-\epsilon)f_B(v)dv + \int_{\epsilon+\Delta\epsilon}^{w_2^{-1}(\epsilon)} \Delta\epsilon f_B(v)dv \\
& - \int_{w_2^{-1}(\epsilon)}^{w_2^{-1}(\epsilon+\Delta\epsilon)} (v-(\epsilon+\Delta\epsilon))f_B(v)dv. \quad (33)
\end{aligned}$$

In the following, we bound each term in (33) one by one:

1. The left-hand side is bounded from below as follows: Recall our assumption that $\Delta\epsilon < \delta_\beta/2$. Because $\phi(\epsilon)-\epsilon > \delta_\beta$ and $v \geq \phi(\epsilon)$ for v in the range of the integration, one can bound the integrand as follows: $v-(\epsilon+\Delta\epsilon) \geq \phi(\epsilon)-\epsilon-\Delta\epsilon > \delta_\beta - \delta_\beta/2 = \delta_\beta/2$. Therefore, we bound the left-hand side by

$$\beta \int_{\phi(\epsilon)}^{\phi(\epsilon+\Delta\epsilon)} (v-(\epsilon+\Delta\epsilon))f_A(v)dv \geq \beta \int_{\phi(\epsilon)}^{\phi(\epsilon+\Delta\epsilon)} \frac{\delta_\beta}{2} f_A(v)dv \geq \frac{\beta\delta_\beta \underline{f}_A}{2} \cdot [\phi(\epsilon+\Delta\epsilon)-\phi(\epsilon)].$$

2. We bound the first term of the right-hand side from above as follows:

$$\beta \int_{\epsilon}^{\epsilon+\Delta\epsilon} (v-\epsilon)f_A(v)dv \leq \beta \int_{\epsilon}^{\epsilon+\Delta\epsilon} 1 \cdot f_A(v)dv \leq \beta\Delta\epsilon \overline{f}_A. \quad (34)$$

3. We bound the second term of the right-hand side as follows:

$$\beta \int_{\epsilon+\Delta\epsilon}^{\phi(\epsilon)} \Delta\epsilon f_A(v)dv \leq \beta \int_0^1 \Delta\epsilon f_A(v)dv = \beta\Delta\epsilon \tag{35}$$

4. We bound the third term of the right-hand side from above as follows:

$$\int_{\epsilon}^{\epsilon+\Delta\epsilon} (v-\epsilon)f_B(v)dv \leq \int_{\epsilon}^{\epsilon+\Delta\epsilon} \Delta\epsilon f_B(v)dv \leq \int_0^1 \Delta\epsilon f_B(v)dv = \Delta\epsilon. \tag{36}$$

5. We bound the fourth term of the right-hand side from above as follows:

$$\int_{\epsilon+\Delta\epsilon}^{w_2^{-1}(\epsilon)} \Delta\epsilon f_B(v)dv \leq \int_0^1 \Delta\epsilon f_B(v)dv = \Delta\epsilon. \tag{37}$$

6. We bound the fifth term of the right-hand side from above as follows:

$$\begin{aligned} -\int_{w_2^{-1}(\epsilon)}^{w_2^{-1}(\epsilon+\Delta\epsilon)} (v-(\epsilon+\Delta\epsilon))f_B(v)dv &\leq -\int_{w_2^{-1}(\epsilon)}^{w_2^{-1}(\epsilon+\Delta\epsilon)} (\epsilon-(\epsilon+\Delta\epsilon))f_B(v)dv \\ &= \int_{w_2^{-1}(\epsilon)}^{w_2^{-1}(\epsilon+\Delta\epsilon)} \Delta\epsilon f_B(v)dv \\ &\leq \int_0^1 \Delta\epsilon f_B(v)dv \\ &\leq \Delta\epsilon, \end{aligned} \tag{38}$$

where the first inequality follows because $v \geq w_2^{-1}(\epsilon)$ for every v in the range of the integration and $w_2^{-1}(\epsilon) \geq \epsilon$ for every ϵ, so we have $v \geq \epsilon$.

Now, by (33)–(38), we obtain

$$\begin{aligned} &\frac{\beta\delta_\beta \underline{f}_A}{2} \cdot [\phi(\epsilon+\Delta\epsilon)-\phi(\epsilon)] \leq \beta\Delta\epsilon\bar{f}_A + \beta\Delta\epsilon + \Delta\epsilon + \Delta\epsilon + \Delta\epsilon \\ \Leftrightarrow &\phi(\epsilon+\Delta\epsilon)-\phi(\epsilon) \leq \frac{2}{\beta\delta_\beta \underline{f}_A}\left[\beta\bar{f}_A+\beta+3\right]\cdot\Delta\epsilon. \end{aligned}$$

Recall that the above bound has been obtained under the assumption that $\Delta\epsilon < \frac{\delta_\beta}{2}$. Now suppose that $\Delta\epsilon \geq \frac{\delta_\beta}{2}$. For any such $\Delta\epsilon$, we have $\phi(\epsilon+\Delta\epsilon)-\phi(\epsilon) \leq 1 = \frac{\Delta\epsilon}{\Delta\epsilon} \leq \frac{2}{\delta_\beta}\Delta\epsilon$. As shown in the Proof of Lemma 7, $\phi(\cdot)$ is weakly increasing, so $\phi(\epsilon+\Delta\epsilon)-\phi(\epsilon) \geq 0$.

Combining the two cases, the function $\phi(\cdot)$ is Lipschitz continuous in ϵ with Lipschitz constant $\max\left\{\frac{2}{\beta\delta_\beta \underline{f}_A}\left[\beta\bar{f}_A+\beta+3\right], \frac{2}{\delta_\beta}\right\} = \frac{2}{\beta\delta_\beta}\left[\beta\bar{f}_A+\beta+3\right]$. By the Stepanov Theorem, this implies that $\frac{\partial\phi(\cdot)}{\partial\epsilon}$ exists for almost every $\epsilon \in [\underline{\epsilon}, 1]$. □

Proof of Part 2. Let us temporarily write $\phi(\epsilon;\beta)$ to be explicit about the dependence of ϕ on β. Fix $\underline{\epsilon}>0$. It suffices to show that for all $\epsilon\in[\underline{\epsilon},1]$, $\frac{\partial\phi(\epsilon;\beta)}{\partial\beta}$ exists. Recall $\beta\geq 1$ and let $\Delta\beta>0$. We compare $\phi(\epsilon;\beta)$ to $\phi(\epsilon;\beta+\Delta\beta)$. By inspection of (3), note that $\phi(\epsilon;\beta)$ is non-increasing in β. Therefore, if $\phi(\epsilon;\beta+\Delta\beta)=1$, then $\phi(\epsilon;\beta)-\phi(\epsilon;\beta+\Delta\beta)=1-1=0$, so there is nothing to show. So suppose $\phi(\epsilon;\beta+\Delta\beta)<1$. Then the No Desegregation Condition conditions for β and $\beta+\Delta\beta$ are, respectively,

$$\int_0^1 [v-w_2(v)]f_B(v)dv\geq\beta\int_\epsilon^{\phi(\epsilon;\beta)}(v-\epsilon)f_A(v)dv+\int_\epsilon^{w_2^{-1}(\epsilon)}(v-\epsilon)f_B(v)dv. \tag{39}$$

and

$$\int_0^1 [v-w_2(v)]f_B(v)dv=(\beta+\Delta\beta)\int_\epsilon^{\phi(\epsilon;\beta+\Delta\beta)}(v-\epsilon)f_A(v)dv+\int_\epsilon^{w_2^{-1}(\epsilon)}(v-\epsilon)f_B(v)dv. \tag{40}$$

Subtracting (40) from (39) we obtain

$$0\geq\beta\int_\epsilon^{\phi(\epsilon;\beta)}(v-\epsilon)f_A(v)dv-(\beta+\Delta\beta)\int_\epsilon^{\phi(\epsilon;\beta+\Delta\beta)}(v-\epsilon)f_A(v)dv$$
$$\Leftrightarrow\beta\int_{\phi(\epsilon;\beta+\Delta\beta)}^{\phi(\epsilon;\beta)}(v-\epsilon)f_A(v)dv\leq\Delta\beta\int_\epsilon^{\phi(\epsilon;\beta+\Delta\beta)}(v-\epsilon)f_A(v)dv. \tag{41}$$

For each $\beta\geq 1$, define δ_β as in (30). As shown in the proof of Claim 11, $\phi(\epsilon;\beta+\Delta\beta)-\epsilon>\delta_{\beta+\Delta\beta}$ for all $\epsilon\geq\underline{\epsilon}$. Therefore, the left-hand side of (41) can be bounded from below by

$$\beta\int_{\phi(\epsilon;\beta+\Delta\beta)}^{\phi(\epsilon;\beta)}(v-\epsilon)f_A(v)dv\geq\beta\int_{\phi(\epsilon;\beta+\Delta\beta)}^{\phi(\epsilon;\beta)}\delta_{\beta+\Delta\beta}f_A(v)dv\geq\beta\delta_{\beta+\Delta\beta}\underline{f}_A[\phi(\epsilon;\beta)-\phi(\epsilon;\beta+\Delta\beta)]. \tag{42}$$

Meanwhile, the right-hand side of (41) can be bounded from above by

$$\Delta\beta\int_\epsilon^{\phi(\epsilon;\beta+\Delta\beta)}(v-\epsilon)f_A(v)dv\leq\Delta\beta\int_0^1 1\cdot f_A(v)dv=\Delta\beta. \tag{43}$$

Therefore, from (41)-(43) we conclude

$$\frac{\phi(\epsilon;\beta)-\phi(\epsilon;\beta+\Delta\beta)}{\Delta\beta}\leq\frac{1}{\beta\delta_{\beta+\Delta\beta}\underline{f}_A}=\frac{1}{\beta\frac{\underline{\epsilon}^2\underline{f}_B}{4(\beta+\Delta\beta)}\underline{f}_A}. \tag{44}$$

By (44) and the fact that $\phi(\epsilon;\beta)-\phi(\epsilon;\beta+\Delta\beta)\geq 0$ for all $\Delta\beta>0$, it follows that

$$\limsup_{\Delta\beta\to 0}\frac{\phi(\epsilon;\beta)-\phi(\epsilon;\beta+\Delta\beta)}{\Delta\beta}\in\left[0,\frac{1}{\frac{\underline{\epsilon}^2\underline{f}_B}{4\beta}\underline{f}_A}\right].$$

By the Stepanov Theorem, this implies that $\frac{\partial\phi(\epsilon;\beta)}{\partial\beta}$ exists for almost every $\beta\in[1,\infty)$. □

Proof of Part 3.

We begin this proof with a technical result.

Claim **12.** $\frac{\partial \hat{w}_1(\hat{w}_1^{-1}(\epsilon))}{\partial v}$ *is continuous in* β *for almost all* $\epsilon \in (0,E)$ *and almost all* $\beta \geq 1$.

Proof. We first note that $w_2^{-1}(\cdot)$ is absolutely continuous (in ϵ) by assumption, and hence its partial derivative with respect to ϵ exists for almost all $\epsilon \in (0,1)$. Solving (3) with equality, it follows that $\beta \int_\epsilon^{\hat{w}_1^{-1}(\epsilon)} (v-\epsilon) f_A(v)dv + \int_\epsilon^{w_2^{-1}(\epsilon)} (v-\epsilon) f_B(v)dv$ is constant in $\epsilon < E$, since $\hat{w}_1^{-1}(\epsilon) = \phi(\epsilon)$ by Lemma 7 for all $\epsilon < E$. This implies that for $\epsilon < E$

$$\beta\left[(\hat{w}_1^{-1}(\epsilon)-\epsilon) f_A(\hat{w}_1^{-1}(\epsilon)) \frac{\partial \hat{w}_1^{-1}(\epsilon)}{\partial \epsilon} - \int_\epsilon^{\hat{w}_1^{-1}(\epsilon)} f_A(v)dv\right] + (w_2^{-1}(\epsilon)-\epsilon) f_B(w_2^{-1}(\epsilon)) \frac{dw_2^{-1}(\epsilon)}{d\epsilon} - \int_\epsilon^{w_2^{-1}(\epsilon)} f_B(v)dv = 0.$$

Simplifying yields

$$\frac{\partial \hat{w}_1^{-1}(\epsilon)}{\partial \epsilon} = \frac{F_B(w_2^{-1}(\epsilon)) - F_B(\epsilon) - (w_2^{-1}(\epsilon)-\epsilon) f_B(w_2^{-1}(\epsilon)) \frac{dw_2^{-1}(\epsilon)}{d\epsilon} + \beta[F_A(\hat{w}_1^{-1}(\epsilon)) - F_A(\epsilon)]}{\beta[\hat{w}_1^{-1}(\epsilon)-\epsilon] f_A(\hat{w}_1^{-1}(\epsilon))},$$

where we note that $\hat{w}_1^{-1}(\epsilon) - \epsilon > 0$ for all $\epsilon \in (0,E)$.[34] Using this equality and the inverse function theorem tells us that

$$\frac{\partial \hat{w}_1(\hat{w}_1^{-1}(\epsilon))}{\partial v} = \frac{\beta[\hat{w}_1^{-1}(\epsilon)-\epsilon] f_A(\hat{w}_1^{-1}(\epsilon))}{F_B(w_2^{-1}(\epsilon)) - F_B(\epsilon) - (w_2^{-1}(\epsilon)-\epsilon) f_B(w_2^{-1}(\epsilon)) \frac{dw_2^{-1}(\epsilon)}{d\epsilon} + \beta[F_A(\hat{w}_1^{-1}(\epsilon)) - F_A(\epsilon)]} \tag{45}$$

(45) is continuous in β because: 1) only the terms multiplied by β are functions of β, 2) the numerator is continuous in β because $\hat{w}_1^{-1}(\epsilon)$ is continuous in β (which we can see by inspection of (3)) and by the fact that $f_A(\cdot)$ is continuous, and 3) the denominator is continuous also because $\hat{w}_1^{-1}(\epsilon)$ is continuous in β and because F_A has no mass points. □

We now return to the *Proof of Part 3* of Lemma 8. Let us temporarily write $\hat{w}_1(\epsilon;\beta)$ and $\hat{w}_1^{-1}(\epsilon;\beta)$ to be explicit about the dependence of these functions on β.

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[34]The proof sketch is as follows. By Lemma 7 and (31), we know that $\int_0^1 [v - w_2(v)] f_B(v)dv = \beta \int_\epsilon^{\hat{w}_1^{-1}(\epsilon)} (v-\epsilon) f_A(v)dv + \int_\epsilon^{w_2^{-1}(\epsilon)} (v-\epsilon) f_B(v)dv > 0$ for all $\epsilon < E$, where the inequality follows because we have excluded the case in which $w_2(v) = v$ for almost all $v \in [0,1]$. For contradiction, assume that there exists $\epsilon' \in (0,E)$ such that $\hat{w}_1^{-1}(\epsilon') = \epsilon'$. Then it must follow that $\int_{\epsilon'}^{w_2^{-1}(\epsilon')} (v-\epsilon') f_B(v)dv = \int_0^1 [v - w_2(v)] f_B(v)dv > 0$. This equality is possible only if $w_2(v) = v$ for all $v < \epsilon'$. Therefore, for this function $w_2(\cdot)$, $\int_{\epsilon''}^{w_2^{-1}(\epsilon'')} (v-\epsilon'') f_B(v)dv = 0$ for every $\epsilon'' < \epsilon'$, a contradiction to the assumption that the term $\int_\epsilon^{w_2^{-1}(\epsilon)} (v-\epsilon) f_B(v)dv$ is non-increasing in ϵ.

Consider a sequence of real numbers, with generic element $\Delta\beta$, that converges to 0. Since $\hat{w}_1^{-1}(\epsilon;\beta)$ is differentiable with respect to β,

$$\hat{w}_1^{-1}(\epsilon;\beta+\Delta\beta)=\hat{w}_1^{-1}(\epsilon;\beta)+\frac{\partial\hat{w}_1^{-1}(\epsilon;\beta)}{\partial\beta}\cdot\Delta\beta+o(\Delta\beta).$$

Letting $\psi(\Delta\beta):=-\frac{\partial\hat{w}_1^{-1}(\epsilon;\beta)}{\partial\beta}\cdot\Delta\beta-o(\Delta\beta)$, we get $\hat{w}_1^{-1}(\epsilon;\beta)=\hat{w}_1^{-1}(\epsilon;\beta+\Delta\beta)+\psi(\Delta\beta)$. Hence,

$$\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta);\beta+\Delta\beta)=\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta+\Delta\beta)+\psi(\Delta\beta);\beta+\Delta\beta). \tag{46}$$

Since $\hat{w}_1(\epsilon;\beta)$ is differentiable with respect to ϵ,

$$\begin{aligned}\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta+\Delta\beta)+\psi(\Delta\beta);\beta+\Delta\beta)&=\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta+\Delta\beta);\beta+\Delta\beta)\\&+\frac{\partial\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta+\Delta\beta);\beta+\Delta\beta)}{\partial v}\cdot\psi(\Delta\beta)+o(\psi(\Delta\beta)).\end{aligned} \tag{47}$$

Because $\hat{w}_1$ and $\hat{w}_1^{-1}$ are inverse functions over the relevant values of ϵ and β it is the case that $\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta+\Delta\beta);\beta+\Delta\beta)=\epsilon=\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta);\beta)$. Therefore, together with (46) and (47),

$$\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta);\beta+\Delta\beta)=\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta);\beta)+\frac{\partial\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta+\Delta\beta);\beta+\Delta\beta)}{\partial v}\cdot\psi(\Delta\beta)+o(\psi(\Delta\beta)).$$

By substituting the definition of $\psi(\Delta\beta)$,[35]

$$\begin{aligned}&\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta);\beta+\Delta\beta)\\&=\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta);\beta)+\frac{\partial\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta+\Delta\beta);\beta+\Delta\beta)}{\partial v}\cdot\Big(-\frac{\partial\hat{w}_1^{-1}(\epsilon;\beta)}{\partial\beta}\cdot\Delta\beta-o(\Delta\beta)\Big)+o(\psi(\Delta\beta))\\&=\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta);\beta)-\frac{\partial\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta+\Delta\beta);\beta+\Delta\beta)}{\partial v}\cdot\frac{\partial\hat{w}_1^{-1}(\epsilon;\beta)}{\partial\beta}\cdot\Delta\beta-\frac{\partial\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta+\Delta\beta);\beta+\Delta\beta)}{\partial v}\cdot o(\Delta\beta)\\&+o(\psi(\Delta\beta))\\&=\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta);\beta)-\frac{\partial\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta+\Delta\beta);\beta+\Delta\beta)}{\partial v}\cdot\frac{\partial\hat{w}_1^{-1}(\epsilon;\beta)}{\partial\beta}\cdot\Delta\beta-\frac{\partial\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta+\Delta\beta);\beta+\Delta\beta)}{\partial v}\cdot o(\Delta\beta)\\&+\frac{o(\psi(\Delta\beta))}{\psi(\Delta\beta)}\cdot\psi(\Delta\beta).\end{aligned}$$

It follows that

$$\begin{aligned}&\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta);\beta+\Delta\beta)-\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta);\beta)\\&=\underbrace{-\frac{\partial\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta+\Delta\beta);\beta+\Delta\beta)}{\partial v}\cdot\frac{\partial\hat{w}_1^{-1}(\epsilon;\beta)}{\partial\beta}\cdot\Delta\beta}_{(A)}\underbrace{-\frac{\partial\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta+\Delta\beta);\beta+\Delta\beta)}{\partial v}\cdot o(\Delta\beta)}_{(B)}\underbrace{+\frac{o(\psi(\Delta\beta))}{\psi(\Delta\beta)}\cdot\psi(\Delta\beta)}_{(C)}.\end{aligned}$$

[35] In the last line of the subsequent display equation, we have $\psi(\Delta\beta)$ in the denominator. Here we choose a subsequence of $\psi(\Delta\beta)$ so that $\psi(\Delta\beta)\neq 0$ for all $\Delta\beta$. More precisely, denoting the originally chosen sequence of $\Delta\beta$s as $\{\Delta\beta_k\}_{k=1}^{\infty}$, there exists a sufficiently large index k' such that $\psi(\Delta\beta_k)\neq 0$ for all $k\geq k'$. To see this, suppose that, for any k', there exists $k''\geq k'$ such that $\psi(\Delta\beta_{k''})=0$. Then, there exists $\{\Delta\beta_\ell\}_{\ell=1}^{\infty}\subseteq\{\Delta\beta_k\}_{k=1}^{\infty}$ such that $\Delta\beta_\ell\to 0$ and $\psi(\Delta\beta_\ell)=0$ for all ℓ. By the definition of $\psi(\cdot)$, we get $\frac{o(\Delta\beta_\ell)}{\Delta\beta_\ell}=-\frac{\partial\hat{w}_1^{-1}(\epsilon;\beta)}{\partial\beta}$. We obtain a contradiction to $\frac{o(\Delta\beta_\ell)}{\Delta\beta_\ell}\to 0$ as $\ell\to\infty$.

Now, divide both sides by $\Delta\beta$ and let $\Delta\beta \to 0$. We consider each term.

Term (A): Since $\frac{\partial \hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta);\beta)}{\partial v}$ is continuous with respect to β at almost all $(\hat{w}_1^{-1}(\epsilon;\beta);\beta)$ by Claim 12,

$$\lim_{\Delta\beta \to 0} \frac{\partial \hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta+\Delta\beta);\beta+\Delta\beta)}{\partial v} = \frac{\partial \hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta);\beta)}{\partial v}.$$

Therefore,

$$\lim_{\Delta\beta \to 0} \frac{(A)}{\Delta\beta} = -\frac{\partial \hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta);\beta)}{\partial v} \cdot \frac{\partial \hat{w}_1^{-1}(\epsilon;\beta)}{\partial \beta}.$$

Term (B): As proven above, $\lim_{\Delta\beta \to 0} \frac{\partial \hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta+\Delta\beta);\beta+\Delta\beta)}{\partial v} = \frac{\partial \hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta);\beta)}{\partial v}$. Together with $\lim_{\Delta\beta \to 0} \frac{o(\Delta\beta)}{\Delta\beta} = 0$, it implies

$$\lim_{\Delta\beta \to 0} \frac{(B)}{\Delta\beta} = 0.$$

Term (C): Since $\psi(\Delta\beta) \to 0$ when $\Delta\beta \to 0$, we get

$$\lim_{\Delta\beta \to 0} \frac{o(\psi(\Delta\beta))}{\psi(\Delta\beta)} = 0. \tag{48}$$

Furthermore,

$$\begin{aligned}
\lim_{\Delta\beta \to 0} \frac{\psi(\Delta\beta)}{\Delta\beta} &= \lim_{\Delta\beta \to 0} \frac{-\frac{\partial \hat{w}_1^{-1}(\epsilon;\beta)}{\partial \beta} \cdot \Delta\beta - o(\Delta\beta)}{\Delta\beta} \\
&= -\frac{\partial \hat{w}_1^{-1}(\epsilon;\beta)}{\partial \beta} - \lim_{\Delta\beta \to 0} \frac{o(\Delta\beta)}{\Delta\beta} \\
&= -\frac{\partial \hat{w}_1^{-1}(\epsilon;\beta)}{\partial \beta}.
\end{aligned}$$

Together with (48), it implies

$$\lim_{\Delta\beta \to 0} \frac{(C)}{\Delta\beta} = 0.$$

Combining the conclusions regarding the three terms, we obtain that for almost all ϵ and almost all β

$$\lim_{\Delta\beta \to 0} \frac{\hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta);\beta+\Delta\beta) - \hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta);\beta)}{\Delta\beta} = -\frac{\partial \hat{w}_1(\hat{w}_1^{-1}(\epsilon;\beta);\beta)}{\partial v} \cdot \frac{\partial \hat{w}_1^{-1}(\epsilon;\beta)}{\partial \beta}.$$

This concludes the *Proof of Part 3*.

□

Proof of Part 4.

Next, we will show that $\hat{w}_1$ is Lipschitz continuous in $v\in[0,1)$. To do so, first note that $\hat{w}_1(v)$ is constant and equal to 0 for $v\in[0,\phi(0)]$. So, consider $v\in[\phi(0),1)$. In that case, $\hat{w}_1(\cdot)$ is the inverse function of $\phi(\cdot)$ defined over $[0,E]$. We will show that there exists a constant $c>0$ such that $\phi(\epsilon_1)-\phi(\epsilon_2)\geq c(\epsilon_1-\epsilon_2)$ for every $\epsilon_1,\epsilon_2\in[0,E]$ with $\epsilon_1>\epsilon_2$.

From the argument in the proof of Claim 11, it follows that for any $\beta<\bar{\beta}$ and any $\underline{\epsilon}>0$, $\phi(\epsilon)-\epsilon>\delta_\beta>\delta_{\bar{\beta}}>0$ for all $\epsilon\in[\underline{\epsilon},1]$. By a similar argument and the assumption that w_2 is continuous and is not the identity function, for any $\beta<\bar{\beta}$, there exists $c''>0$ such that $\phi(0)-0>c''$. Letting $\underline{\epsilon}=c''/2$, the non-decreasing property of $\phi(\cdot)$ implies that $\phi(\epsilon)-\epsilon>c''/2$ for all $\epsilon\in[0,\underline{\epsilon})$. Therefore, letting $c'=\min\{c''/2,\delta_{\bar{\beta}}\}$, $\phi(\epsilon)-\epsilon>c'$ for all $\epsilon\in[0,1]$.

Next, we will complete the proof of the claim that there exists a constant $c>0$ such that $\phi(\epsilon_1)-\phi(\epsilon_2)\geq c(\epsilon_1-\epsilon_2)$ for every $\epsilon_1,\epsilon_2\in[0,E]$ with $\epsilon_1>\epsilon_2$. To do so, we first note that it suffices to establish the desired conclusion for ϵ_1,ϵ_2 such that $\epsilon_1\leq\phi(\epsilon_2)$.[36] Denote $\Delta\epsilon=\epsilon_1-\epsilon_2$ and subtract (31) from (32) holding with equality to obtain (recalling the assumption that $\int_\epsilon^{w_2^{-1}(\epsilon)}(v-\epsilon)f_B(v)dv$ is non-increasing in ϵ),

$$\int_{\epsilon_2+\Delta\epsilon}^{\phi(\epsilon_2+\Delta\epsilon)}[v-(\epsilon_2+\Delta\epsilon)]f_A(v)dv-\int_{\epsilon_2}^{\phi(\epsilon_2)}[v-\epsilon_2]f_A(v)dv\geq 0$$

$$\iff\int_{\epsilon_2}^{\epsilon_2+\Delta\epsilon}[v-\epsilon_2]f_A(v)dv+\int_{\epsilon_2+\Delta\epsilon}^{\phi(\epsilon_2)}\Delta\epsilon f_A(v)dv\leq\int_{\phi(\epsilon_2)}^{\phi(\epsilon_2+\Delta\epsilon)}[v-(\epsilon_2+\Delta\epsilon)]f_A(v)dv. \tag{49}$$

To bound the left-hand side of (49), recall that $c'<\phi(\epsilon)-\epsilon$ for all $\epsilon\in[0,E]$. Suppose that $\Delta\epsilon\geq\frac{c'}{2}$. Then, the left-hand side is lower bounded by $\int_{\epsilon_2}^{\epsilon_2+\Delta\epsilon}[v-\epsilon_2]f_A(v)dv$ because the other term is nonnegative. The term $\int_{\epsilon_2}^{\epsilon_2+\Delta\epsilon}[v-\epsilon_2]f_A(v)dv$ can be lower bounded as follows:

$$\begin{aligned}\int_{\epsilon_2}^{\epsilon_2+\Delta\epsilon}[v-\epsilon_2]f_A(v)dv&\geq\int_{\epsilon_2}^{\epsilon_2+\Delta\epsilon}[v-\epsilon_2]\underline{f}_A dv\\&=\frac{(\Delta\epsilon)^2\underline{f}_A}{2}\\&\geq\frac{c'\Delta\epsilon\underline{f}_A}{4}.\end{aligned} \tag{50}$$

Suppose that $\Delta\epsilon<\frac{c'}{2}$. Then, the left-hand side of (49) is lower bounded by $\int_{\epsilon_2+\Delta\epsilon}^{\phi(\epsilon_2)}\Delta\epsilon f_A(v)dv$ because the other term is nonnegative. The term $\int_{\epsilon_2+\Delta\epsilon}^{\phi(\epsilon_2)}\Delta\epsilon f_A(v)dv$ is lower bounded as follows:

$$\begin{aligned}\int_{\epsilon_2+\Delta\epsilon}^{\phi(\epsilon_2)}\Delta\epsilon f_A(v)dv&\geq\int_{\epsilon_2+\Delta\epsilon}^{\phi(\epsilon_2)}\Delta\epsilon\underline{f}_A dv\\&=[\phi(\epsilon_2)-\epsilon_2-\Delta\epsilon]\Delta\epsilon\underline{f}_A\\&\geq\frac{c'\Delta\epsilon\underline{f}_A}{2}.\end{aligned} \tag{51}$$

[36] To see this, consider any ϵ_1, ϵ_2 such that $\epsilon_1>\epsilon_2$. Then, there exists $n\in\mathbb{Z}_+$ such that $\epsilon_1\in(\epsilon_2+n(\phi(\epsilon_2)-\epsilon_2),\epsilon_2+(n+1)(\phi(\epsilon_2)-\epsilon_2)]$. Then, applying the conclusion for ϵ_2 and $\epsilon_2+(\phi(\epsilon_2)-\epsilon_2)$, $\epsilon_2+(\phi(\epsilon_2)-\epsilon_2)$ and $\epsilon_2+2(\phi(\epsilon_2)-\epsilon_2)$,..., and $\epsilon_2+n(\phi(\epsilon_2)-\epsilon_2)$ and ϵ_1, we obtain the desired conclusion for ϵ_2 and ϵ_1.

From (50) and (51), the left-hand side of (49) is lower bounded by $\frac{c'\Delta\epsilon \underline{f}_A}{4}$.

We provide an upper bound of (49) as follows:

$$\begin{aligned}\int_{\phi(\epsilon_2)}^{\phi(\epsilon_2+\Delta\epsilon)}[v-(\epsilon_2+\Delta\epsilon)]f_A(v)dv &\le \int_{\phi(\epsilon_2)}^{\phi(\epsilon_2+\Delta\epsilon)}[v-(\epsilon_2+\Delta\epsilon)]\bar{f}_A dv\\ &\le \int_{\phi(\epsilon_2)}^{\phi(\epsilon_2+\Delta\epsilon)}\bar{f}_A dv\\ &\le [\phi(\epsilon_2+\Delta\epsilon)-\phi(\epsilon_2)]\bar{f}_A.\end{aligned}$$

Therefore, combining the lower and upper bounds, we obtain

$$\phi(\epsilon_1)-\phi(\epsilon_2)\ge \frac{c'\underline{f}_A}{4\bar{f}_A}\cdot(\epsilon_1-\epsilon_2),$$

which shows the desired claim with $c:=\frac{c'\underline{f}_A}{4\bar{f}_A}$.

Recalling that $\hat{w}_1$ is the inverse function of ϕ for $\phi([0,E])$, the preceding claim implies that $\hat{w}_1$ is Lipschitz continuous with constant $1/c$.[37]

□

The previous Lemmas establish technical conditions. Building upon those results, we now present the following Lemma which completes the proof of Remark 6.

Lemma **9.** *Let $w_2(\cdot)$ be a wage function such that $w_2^{-1}(\cdot)$ is absolutely continuous and $\int_{\epsilon}^{w_2^{-1}(\epsilon)}(v-\epsilon)f_B(v)dv$ is non-increasing in ϵ. Also, let $f_A(\cdot)$ be continuous. Finally, suppose that* (19) *holds for $\beta>0$. Then the inequality holds for all $\beta'>\beta$.*

Proof. We temporarily use notation $\hat{\pi}_1^{\beta}:=\beta\int_0^1(v-\hat{w}_1(v;\beta))f_A(v)dv$ to make the dependence of $\hat{\pi}_1$ on β explicit. We will show that for any $\beta_1>\beta_2$, $\hat{\pi}_1^{\beta_1}-\hat{\pi}_1^{\beta_2}\ge 0$.

Claim **13.** *Let $w_2(\cdot)$ be a wage function such that $w_2(v)\le v$ for all $v\in[0,1)$, $w_2^{-1}(\cdot)$ is absolutely continuous, and $\int_{\epsilon}^{w_2^{-1}(\epsilon)}(v-\epsilon)f_B(v)dv$ is non-increasing in ϵ. Then, $\hat{\pi}_1^{\beta}$ is absolutely continuous in β.*

Proof. Let $\beta_1>\beta_2$ and define

$$\tilde{w}_1^{\beta_1}(v):=\begin{cases}\hat{w}_1^{\beta_2}(v+\gamma) & v<1-\gamma,\\ 1 & \text{otherwise.}\end{cases}$$

where $\gamma:=\frac{\beta_1-\beta_2}{\beta_2}\cdot\frac{\bar{f}_A}{\underline{f}_A}$. We will first show that, $\tilde{w}_1^{\beta_1}$ satisfies (2) for β_1 and each $\epsilon\in[0,1]$. First note that it suffices to show (2) for $\epsilon<1-\gamma$ because the right-hand side is nonincreasing in $\epsilon\ge 1-\gamma$

[37] The proof is as follows. Consider any $v_1,v_2\in\phi([0,E])$ with $v_1>v_2$. Because ϕ is increasing and continuous, there exist unique $\epsilon_1,\epsilon_2\in[0,E]$ with $\epsilon_1>\epsilon_2$ such that $\phi(\epsilon_1)=v_1$ and $\phi(\epsilon_2)=v_2$. By the preceding claim, $v_1-v_2=\phi(\epsilon_1)-\phi(\epsilon_2)\ge c(\epsilon_1-\epsilon_2)=c(\hat{w}_1(v_1)-\hat{w}_1(v_2))$, which completes the argument.

as $\int_{\epsilon}^{w_2^{-1}(\epsilon)}(v-\epsilon)f_B(v)dv$ is non-increasing in ϵ while the left-hand side is constant in ϵ. Then, we note that to show the desired inequality for $\epsilon<1-\gamma$, it suffices to show that, for any $\epsilon\in[0,1]$,

$$\beta_2\int_{\epsilon}^{(\hat{w}_1^{\beta_2})^{-1}(\epsilon)}(v-\epsilon)f_A(v)dv-\beta_1\int_{\epsilon}^{(\tilde{w}_1^{\beta_1})^{-1}(\epsilon)}(v-\epsilon)f_A(v)dv=\beta_2\int_{\epsilon}^{\phi(\epsilon;\beta_2)}(v-\epsilon)f_A(v)dv-\beta_1\int_{\epsilon}^{\phi(\epsilon;\beta_2)-\gamma}(v-\epsilon)f_A(v)dv\geq 0.$$

where the equality follows from Lemma 7 and the construction of $\tilde{w}_1^{\beta_1}$. Note that this inequality holds for all ϵ such that $\phi(\epsilon;\beta_2)-\gamma-\epsilon\leq 0$ because in that case the second term is nonpositive. Therefore, without loss of generality we assume $\phi(\epsilon;\beta_2)-\gamma-\epsilon>0$. By rearranging terms, this inequality is equivalent to

$$\int_{\phi(\epsilon;\beta_2)-\gamma}^{\phi(\epsilon;\beta_2)}(v-\epsilon)f_A(v)dv\geq\frac{\beta_1-\beta_2}{\beta_2}\int_{\epsilon}^{\phi(\epsilon;\beta_2)-\gamma}(v-\epsilon)f_A(v)dv.$$

This inequality holds because

$$\begin{aligned}\int_{\phi(\epsilon;\beta_2)-\gamma}^{\phi(\epsilon;\beta_2)}(v-\epsilon)f_A(v)dv&\geq\int_{\phi(\epsilon;\beta_2)-\gamma}^{\phi(\epsilon;\beta_2)}(\phi(\epsilon;\beta_2)-\gamma-\epsilon)\underline{f}_A dv\\&=\gamma(\phi(\epsilon;\beta_2)-\gamma-\epsilon)\underline{f}_A\\&=\frac{\beta_1-\beta_2}{\beta_2}(\phi(\epsilon;\beta_2)-\gamma-\epsilon)\bar{f}_A\\&=\frac{\beta_1-\beta_2}{\beta_2}\int_{\epsilon}^{\phi(\epsilon;\beta_2)-\gamma}\bar{f}_A dv\\&\geq\frac{\beta_1-\beta_2}{\beta_2}\int_{\epsilon}^{\phi(\epsilon;\beta_2)-\gamma}(\phi(\epsilon;\beta_2)-\gamma-\epsilon)\bar{f}_A dv\\&\geq\frac{\beta_1-\beta_2}{\beta_2}\int_{\epsilon}^{\phi(\epsilon;\beta_2)-\gamma}(v-\epsilon)f_A(v)dv,\end{aligned}$$

where the second equality holds by construction of γ.

Now we proceed to provide bounds on $\hat{\pi}_1^{\beta_1}$. For an upper bound, since $\beta_1>\beta_2$, we obtain

$$\begin{aligned}\hat{\pi}_1^{\beta_1}:=&\beta_1\int_0^1(v-\hat{w}_1^{\beta_1}(v))f_A(v)dv\\\leq&\beta_1\int_0^1(v-\hat{w}_1^{\beta_2}(v))f_A(v)dv\\=&\hat{\pi}_1^{\beta_2}+(\beta_1-\beta_2)\int_0^1(v-\hat{w}_1^{\beta_2}(v))f_A(v)dv\\\leq&\hat{\pi}_1^{\beta_2}+(\beta_1-\beta_2),\end{aligned}\tag{52}$$

where the first inequality follows because $\hat{w}_1^{\beta_1}(v)\geq\hat{w}_1^{\beta_2}(v)$ for all v.[38]

Next, we will provide a lower bound of $\pi_1^{\beta_1}$. First, observe that

$$\hat{\pi}_1^{\beta_1}\geq\tilde{\pi}_1^{\beta_1}:=\beta_1\int_0^1(v-\tilde{w}_1^{\beta_1}(v))f_A(v)dv,\tag{53}$$

[38]The claim that $\hat{w}_1^{\beta_1}(v)\geq\hat{w}_1^{\beta_2}(v)$ for all v follows immediately from the fact that $\phi(\epsilon;\beta)$ is non-increasing in β.

where the inequality holds by Claim 5 because $\tilde{w}_1^{\beta_1}(v)$ satisfies (2). Now observe that

$$
\begin{aligned}
\tilde{\pi}_1^{\beta_1} &:= \beta_1 \int_0^1 (v - \tilde{w}_1^{\beta_1}(v)) f_A(v) dv \\
&= \hat{\pi}_1^{\beta_2} - \hat{\pi}_1^{\beta_2} + \beta_1 \int_0^1 (v - \tilde{w}_1^{\beta_1}(v)) f_A(v) dv \\
&= \hat{\pi}_1^{\beta_2} - \beta_2 \int_0^1 (v - \hat{w}_1^{\beta_2}(v)) f_A(v) dv + \beta_1 \int_0^1 (v - \tilde{w}_1^{\beta_1}(v)) f_A(v) dv \\
&= \hat{\pi}_1^{\beta_2} + (\beta_1 - \beta_2) \int_0^1 (v - \hat{w}_1^{\beta_2}(v)) f_A(v) dv + \beta_1 \int_0^1 (\hat{w}_1^{\beta_2}(v) - \tilde{w}_1^{\beta_1}(v)) f_A(v) dv \\
&\geq \hat{\pi}_1^{\beta_2} + \beta_1 \int_0^1 (\hat{w}_1^{\beta_2}(v) - \tilde{w}_1^{\beta_1}(v)) f_A(v) dv \\
&\geq \hat{\pi}_1^{\beta_2} + \beta_1 \int_0^1 (\hat{w}_1^{\beta_2}(v) - \tilde{w}_1^{\beta_1}(v)) \bar{f}_A dv \\
&\geq \hat{\pi}_1^{\beta_2} + \beta_1 \int_0^{\max\{1-\gamma,0\}} (\hat{w}_1^{\beta_2}(v) - \tilde{w}_1^{\beta_1}(v)) \bar{f}_A dv - \beta_1 \int_{1-\gamma}^1 \bar{f}_A dv \\
&\geq \hat{\pi}_1^{\beta_2} - \beta_1 \int_0^{\max\{1-\gamma,0\}} \frac{\gamma}{c} \bar{f}_A dv - \beta_1 \int_{1-\gamma}^1 \bar{f}_A dv \\
&\geq \hat{\pi}_1^{\beta_2} - \beta_1 \int_0^1 \frac{\gamma}{c} \bar{f}_A dv - \beta_1 \int_{1-\gamma}^1 \bar{f}_A dv \\
&= \hat{\pi}_1^{\beta_2} - \beta_1 \gamma \bar{f}_A \left[\frac{1}{c} + 1\right] \\
&= \hat{\pi}_1^{\beta_2} - \beta_1 \left(\frac{\beta_1 - \beta_2}{\beta_2} \frac{\bar{f}_A}{\underline{f}_A} \right) \bar{f}_A \left[\frac{1}{c} + 1\right] \\
&\geq \hat{\pi}_1^{\beta_2} - \bar{\beta} \left(\frac{\beta_1 - \beta_2}{1} \frac{\bar{f}_A}{\underline{f}_A} \right) \bar{f}_A \left[\frac{1}{c} + 1\right] \\
&= \hat{\pi}_1^{\beta_2} - (\beta_1 - \beta_2) \bar{\beta} \left(\frac{\bar{f}_A}{\underline{f}_A} \right) \bar{f}_A \left[\frac{1}{c} + 1\right] \qquad (54)
\end{aligned}
$$

where the first inequality follows because $v - \hat{w}_1^{\beta_2}(v) \geq 0$ by Claim 6, the second inequality follows because $\hat{w}_1^{\beta_2}(v) \leq \tilde{w}_1^{\beta_1}(v)$ for all v by construction of $\tilde{w}_1^{\beta_1}(v)$, the third inequality follows because $\hat{w}_1^{\beta_2}(v) \geq 0$ and $\tilde{w}_1^{\beta_1}(v) = 1$ for all $v \in [1-\gamma, 1]$, the fourth inequality follows from Lemma 8 Part 4 and the construction of γ, the second-to-last equality follows from substituting in for γ, and the final inequality follows from the assumption that $1 \leq \beta_2 < \beta_1 \leq \bar{\beta}$.

(53) and (54) imply that $\tilde{\pi}_1^{\beta_1} \geq \hat{\pi}_1^{\beta_2} - (\beta_1 - \beta_2) \bar{\beta} \left(\frac{\bar{f}_A}{\underline{f}_A} \right) \bar{f}_A \left[\frac{1}{c} + 1\right]$. Combined with (52) implies that $\hat{\pi}_1^{\beta}$ is Lipschitz continuous (in β) with constant $\max\{1, (\beta_1 - \beta_2) \bar{\beta} \left(\frac{\bar{f}_A}{\underline{f}_A} \right) \bar{f}_A \left[\frac{1}{c} + 1\right]\}$. Hence, $\hat{\pi}_1^{\beta}$ is absolutely continuous in β. □

Since $\hat{\pi}_1^{\beta}$ is absolutely continuous in β by Claim 13,

$$
\hat{\pi}_1^{\beta_1} - \hat{\pi}_1^{\beta_2} = \int_{\beta_2}^{\beta_1} \frac{\partial \hat{\pi}_1^{\beta}}{\partial \beta} d\beta.
$$

Therefore, for the desired conclusion, it suffices to show $\frac{\partial\hat{\pi}_1^\beta}{\partial\beta}\geq 0$ for almost all β because $\pi_2^{O_2}$ is constant in β. By Lemma 8, $\frac{\partial\hat{w}_1(v;\beta)}{\partial\beta}$ exists almost everywhere, and hence we can apply the Leibniz integral rule to show

$$\frac{\partial\hat{\pi}_1^\beta}{\partial\beta}=\int_0^1(v-\hat{w}_1(v;\beta))f_A(v)dv-\beta\int_0^1\frac{\partial\hat{w}_1(v;\beta)}{\partial\beta}f_A(v)dv=\int_0^1(v-\hat{w}_1(v;\beta))f_A(v)dv-\beta\int_{\hat{w}_1^{-1}(0;\beta)}^1\frac{\partial\hat{w}_1(v;\beta)}{\partial\beta}f_A(v)dv \tag{55}$$

where the second equality follows since $\hat{w}_1(\cdot)$ is constant on $[0,\hat{w}_1^{-1}(0;\beta))$.

In order to evaluate the partial derivative term in the previous equation, we turn to Lemma 8. Letting $v=\hat{w}_1^{-1}(\epsilon;\beta)$, we have that

$$\frac{\partial\hat{w}_1(v;\beta)}{\partial\beta}=-\frac{\partial\hat{w}_1(v;\beta)}{\partial v}\cdot\frac{\partial\hat{w}_1^{-1}(\epsilon;\beta)}{\partial\beta} \tag{56}$$

Next, we will find $\frac{\partial\hat{w}_1^{-1}(\epsilon;\beta)}{\partial\beta}$. For any $\epsilon<E$, (3) holds with equality at $\tilde{v}=\phi(\epsilon;\beta)=\hat{w}_1^{-1}(\epsilon;\beta)$, that is,

$$\int_0^1(v-w_2(v))f_B(v)dv=\beta\int_\epsilon^{\hat{w}_1^{-1}(\epsilon;\beta)}(v-\epsilon)f_A(v)dv+\int_\epsilon^{w_2^{-1}(\epsilon)}(v-\epsilon)f_B(v)dv.$$

Noticing that only the first term on the right-hand side depends on β (through both β and $\hat{w}_1^{-1}(\epsilon;\beta)$) we note that this term must be constant in β, i.e.

$$\int_\epsilon^{\hat{w}_1^{-1}(\epsilon;\beta)}(v-\epsilon)f_A(v)dv+\beta\left[(\hat{w}_1^{-1}(\epsilon;\beta)-\epsilon)f_A(\hat{w}_1^{-1}(\epsilon;\beta))\cdot\frac{\partial\hat{w}_1^{-1}(\epsilon;\beta)}{\partial\beta}\right]=0,$$

or, equivalently, for almost all $\epsilon\in[0,E]$ and almost all $\beta\geq 1$,

$$\frac{\partial\hat{w}_1^{-1}(\epsilon;\beta)}{\partial\beta}=\frac{-\int_\epsilon^{\hat{w}_1^{-1}(\epsilon;\beta)}(v-\epsilon)f_A(v)dv}{\beta(\hat{w}_1^{-1}(\epsilon;\beta)-\epsilon)f_A(\hat{w}_1^{-1}(\epsilon;\beta))}, \tag{57}$$

where it is straightforward to see that the denominator is strictly positive.

(45) provides an expression for the remaining partial derivative term in (56). We therefore substitute (45) and (57) into (56), and change variables using the identity $\hat{w}_1(v;\beta)=\epsilon$ to yield that for almost all $\epsilon\in[0,E)$,

$$\begin{aligned}\frac{\partial\hat{w}_1(v;\beta)}{\partial\beta}&=\frac{\int_{\hat{w}_1(v;\beta)}^v(x-\hat{w}_1(v;\beta))f_A(x)dx}{F_B(w_2^{-1}(\hat{w}_1(v;\beta)))-F_B(\hat{w}_1(v;\beta))-(w_2^{-1}(\hat{w}_1(v;\beta))-\hat{w}_1(v;\beta))f_B(w_2^{-1}(\hat{w}_1(v;\beta)))\frac{dw_2^{-1}(\hat{w}_1(v;\beta))}{d\epsilon}+\beta[F_A(v)-F_A(\hat{w}_1(v;\beta))]}\\&\leq\frac{\int_{\hat{w}_1(v;\beta)}^v(x-\hat{w}_1(v;\beta))f_A(x)dx}{\beta[F_A(v)-F_A(\hat{w}_1(v;\beta))]},\end{aligned} \tag{58}$$

where the inequality is due to the fact that
$F_B(w_2^{-1}(\hat{w}_1(v;\beta))) - F_B(\hat{w}_1(v;\beta)) - (w_2^{-1}(\hat{w}_1(v;\beta)) - \hat{w}_1(v;\beta)) f_B(w_2^{-1}(\hat{w}_1(v;\beta))) \frac{dw_2^{-1}(\hat{w}_1(v;\beta))}{d\epsilon} \geq 0$
by the ongoing assumption that $\int_{\epsilon}^{w_2^{-1}(\epsilon)} (v-\epsilon) f_B(v) dv$ is nonincreasing in ϵ.[39] Therefore, returning to (55),

$$\begin{aligned}
\frac{\partial \hat{\pi}_1^{\beta}}{\partial \beta} &= \int_0^1 (v - \hat{w}_1(v;\beta)) f_A(v) dv - \beta \int_{\hat{w}_1^{-1}(0;\beta)}^1 \frac{\partial \hat{w}_1(v;\beta)}{\partial \beta} f_A(v) dv \\
&\geq \int_{\hat{w}_1^{-1}(0;\beta)}^1 (v - \hat{w}_1(v;\beta)) f_A(v) dv - \beta \int_{\hat{w}_1^{-1}(0;\beta)}^1 \frac{\partial \hat{w}_1(v;\beta)}{\partial \beta} f_A(v) dv \\
&\geq \int_{\hat{w}_1^{-1}(0;\beta)}^1 (v - \hat{w}_1(v;\beta)) f_A(v) dv - \beta \int_{\hat{w}_1^{-1}(0;\beta)}^1 \frac{\int_{\hat{w}_1(v;\beta)}^{v} (x - \hat{w}_1(v;\beta)) f_A(x) dx}{\beta [F_A(v) - F_A(\hat{w}_1(v;\beta))]} f_A(v) dv \\
&= \int_{\hat{w}_1^{-1}(0;\beta)}^1 (v - \hat{w}_1(v;\beta)) f_A(v) dv - \int_{\hat{w}_1^{-1}(0;\beta)}^1 \frac{\int_{\hat{w}_1(v;\beta)}^{v} (x - \hat{w}_1(v;\beta)) f_A(x) dx}{[F_A(v) - F_A(\hat{w}_1(v;\beta))]} f_A(v) dv \\
&> 0
\end{aligned}$$

where the first inequality follows because $v - \hat{w}_1(v;\beta) \geq 0$ (see Claim 6), the second inequality follows from (58), and the third inequality follows from

$$\frac{\int_{\hat{w}_1(v;\beta)}^{v} (x - \hat{w}_1(v;\beta)) f_A(x) dx}{[F_A(v) - F_A(\hat{w}_1(v;\beta))]} = \mathbb{E}_x \left[x - \hat{w}_1(v;\beta) \middle| x \in (\hat{w}_1(v;\beta), v) \right] < v - \hat{w}_1(v;\beta),$$

for every $v \in [0,1]$.

Therefore, we obtain that for any $\beta_1 > \beta_2$, $\hat{\pi}_1^{\beta_1} - \hat{\pi}_1^{\beta_2} \geq 0$, as desired.

□

Combining the conclusions of Lemmas 2 and 9 completes the Proof of Remark 6. □

[39] To see this claim, recall that $w_2^{-1}(\cdot)$ is absolutely continuous in ϵ by assumption, and therefore its partial derivative with respect to ϵ exists for almost all $\epsilon \in [0,1]$. Therefore, we can differentiate $\int_{\epsilon}^{w_2^{-1}(\epsilon)} (v-\epsilon) f_B(v) dv$ with respect to ϵ, which yields (by Leibniz' integral rule) $-F_B(w_2^{-1}(\epsilon)) + F_B(\epsilon) + (w_2^{-1}(\epsilon) - \epsilon) f_B(w_2^{-1}(\epsilon)) \frac{dw_2^{-1}(\epsilon)}{d\epsilon}$. By the assumption that $\int_{\epsilon}^{w_2^{-1}(\epsilon)} (v-\epsilon) f_B(v) dv$ is nonincreasing in ϵ, it must be that this derivative term is weakly negative for (almost) all ϵ. Evaluating this derivative at $\hat{w}_1(v;\beta)$ yields the desired claim.

Proof of Proposition 6

Proof of Part 1. Fix w_1 and w_2 such that $w_1 \le w_2$. Individual rationality implies that firm $i \in \{1,2\}$ will hire almost no workers of productivity less than w_i. Moreover, each firm i is willing to hire all workers with productivities weakly greater than w_i, however, if $w_1 < w_2$, Condition 1 of the definition of block implies that almost all workers with productivities weakly above w_2 must be employed by firm 2. Combining these points, in any core outcome with $w_1 < w_2$, it is the case that

$$f^1(v) = \begin{cases} f(v) & \text{for almost all } v \in [w_1, w_2), \\ 0 & \text{for almost all } v \notin [w_1, w_2). \end{cases}$$

$$f^2(v) = \begin{cases} f(v) & \text{for almost all } v \in [w_2, 1], \\ 0 & \text{for almost all } v \notin [w_2, 1]. \end{cases}$$

Next, we will show that a wage profile (w_1, w_2) such that $w_1 = w_2$ is not part of any core outcome. To show this, suppose for contradiction that outcome (O_1, O_2) associated with (w_1, w_2) is a core outcome. From the argument in the previous paragraph, it must be that

$$f^1(v) + f^2(v) = \begin{cases} 0 & \text{for almost all } v \in [0, w_1), \\ f(v) & \text{for almost all } v \in [w_1, 1]. \end{cases}$$

Therefore, total profit of the firms is

$$\pi_1^{O_1} + \pi_2^{O_2} = \int_{w_1}^{1} (v - w_1) f(v) dv.$$

Moreover, note that $\pi_1^{O_1} = \pi_2^{O_2}$ by the Equal Profit Condition (Remark 3).

Consider the following two cases. First, suppose that $\pi_1^{O_1} = \pi_2^{O_2} = 0$. This implies that $w_1 = w_2 = 1$. Then, firm 1 can block (O_1, O_2) by setting wage $\tilde{w}_1(v) = 0$ for all v and hiring almost all workers, i.e., $\tilde{f}^1(v) = f(v)$ for almost all $v \in [0,1]$ (where $\tilde{O}_1 = (\tilde{f}^1, \tilde{w}_1)$ is a valid block because it results in a strictly positive profit for firm 1 and Condition 4 of the definition of a block is satisfied). Thus, (O_1, O_2) is not a core outcome, a contradiction.

Second, suppose that $\pi_1^{O_1} = \pi_2^{O_2} > 0$. This implies that $w_1 = w_2 < 1$. Then, firm 1 can block (O_1, O_2) by setting wage $\tilde{w}_1(v) = w_1 + \epsilon$ for all $v \ge w_1 + \epsilon$ and hiring almost all workers with productivity at least $w_1 + \epsilon$, i.e., $\tilde{f}^1(v) = f(v)$ for almost all $v \in [w_1 + \epsilon, 1]$ for any sufficiently small $\epsilon > 0$ (where $\tilde{O}_1 = (\tilde{f}^1, \tilde{w}_1)$ is a valid block because it results in a profit arbitrarily close to $2\pi_1^{O_1} > \pi_1^{O_1}$ for firm 1 and Condition 1 of the definition of block is satisfied). Thus, (O_1, O_2) is not a core outcome, a contradiction.

Therefore, we proceed by considering $w_1 < w_2$. For any $w_1 \in [0,1)$, let $w_2(w_1)$ be the unique wage such that the profits of firms 1 and 2, $\pi_1^{O_1}$ and $\pi_2^{O_2}$, are equal to each other under wages w_1 and $w_2(w_1)$. The function $w_2(\cdot)$ is well defined in the sense that for any w_1, there exists a unique $w_2 := w_2(w_1)$ such that the firm profits $\pi_1^{O_1}$ and $\pi_2^{O_2}$ are equal to each other: To see this, note that, for any $w_1 < 1$, if $w_2 > w_1$ is sufficiently close to w_1, then firm 2's profit is higher than the profit of firm 1 because firm 1 will only hire workers of productivities $[w_1, w_2)$ so its profit goes to zero as w_2 approaches w_1 from above while firm 2 hires all workers from $[w_2, 1]$ resulting in a profit bounded away from zero. Moreover, when $w_2 = 1$, $\pi_1^{O_1} > 0 = \pi_2^{O_2}$. Finally, $\pi_2^{O_2} - \pi_1^{O_1}$ is continuous and strictly decreasing in w_2. Therefore, by the intermediate value theorem, there exists a unique w_2 such that $\pi_1^{O_1} = \pi_2^{O_2}$. Thus, we have shown the well-definedness of $w_2(\cdot)$.

Now, define

$$\pi_1(w_1) := \int_{w_1}^{w_2(w_1)} (v - w_1) f(v) dv,$$
$$\pi_2(w_2) := \int_{w_2}^{1} (v - w_2) f(v) dv,$$
$$\pi_0(w_1) := \int_{0}^{w_1} v f(v) dv.$$

Note that $\pi_1(0) > 0 = \pi_0(0)$. Moreover, both $\pi_1(\cdot)$ and $\pi_0(\cdot)$ are continuous, and $\pi_0(\cdot)$ is strictly increasing while $\pi_1(\cdot)$ is strictly decreasing.[40] Therefore, $w_1^* := \sup\{w_1 \in [0,1] : \pi_1(w_1) \geq \pi_0(w_1)\}$ is strictly positive, and $\pi_1(w_1) \geq \pi_0(w_1)$ if and only if $w_1 \in [0, w_1^*]$.

We demonstrate that a pair (w_1, w_2) are supported in a core outcome if and only if $w_1 \in [0, w_1^*]$ and $w_2 = w_2(w_1)$.

We first discuss the necessity of these two conditions. First, note that $w_2 = w_2(w_1)$ is necessary by the Equal Profit Condition. Second, to see the necessity of $w_1 \in [0, w_1^*]$, assume for contradiction that there is a core outcome O such that $w_1 > w_1^*$. Then, since in any core outcome $w_2 = w_2(w_1)$, firm 1's profit is equal to $\pi_1(w_1)$. Let $\tilde{w}_1(v) = 0$ for all v and let

$$\tilde{f}^1(v) = \begin{cases} f(v) & \text{for almost all } v \in [0, w_1), \\ 0 & \text{for almost all } v \in [w_1, 1]. \end{cases}$$

Then outcome $\tilde{O}_1 := (\tilde{f}^1, \tilde{w}_1)$ blocks outcome O because $\pi_1^{\tilde{O}_1} = \pi_0(w_1) > \pi_1(w_1)$ by the definition of w_1^* and the assumption $w_1 > w_1^*$, and Condition 4 of the definition of block is satisfied for almost all $v \in [0, w_1)$. This concludes the necessity argument.

To show sufficiency, take any outcome $O := (f^i, w_i)$, $i \in \{1, 2\}$ such that $w_1 \leq w_1^*$, and $w_2 = w_2(w_1)$. Consider for contradiction that, without loss of generality, firm 1 has a blocking outcome $\tilde{O}_1 := (\tilde{f}^1, \tilde{w}_1)$ where $\tilde{w}_1(v) = w$ for all v such that $\tilde{f}^1(v) > 0$. We evaluate three exhaustive cases for $\tilde{w}_1$ and the attendant functions $\tilde{f}^1$:

1. Suppose $w \in [0, w_1)$. Then

$$\pi_1^{\tilde{O}_1} = \int_0^{w_1} (v - w) \tilde{f}^1(v) dv \leq \int_w^{w_1} (v - w) \tilde{f}^1(v) dv \leq \int_w^{w_1} (v - w) f(v) dv \leq \int_0^{w_1} v f(v) dv = \pi_0(w_1) \leq \pi_1(w_1),$$

   where the first equality is due to the fact that $\tilde{f}^1(v) = 0$ for almost all $v \geq w_1$ by the definition of block, and the second inequality follows because $\tilde{f}^1(v) \in [0, f(v)]$ for all v. Therefore, $\tilde{O}_1$ is not a block, a contradiction.

2. Suppose $w \in [w_1, w_2(w_1))$. Then

$$\pi_1^{\tilde{O}_1} = \int_0^{w_2(w_1)} (v - w) \tilde{f}^1(v) dv \leq \int_w^{w_2(w_1)} (v - w) \tilde{f}^1(v) dv \leq \int_w^{w_2(w_1)} (v - w) f(v) dv \leq \int_{w_1}^{w_2(w_1)} (v - w_1) f(v) dv = \pi_1(w_1),$$

[40] That $\pi_0(\cdot)$ is strictly increasing is obvious from the definition. To see that $\pi_1(\cdot)$ is strictly decreasing, let us first note that $w_2(\cdot)$ is a strictly increasing fucntion because, if not, $\pi_2(w_2(w_1))$ weakly increases while $\pi_1(w_1)$ strictly decreases at some w_1, a contradiciton to the construction of $w_2(\cdot)$ to satisfy the Equal Profit Condition. This implies that $\pi_2(w_2(\cdot))$ is strictly decreasing. Again by the construction of $w_2(\cdot)$ to satisfy the Equal Profit Condition, $\pi_1(\cdot)$ is also strictly decreasing, as desired.

where the first equality is due to the fact that $\tilde{f}^1(v)=0$ for almost all $v \geq w_2(w_1)$ by the definition of block, and the second inequality follows because $\tilde{f}^1(v) \in [0, f(v)]$ for all v. Therefore, $\tilde{O}_1$ is not a block, a contradiction.

3. Suppose $w \in [w_2(w_1),1]$. Then

$$\pi_1^{\tilde{O}_1} = \int_0^1 (v-w)\tilde{f}^1(v)dv \leq \int_w^1 (v-w)\tilde{f}^1(v)dv \leq \int_w^1 (v-w)f(v)dv \leq \int_{w_2(w_1)}^1 (v-w_2(w_1))f(v)dv = \pi_2(w_2(w_1)) = \pi_1(w_1),$$

where the second inequality follows because $\tilde{f}^1(v) \in [0,f(v)]$ for all v, and the final equality follows from the Equal Profit Condition. Therefore, $\tilde{O}_1$ is not a block, a contradiction.

Therefore, we have shown that each $w_1 \leq w_1^*$ supports a distinct core outcome. □

Proof of Part 2. In the proof of Part 1, we have established that the set of core outcomes is characterized by $w_1 \in [0,w_1^*]$. Furthermore, we have shown that in any core outcome,

$$f^1(v)+f^2(v) = \begin{cases} 0 & \text{for almost all } v \in [0,w_1), \\ f(v) & \text{for almost all } v \in [w_1,1]. \end{cases}$$

Therefore, for any $w_1 > 0$, a positive measure of workers are unemployed in attendant core outcomes, and for $w_1 = 0$ almost no workers are unemployed in attendant core outcomes. □

Proof of Part 3. We have shown in the *Proof of Part 1* that the set of (non-equivalent) core outcomes is characterized by wage w_1, and $w_2 = w_2(w_1)$ that is strictly increasing in w_1. Moreover, the profit of both firms in a core outcome characterized by w_1 is given by $\int_{w_2(w_1)}^1 (v-w_2(w_1))f(v)dv$ by the equal profit condition, which is strictly decreasing in w_1 since $w_2(\cdot)$ is strictly increasing in w_1. To complete the claim, note that in *Proof of Part 2* we showed that unemployment is strictly increasing in w_1 in the set of core outcomes. □

Proof of Remark 7

Proof of Part 1. Let $\epsilon \in (0,\frac{1}{8}]$, $\beta \geq 1$, and F_A, F_B be such that $F(1-\frac{\epsilon}{2}) < \frac{\epsilon^2}{2}$ and $f(v) = f(v')$ for all $v,v' \in [1-\frac{\epsilon}{2},1)$.[41]

1. Proposition 6 finds that there is an essentially unique core outcome O in which $w_1 = 0$. We show that in such a core outcome $w_2 > 1-\epsilon$. Suppose $w_2 \leq 1-\epsilon$ in this core outcome. Then firm 1's profit is upper bounded as follows

$$\pi_1^{O_1} = \int_0^{w_2} vf(v)dv \leq \int_0^{1-\epsilon} vf(v)dv < \frac{\epsilon^2}{2}(1-\epsilon) < \frac{\epsilon}{2}\left(1-\frac{\epsilon}{2}\right)$$

where the second inequality is due to the fact that $F(1-\epsilon) < F(1-\frac{\epsilon}{2}) < \frac{\epsilon^2}{2}$ and the fact that $v \leq 1-\epsilon$ over the range of integration, and the final inequality follows from $\epsilon \in [0,1)$.

By contrast, firm 2's profit is lower bounded as follows

[41]We let $\epsilon \leq \frac{1}{8}$ here. Doing so is without loss of generality for our purposes because if the conclusion holds for $\epsilon > 0$, then clearly the same conclusion holds for every $\epsilon' > \epsilon$.

$$\pi_2^{O_2} = \int_{w_2}^{1} (v - w_2) f(v) dv \geq \int_{1-\epsilon}^{1} (v - 1 + \epsilon) f(v) dv > \int_{1-\frac{\epsilon}{2}}^{1} (v - 1 + \epsilon) f(v) dv > \frac{\epsilon}{2}\left(1 - \frac{\epsilon^2}{2}\right) > \frac{\epsilon}{2}\left(1 - \frac{\epsilon}{2}\right) > \pi_1^{O_1}$$

where the third inequality follows because $F(1-\frac{\epsilon}{2}) < \frac{\epsilon^2}{2}$ and the fact that $v - 1 + \epsilon \geq \frac{\epsilon}{2}$ over the range of integration. Thus, $w_2 \leq 1 - \epsilon$ is a contradiction with the Equal Profit Condition. Therefore, it must be the case that $w_2 > 1 - \epsilon$ in any core outcome O such that $w_1 = 0$.

2. We show that there exists a core outcome O in which $w_1 = 1 - \frac{\epsilon}{2}$, and combined with the fact that $w_2 < 1$, this implies that $w_2 - w_1 < \frac{\epsilon}{2} < \epsilon$. For $w_1 = 1 - \frac{\epsilon}{2}$, $\pi_1^{O_1} = \pi_2^{O_2}$ if and only if $w_2 = 1 - \frac{\epsilon}{4}$ by the assumption that $f(v) = f(v')$ for all $v, v' \in [1-\frac{\epsilon}{2}, 1)$. Therefore, to verify that O is a core outcome, it remains to show that $w_1 = 1 - \frac{\epsilon}{2} < w_1^*$ where w_1^* is defined in the proof of Proposition 6, i.e.

$$\pi_1^{O_1} \geq \int_0^{w_1} v f(v) dv. \tag{59}$$

We can see that (59) is satisfied because

$$\pi_1^{O_1} = \int_{w_1}^{w_2} (v - w_1) f(v) dv > \left[\left(1 - \frac{3\epsilon}{8}\right) - \left(1 - \frac{\epsilon}{2}\right)\right] \frac{(1-\frac{\epsilon}{2})}{2} = \frac{\epsilon}{8} \frac{(1-\frac{\epsilon}{2})}{2} \geq \frac{\epsilon^2}{2}\left(1 - \frac{\epsilon}{2}\right) > \int_0^{w_1} v f(v) dv$$

where the first inequality follows because the bracketed term is the averageproductivity of workers hired by firm 1 minus the wage paid to each worker (which is true from the assumption that $f(v) = f(v')$ for all $v, v' \in [1-\frac{\epsilon}{2}, 1)$) and $\frac{(1-\frac{\epsilon}{2})}{2}$ is a lower bound on the measure of workers firm 1 hires (which follows from the assumption that $F(1-\frac{\epsilon}{2}) < \frac{\epsilon^2}{2} < \frac{\epsilon}{2}$), the second inequality is true by inspection for all $\epsilon \leq \frac{1}{8}$, and the final inequality follows from the assumption that $F(w_1) = F(1-\frac{\epsilon}{2}) < \frac{\epsilon^2}{2}$ and the fact that $v \leq w_1 = 1 - \frac{\epsilon}{2}$ over the range of integration. Therefore, O is a core outcome such that $w_2 - w_1 < \epsilon$.

□

Proof of Part 2. Let F_A and F_B be defined implicitly by their densities, f_A and f_B, respectively, as follows:

$$f_A(v) = \begin{cases} 2\epsilon & v \in [0, \frac{1}{2}] \\ 2(1-\epsilon) & v \in (\frac{1}{2}, 1] \end{cases}$$

and

$$f_B(v) = \begin{cases} 2(1-\epsilon) & v \in [0, \frac{1}{2}] \\ 2\epsilon & v \in (\frac{1}{2}, 1]. \end{cases}$$

Note that in any core outcome without EPSW, the wage gap is equal to $\mathbb{E}_A(v) - \mathbb{E}_B(v) = \frac{3}{4}(1-\epsilon) + \frac{1}{4}\epsilon - \left(\frac{1}{4}(1-\epsilon) + \frac{3}{4}\epsilon\right) = \frac{1}{2} - \epsilon$. We claim that there exists $\beta^* > 0$ such that for any $\beta > \beta^*$ and $\epsilon < \frac{1}{10}$, under a non-group-based EPSW

1. $w_1 = \frac{1}{2}$ is supported in a core outcome O which leads to $AW_A^O - AW_B^O > \frac{1}{2}$, and

2. $w_1 = 0$ is supported in a core outcome O' which leads to $AW_A^{O'} - AW_B^{O'} < \frac{1}{2} - \epsilon$.

To show the first point, we first argue that for sufficiently large β^*, $w_1 = \frac{1}{2}$ can be supported by a core outcome O. By our assumption on $F_A(\cdot)$ and $F_B(\cdot)$, such a core outcome would require $w_2 = \frac{3}{4}$ so that $\pi^{O_1} = \pi^{O_2}$. For any $\beta > 0$, (4) allows us to represent each firm's profits as follows:

$$\pi_1^{O_1} = \frac{\beta}{1+\beta} 2(1-\epsilon) \int_{1/2}^{3/4} (v - \frac{1}{2}) dv + \frac{1}{1+\beta} 2\epsilon \int_{1/2}^{3/4} (v - \frac{1}{2}) dv,$$
$$\pi_2^{O_2} = \frac{\beta}{1+\beta} 2(1-\epsilon) \int_{3/4}^{1} (v - \frac{3}{4}) dv + \frac{1}{1+\beta} 2\epsilon \int_{3/4}^{1} (v - \frac{3}{4}) dv,$$
$$\pi_0(w_1) = \frac{\beta}{1+\beta} 2\epsilon \int_0^{1/2} v dv + \frac{1}{1+\beta} 2(1-\epsilon) \int_0^{1/2} v dv,$$

where $\pi_0(w_1)$ is defined in the proof of Proposition 6 as the total surplus possible by employing all unemployed workers.

To show that O is a core outcome, from the Proof of Proposition 6, it suffices to show that $\pi_1^{O_1} \geq \pi_0(w_1)$. Given $w_1 = \frac{1}{2}$ and $w_2 = \frac{3}{4}$, $\pi_1^{O_1} \to 2(1-\epsilon) \int_{1/2}^{3/4} (v - 1/2) dv = 2(1-\epsilon) \int_0^{1/4} v dv = 2(1-\epsilon) \frac{1}{32}$ as $\beta \to \infty$. Similarly, $\pi_0(w_1) \to 2\epsilon \int_0^{1/2} v dv = 2\epsilon \frac{1}{8}$. Therefore, for sufficiently large β, $\pi_1^{O_1} \geq \pi_0(w_1)$ if $2(1-\epsilon)\frac{1}{32} > 2\epsilon \frac{1}{8}$ which holds if $\epsilon < \frac{1}{5}$. Therefore, for any sufficiently large β^* and $\epsilon < \frac{1}{5}$, $w_1 = \frac{1}{2}$ is supported by a core outcome.

In any core outcome O in which $w_1 = \frac{1}{2}$, among the $A-$group workers, an ϵ share of them receive a wage of 0, a $(\frac{1-\epsilon}{2})$ share receive a wage of $\frac{1}{2}$, and a $\frac{1-\epsilon}{2}$ share receive a wage of $\frac{3}{4}$. Therefore, $AW_A^O = (1-\epsilon)\frac{5}{8}$. Among the $B-$group workers, a $(1-\epsilon)$ share of them receive a wage of 0, an $\frac{\epsilon}{2}$ share receive a wage of $\frac{1}{2}$, and an $\frac{\epsilon}{2}$ share receive a wage of $\frac{3}{4}$. Therefore, $AW_B^O = \epsilon \frac{5}{8}$. Combining these calculations, $AW_A^O - AW_B^O = (1-\epsilon)\frac{5}{8} - \epsilon\frac{5}{8} = \frac{5(1-2\epsilon)}{8}$. This implies that $AW_A^O - AW_B^O > \frac{1}{2}$ if and only if $\epsilon < \frac{1}{10}$, establishing the first point above.

To show the second point, recall from Proposition 6 that there always exists a core outcome O' in which $w_1 = 0$. We now consider the wage gap in O'. For any $\beta > 1$ it must be the case that $w_2 > \frac{1}{2}$ or else the Equal Profit Condition would be violated. Therefore, it remains only to show the wage gap between $A-$ and $B-$group workers in this core outcome. Since $w_1 = 0$ and $w_2 > \frac{1}{2}$, $AW_A^{O'} - AW_B^{O'} = 2(1-\epsilon)(1-w_2)w_2 - 2\epsilon(1-w_2)w_2 = 2(1-2\epsilon)(1-w_2)w_2$. We want to show that $2(1-2\epsilon)(1-w_2)w_2 < \frac{1}{2} - \epsilon$. For all $\epsilon < \frac{1}{2}$ this is equivalent to the condition that $(1-w_2)w_2 < \frac{1}{4}$. But note that this must be true since $w_2 > \frac{1}{2}$. Therefore, $AW_A^{O'} - AW_B^{O'} < \frac{1}{2} - \epsilon$, establishing the second point above.

□

B Extensions

In this section, we offer extensions to our model, as discussed in Section 4.

B.1 Effect of EPSW with multiple groups and/or firms

We consider a setting in which there are n identical firms 1,...,n as well as m groups g_1,...,g_m of workers. The m groups may represent different genders, races, religions, and other protected statuses. We consider the impact of EPSW (both group-based and non-group-based) on core outcomes.

We define several primitives of the model, which are analogous to the case of $n=m=2$. For each group $g\in\{g_1,...,g_m\}$, there is a $\beta_g>0$ measure of workers of group g, where we normalize $\sum_k\beta_{g_k}=1$. Each group g is endowed with a cumulative distribution function of productivity F_g, which admits a density function f_g. We assume that for each g, $0<\underline{f}_g\leq\bar{f}_g<+\infty$ where $\underline{f}_g=\inf\{f_g(v)|v\in[0,1]\}$ and $\bar{f}_g=\sup\{f_g(v)|v\in[0,1]\}$.

Define a labor market by $\Gamma:=(\{1,...,n\},(g_k)_{k=1,...,m},(\beta_{g_k})_{k=1,...,m},(F_{g_k})_{k=1,...,m})$. Throughout the remainder of Appendix B.1, we fix an arbitrary labor market Γ and present results within this labor market, except where explicitly stated otherwise.

An outcome for firm i is $O_i:=\{(f_g^i(v),w_i^g(v))\}_{v\in[0,1],g=g_1,...,g_m}$. The interpretations are

1. $f_g^i(v)\in[0,f_g(v)]$ is the density of workers of type $e=(g,v)$ hired by firm i,
2. $w_i^g(v)\in[0,\infty)$ is the wage firm i pays to workers of type $e=(g,v)$ it hires.

An outcome is a tuple $O:=(O_1,O_2,...,O_n)$ where O_i is the outcome for firm i such that $f_g^1(v)+f_g^2(v)+...+f_g^n(v)\leq f_g(v)$ for each v and g. That is, the (overall) outcome specifies the outcome for all firms such that the total hiring does not exceed the supply of workers (a feasibility requirement). We assume that f_g^i and w_i^g are measurable functions for each i and g. We also assume that wages must be monotone non-decreasing in workerproductivity within each firm. Formally, for each $i\in\{1,2,...,n\}$, $g\in\{g_1,...,g_m\}$, and any $v,v'\in[0,1]$, $w_i^g(v)\geq w_i^g(v')$ if $v\geq v'$ and $f_g^i(v)>0$. If $f_g^i(v)=0$, then we fix $w_i^g(v)=0$.

Each firm i's profit in outcome O_i is

$$\pi_i^{O_i}:=\beta_1\int_0^1[v-w_i^{g_1}(v)]f_{g_1}^i(v)dv+\beta_2\int_0^1[v-w_i^{g_2}(v)]f_{g_2}^i(v)dv+...+\beta_n\int_0^1[v-w_i^{g_m}(v)]f_{g_m}^i(v)dv.$$

The definition of a core outcome extends from earlier sections in a natural manner.

We now characterize core outcomes without EPSW.

Proposition **7.** *Without EPSW, there exist a continuum of (non-equivalent) core outcomes. In any core outcome, almost every worker is employed and earns a wage equal to her productivity (formally, for all $i\in\{1,...,n\}$, all $g\in\{g_1,...,g_m\}$, and almost all $v\in[0,1]$: $f_g^1(v)+...+f_g^n(v)=f_g(v)$ and $w_i^g(v)=v$ if $f_g^i(v)>0$).*

Proof. The proof follows similar logic to the proof of Proposition 1.

□

Non-group-based EPSW requires that a firm pays the same wage to almost all workers it hires. Formally, we modify the definition of outcome $O_i=\{(f_g^i(v),w_i^g(v))\}_{v\in[0,1],g=g_1,...,g_m}$ for all $i\in\{1,...,n\}$ to include the following restriction:

> There exists $w_i\in[0,1]$ such that $w_i^g(v)=w_i$ for all g and almost all v such that $\beta_{g_1}f_{g_1}^i(v)+...+\beta_{g_m}f_{g_m}^i(v)>0$.

We now study the effect of non-group-based EPSW. As before, it is convenient to proceed with the distribution of productivities of the entire population $F(v)=\sum_k\beta_{g_k}F_{g_k}(v)$ and the associated density function denoted as $f(v)$. By the assumption that for each g, $0<\underline{f}_g\leq\bar{f}_g<+\infty$, we observe that $\underline{f}:=\inf\{f(v)|v\in[0,1]\}>0$ and $\bar{f}:=\sup\{f(v)|v\in[0,1]\}<+\infty$. In any outcome we denote the density of workers hired by each firm $i\in\{1,2,...,n\}$ as $f^i(v):=\beta_{g_1}f_{g_1}^i(v)+...+\beta_{g_m}f_{g_m}^i(v)$.

Proposition **8.** *Suppose there is a non-group-based EPSW. Without loss of generality, assume $0\leq w_1\leq w_2\leq\cdots\leq w_n\leq1$.*

1. *There exist a continuum of non-equivalent core outcomes. In any core outcome,* $0 \leq w_1 < w_2 < ... < w_n < 1$.

2. *There exists one core outcome (and its equivalent outcomes) in which almost all workers are employed. In all other core outcomes, a strictly positive measure of workers are unemployed.*

3. *Consider any two core outcomes. The measure of unemployed workers is higher in the first outcome if and only if the firm profit is lower in the first outcome.*

Proof. We prove only Part 1 because Parts 2 and 3 follow from the algorithm constructed below, and arguments analogous to those in Proposition 6.

As in the case of two firms, notice that in any core outcome such that $w_i \neq w_j$ for any $i \neq j$, it must be the case that

$$f^1(v) = \begin{cases} f(v) & \text{for almost all } v \in [w_1, w_2), \\ 0 & \text{for almost all } v \notin [w_1, w_2). \end{cases}$$

$$f^2(v) = \begin{cases} f(v) & \text{for almost all } v \in [w_2, w_3), \\ 0 & \text{for almost all } v \notin [w_2, w_3). \end{cases}$$

$$\vdots$$

$$f^n(v) = \begin{cases} f(v) & \text{for almost all } v \in [w_n, 1], \\ 0 & \text{for almost all } v \notin [w_n, 1]. \end{cases} \tag{60}$$

By an argument analogous to the case of two firms, it is the case that $w_i \neq w_j$ for any $i \neq j$ in any core outcome. Therefore, the above formulation pins down each firm's hiring in a core outcome as a function of the wage profile $w_1, ..., w_n$.

Let $w_{n+1} = 1$. For any firm i and a pair of wages w_i and $w_{i+1} > w_i$, let

$$\pi(w_i, w_{i+1}) := \int_{w_i}^{w_{i+1}} (v - w_i) f(v) dv.$$

$\pi(w_i, w_{i+1})$ is a firm's profit when it hires almost all workers whose productivities lie within $[w_i, w_{i+1})$ at wage w_i, and almost none of the workers whose productivities lie outside $[w_i, w_{i+1})$.

We turn to the question of which wage profiles $(w_1, w_2, ..., w_n)$ can be supported in a core outcome. To do so, we define an algorithm to produce (what we will show are) upper bounds on the wages of each firm.

Let $p \in \mathbb{R}_+$, $w_0^p := 0$, and $w_{n+1}^p = 1$. For each $i \in \{1, 2, ..., n\}$, define w_i^p inductively by

$$w_i^p := \sup\{w_i \in [w_{i-1}^p, 1] : \pi(w_{i-1}^p, w_i) \leq p\}.$$

Let $\eta(p) := \pi(w_n^p, 1)$. Observe that $\eta(0) > 0$, $\eta(1) = 0$, and $\eta(p)$ is strictly decreasing for any p such that $\eta(p) \neq 0$. Moreover, the function $\eta(\cdot)$ is clearly continuous. Therefore, by the intermediate value theorem, there exists a unique solution to $\eta(p) = p$. Let the unique solution be denoted by p^* and $w_1^* := w_1^{p^*}$.

The following claim completes the argument, and characterizes the set of core outcomes in a similar way as in Proposition 6.

Claim **14.**

1. *For any $w_1 \in [0,w_1^*]$, there exists an essentially unique core outcome in which firm 1 sets wage w_1.*

2. *For any $w_1 \notin [0,w_1^*]$, there exists no core outcome in which firm 1 sets wage w_1.*

Proof of Part 1. We construct a core outcome for each $w_1 \leq w_1^*$. To do so, for any $w_1 \in [0,1]$ and any $p \in \mathbb{R}_+$ let $\bar{w}_{n+1}^p = 1$. We define an inductive algorithm which outputs a wage for each firm $i \in \{2,...,n\}$. For $i \in \{2,...,n\}$, define $\bar{w}_i^p$ inductively by

$$\bar{w}_i^p := \sup\{w_i \in [\bar{w}_{i-1}^p,1] : \pi(\bar{w}_{i-1}^p,w_i) \leq p\}.$$

Let $\eta^{w_1}(p) := \pi(\bar{w}_n^p,1)$. Observe that $\eta^{w_1}(0) \geq 0$ (with the inequality strict if $w_1 \neq 1$), and $\eta^{w_1}(1)=0$, and $\eta^{w_1}(p)$ is strictly decreasing in p for any p such that $\eta^{w_1}(p) \neq 0$. Moreover, the function η^{w_1} is clearly continuous. Therefore, by the intermediate value theorem, there exists a unique solution to $\eta^{w_1}(p)=p$.

For any $w_1 \in [0,w_1^*]$, let p^{w_1} be the unique solution to the equation $\eta^{w_1}(p)=p$ (which we have shown to exist). For each $i \in \{2,...,n\}$ let $w_i = w_i^{p^{w_1}}$, and let $w_{n+1}=1$.

Consider an outcome O in which for each $i \in \{1,2,...,n\}$, O_i is characterized by (60) and $w_i = w_i^{p^{w_1}}$ for all i. By construction, at the wage profile $\mathbf{w}=(w_i)_i$ the profits of all firms are equalized at p^{w_1}, i.e. $\pi_i^{O_i} = \pi(w_i,w_{i+1}) = p^{w_1}$ for all i.

Consider for contradiction that some firm i has a blocking outcome $\tilde{O}_i := (\tilde{f}^i,\tilde{w}_i)$ where $\tilde{w}_i(v)=w$ for almost all v such that $\tilde{f}^i(v)>0$. We evaluate two exhaustive cases for the value w and the attendant function $\tilde{f}^i$:

1. Suppose $w \in [0,w_1)$. Then

$$\pi_i^{\tilde{O}_i} = \int_0^{w_1} (v-w)\tilde{f}^i(v)dv \leq \int_w^{w_1} (v-w)\tilde{f}^i(v)dv \leq \int_w^{w_1} (v-w)f(v)dv \leq \int_0^{w_1} vf(v)dv \leq \pi_1^{O_1} = \pi_i^{O_i},$$

   where the first equality is due to the fact that $\tilde{f}^i(v)=0$ for almost all $v \geq w_1$ by the definition of block, the second inequality follows because $\tilde{f}^i(v) \in [0,f(v)]$ for all v, the final inequality follows because $w_1 < w_1^*$ by assumption, and the final equality follows as the profits of all firms are equal by construction in outcome O. Therefore, $\tilde{O}_i$ is not a block, a contradiction.

2. Suppose for some $j \in \{1,...,n\}$, $w \in [w_j,w_{j+1})$. Then

$$\pi_i^{\tilde{O}_i} = \int_0^{w_{j+1}} (v-w)\tilde{f}^i(v)dv \leq \int_w^{w_{j+1}} (v-w)\tilde{f}^i(v)dv \leq \int_w^{w_{j+1}} (v-w)f(v)dv \leq \int_{w_j}^{w_{j+1}} (v-w_j)f(v)dv = \pi_j^{O_j} = \pi_i^{O_i},$$

   where the first equality is due to the fact that $\tilde{f}^i(v)=0$ for almost all $v \geq w_{j+1}$ by the definition of block, the second inequality follows because $\tilde{f}^i(v) \in [0,f(v)]$ for all v, and the final equality follows as the profits of all firms are equal by construction in outcome O. Therefore, $\tilde{O}_i$ is not a block, a contradiction.

Therefore, we have shown that any $w_1 \leq w_1^*$ supports a core outcome. Note that for each w_1 the associated core outcome is essentially unique as the equal profit condition pins down wages $w_2,...,w_n$ paid by firms $2,...,n$, respectively, to all hired workers.

□

Proof of Part 2. Suppose for contradiction that there exists a core outcome O in which $w_1 > w_1^*$. The Equal Profit Condition implies $\pi^{O_i} = p^{w_1}$ for every firm i. However, consider any firm i and an outcome $\tilde{O}_i = (\tilde{f}^i, \tilde{w}_i)$ such that

$$\tilde{f}^i(v) = \begin{cases} f(v) & \text{for almost all } v \in [0, w_1), \\ 0 & \text{for almost all } v \notin [0, w_1). \end{cases}$$

and $\tilde{w}_i(v) = 0$ for all v. For almost all $v \in [0, w_1)$, Condition 4 of the definition of block is satisfied. Moreover, firm i's profit under outcome $\tilde{O}_i$ is

$$\pi_i^{\tilde{O}_i} = \int_0^{w_1} v f(v) dv = \pi(w_0, w_1) > \pi_1^{O_1} = \pi_i^{O_i}$$

where the second equality follows from the definition of $\pi(w_0, w_1)$, the inequality follows by construction of w_1^*, and the final inequality follows from the Equal Profit Condition. Therefore, firm i blocks outcome O via $\tilde{O}_i$, which contradicts that O is a core outcome. □

□

This result provides a characterization of the set of core outcomes under non-group-based EPSW.

Group-based EPSW requires that if a firm hires workers from more than one group, then the firm must pay the same wage to almost all workers it hires, that is, we modify the definition of outcome $O_i = \{(f_g^i(v), w_i^g(v))\}_{v \in [0,1], g = g_1, \ldots, g_m}$ for all $i \in \{1, \ldots, n\}$ to include the following restriction:

> For any distinct groups g, g', there exist no positive Lebesgue measure sets $V_g \subset [0,1]$ and $V_{g'} \subset [0,1]$ such that:
>
> 1. $f_g^i(v) > 0$ for all $v \in V_g$,
> 2. $f_{g'}^i(v) > 0$ for all $v \in V_{g'}$, and
> 3. $\inf_{v \in V_{g'}} w_i^{g'}(v) > \sup_{v \in V_g} w_i^g(v)$.

PROPOSITION **9.** *Suppose $n > m$. Then in any core outcome with a group-based EPSW:*

1. *Firms completely segregate. Specifically, no firm hires positive measures of workers from more than one group (i.e. there is no firm $i \in \{1, \ldots, n\}$, two distinct groups $g, g' \in \{g_1, \ldots, g_m\}$, and two positive Lebesgue-measure sets $V_g, V_{g'} \subset [0,1]$ such that $f_g^i(v) > 0$ for all $v \in V_g$, and $f_{g'}^i(v) > 0$ for all $v \in V_{g'}$).*

2. *Each firm earns zero profit (i.e. $\pi_i^{O_i} = 0$ for all firms $i \in \{1, \ldots, n\}$), and*

3. *Almost every worker is employed at a wage equal to her productivity (i.e. for all $i \in \{1, \ldots, n\}$, all $g \in \{g_1, \ldots, g_m\}$, and almost all $v \in [0,1]$: $f_g^1(v) + f_g^2(v) + \ldots + f_g^n(v) = f_g(v)$ and $w_i^g(v) = v$ if $f_g^i(v) > 0$).*

Proof of Part 1: This follows by an argument analogous to that in the proof of Proposition 2. □

Proof of Parts 2 and 3: Given the finding of Part 1, the arguments for Parts 2 and 3 follow from analogous arguments in the proof of Proposition 1. □

For the next result, let us introduce some more structure. We will consider a varying number of groups m. Fix n arbitrarily, and consider a sequence of markets $\Gamma^m := (\{1,...,n\},(g_k^m)_{k=1,...,m},(\beta_{g_k^m})_{k=1,...,m},(F_{g_k^m})_{k=1,...,m})$ indexed by $m\in\{1,2,...\}$. We assume that there exist $\underline{\underline{f}}>0$ and $\overline{\overline{f}}<\infty$ such that, for any m, k, and $v\in[0,1]$ it is the case that $\underline{\underline{f}}\leq f_{g_k^m}(v)\leq\overline{\overline{f}}$. We also assume that $\lim_{m\to\infty}\max_{k=1,...,m}\beta_{g_k^m}=0$ and $F:=\sum_{k\in\{1,\cdots,m\}}\beta_{g_k}F_{g_k}$ is constant across m.

PROPOSITION **10.** *For any n, consider a sequence of markets $(\Gamma^m)_{m=1}^\infty$. There exists m^* such that for any $m>m^*$, the set of core outcomes in market Γ^m under group-based-EPSW is equivalent to the set of core outcomes in Γ^m under non-group-based EPSW.*

Proof. Consider any core outcome O under group-based EPSW. Let N^* be the set of firms that hire a positive measures of workers from no more than one group (i.e. the set of firms N^* such that for any $i\in N^*$ there does not exist two groups k,k' and positive Lebesgue measure sets $V_k^i\subset[0,1]$ and $V_{k'}^i\subset[0,1]$ with $f^i_{g_k^m}(v)>0$ for all $v\in V_k^i$ and $f^i_{g_{k'}^m}(v)>0$ for all $v\in V_{k'}^i$). Let $|N^*|:=n^*$.

Suppose that $n^*=0$. Then, O is also a core outcome under non-group-based EPSW because any blocking outcome under non-group-based EPSW is also a blocking outcome under group-based EPSW.

Next, suppose $n^*>0$. Then, any firm $i\notin N^*$ must set a constant wage w_i for almost all workers it hires. Index the firms not in N^* as $\{1,2,...,n-n^*\}$ and, without loss, assume that $w_i\leq w_{i+1}$ for all $i\notin N^*$. Denote $w_0:=0$ and $w_{n-n^*+1}=1$. Because $w_i\in[0,1]$ for every $i\in\{1,...,n-n^*\}$, there exists $i\in\{1,...,n-n^*+1\}$ such that $w_i-w_{i-1}\geq\frac{1}{n-n^*+1}$. Now, consider any sufficiently large m such that $\sum_{g\notin G^m}\beta_g f_g(v)\geq\frac{f(v)}{2}$ for every $v\in[0,1]$, where G^m denotes the set of groups whose workers are hired by firms in N^*: Note that because $|G^m|\leq n^*$, $0<\underline{\underline{f}}\leq\overline{\overline{f}}<\infty$ and $\max_{k\in\{1,...,m\}}\beta_{g_k^m}\to 0$ as $m\to\infty$, for any sufficiently large m, the said inequality holds. Then, a firm can earn profit (arbitrarily close to) $(\frac{1}{2})^2(\frac{1}{n-n^*+1})^2\underline{f}$ or more by setting wage (arbitrarily close to, but strictly greater than) w_{i-1} and hiring all workers whose productivities lie in $[w_{i-1},w_i]$ from groups other than those in G^m, and that lower bound of the profit is positive and independent of m. Meanwhile, the profit of any segregating firm $i\in N^*$ is bounded from above by $\max_{k\in\{1,...,m\}}\beta_{g_k^m}$, which converges to zero as $m\to\infty$. Therefore, for any sufficiently large m, outcome O is not in the core under group-based EPSW.

Moreover, any core outcome O under non-group-based EPSW is also a core outcome under group-based EPSW because each firm's profit $\pi_i^{O_i}$ is at least $p^*>0$ (see Proposition 8) for every m, while the profit from hiring from one group g for a given m is bounded by $\max_{k=1,...,m}\beta_{g_k^m}\int_0^1 vf_{g_k^m}(v)dv\leq\max_{k=1,...,m}\beta_{g_k^m}$, and the last expression converges to 0 as $m\to\infty$ by assumption.

B.2 Heterogeneous treatment EPSW

We study core outcomes of the game when the group-based EPSW applies to only one of the two firms, a situation we refer to as *heterogeneous treatment*. Cases of heterogeneous treatment have been documented and studied by economists; a U.S. federal EPEW policy restricted only federal contractors in the 1970s (see Donohue III and Heckman (1991)). As we discuss in our empirical application below, this model offers important predictions in the study of EPSW in Chile.

Consider again a model with two firms, where we present results in a fixed labor market (β,F_A,F_B). Now we assume that firm 1 is subject to the group-based EPSW while firm 2 is not. Formally, we modify the definition of outcome $O_1=\{(f_g^1(v),w_1^g(v))\}_{v\in[0,1],g=A,B}$ to include the following restriction:

There exist no positive Lebesgue measure sets $V_g \subset [0,1]$ and $V_{-g} \subset [0,1]$ such that:

1. $f_g^1(v) > 0$ for all $v \in V_g$,
2. $f_{-g}^1(v) > 0$ for all $v \in V_{-g}$, and
3. $\inf_{v \in V_{-g}} w_1^{-g}(v) > \sup_{v \in V_g} w_1^g(v)$.

Meanwhile, we do not make any such assumption on firm 2.

Proposition **11.** *With heterogeneous treatment EPSW, there exist a continuum of (non-equivalent) core outcomes. In any core outcome O:*

1. *Almost every worker is employed and earns a wage equal to her productivity (formally, for all $i \in \{1,2\}$, all $g \in \{A,B\}$, and almost all $v \in [0,1]$: $f_g^1(v) + f_g^2(v) = f_g(v)$, and $w_i^g(v) = v$),*
2. *Firm 1 is "fully segregated" in that it does not hire a positive measure of workers from both groups (formally, for at least one $g \in \{A,B\}$ and for almost all $v \in [0,1]$, $f_g^1(v) = 0$), and*
3. *The wage gap between groups is unchanged compared to the case without any EPSW (formally, $AW_A^O - AW_B^O = \mathbb{E}_A[v] - \mathbb{E}_B[v]$.)*

Proof. We first construct a continuum of (non-equivalent) core outcomes. To do so, consider a positive Lebesgue measure set $V_A \subset [0,1]$, and consider outcome $O = \{(f_g^i(v), w_i^g(v))\}_{v \in [0,1], i=1,2, g=A,B}$ characterized by:

$$f_A^1(v) := \begin{cases} f_A(v) & \text{if } v \in V_A, \\ 0 & \text{otherwise.} \end{cases} \qquad w_1^A(v) := \begin{cases} v & \text{if } v \in V_A, \\ 0 & \text{otherwise.} \end{cases}$$

$$f_B^1(v) := 0 \text{ for all } v \qquad w_1^B(v) := 0 \text{ for all } v$$

$$f_g^2(v) := \begin{cases} f_g(v) & \text{if } f_g^1(v) \neq f_g(v), \\ 0 & \text{otherwise.} \end{cases} \qquad w_2^g(v) := \begin{cases} v & \text{if } f_g^2(v) = f_g(v), \\ 0 & \text{otherwise.} \end{cases}$$

O is a core outcome by a similar argument as presented in the proof of Proposition 1: no firm wishes to fire any employed workers, and poaching workers from the other firm requires paying higher wages which would violate the blocking firm's individual rationality condition (see conditions 1 and 3 of the definition of block). Importantly, note that firm 1 is unconstrained by EPSW in outcome O as it hires only $A-$group workers. As O is characterized by V_A, there are uncountably many such core outcomes for different sets V_A, completing the claim.

Proof of Part 1. First, to show that almost all workers are hired at any core outcome, consider an outcome in which some positive measure of workers are unemployed. Then, following the argument as in the proof of Proposition 1, firm 2 can block the outcome: firm 2 can hire all unemployed workers at the wage equal to the essential supremum of the wage for all workers of the same group with weakly lower productivity.

Suppose that there exists a firm i, a group g, and a set of workers V_g with positive measure such that firm i hires positive measure of $g-$group workers from V_g and pays $w_g^i(v) < v$ to almost all of them. If $i=1$, following the argument in the proof of Proposition 1, firm 2 can block the outcome. Thus, in the remainder, we restrict attention to the case in which the profit of firm 1 is zero. For the case of $i=2$, because the profit for firm 1 is zero, firm 1 can block the outcome by firing all its existing workers and poaching $g-$group workers with $v \in V_g$ from firm 2 at a wage $\tilde{w}_g^1(v) \in (w_g^2(v), v)$ such that $\tilde{w}_g^1(\cdot)$ is non-decreasing in v. □

Proof of Part 2. Consider any outcome O in which there exist positive Lebesgue measure sets $V_A \subset [0,1]$ and $V_B \subset [0,1]$ such that $f_A^1(v) > 0$ for all $v \in V_A$ and $f_B^1(v) > 0$ for all $v \in V_B$. Then, because firm 1 is subject to group-based EPSW, there exists $w \geq 0$ such that $w_1^g(v) = w$ for almost all g and v such that $v \in V_g$. Then, by the individual rationality for firm 1, for all g and almost all v such that $v \in V_g$, $v > w$. Therefore, by an argument analogous to the proof of Proposition 1, it follows that there exists a block by firm 2. Therefore, O is not a core outcome. □

Proof of Part 3. This claim follows directly from Part 1. □

This completes the proof. □

□

B.3 Taste-Based Bias

Our base model did not explicitly incorporate (taste-based) bias against the minority group, nor did it consider differences in bias across firms. This section specifically models bias against one group and shows how bias can lead to discrimination affecting job segregation and the wage gap.

It is worth noting that the basic model allows for bias against the minority group, as long as there is no heterogeneity in the two firms in terms of their bias. Specifically, we allow the distribution of productivities for $A-$group workers to be different from that of $B-$group workers workers. By interpreting productivity of $B-$group workers as net of the disutility that (a manager of) the firm incurs when hiring a $B-$group worker, then the model becomes one without any explicit disutility term associated with $B-$group workers, while their productivity distributions are shifted to reflect the effect of the disutility for the firms.

In this subsection, we consider the case in which one firm (let it be firm 1) has biased preferences while the other firm (firm 2) is purely profit motivated.[42] Specifically, firm 1 incurs a constant per-worker disutility $\lambda \in (0,1)$ for hiring a $B-$group worker, while firm 2 does not experience any disutility from hiring a $B-$group worker. We refer to firm 1 as having "biased" preferences. All other model preliminaries–including the definitions of outcome, core outcome, and EPSW–continue to hold without modification. We present results in a fixed labor market $(\lambda, \beta, F_A, F_B)$, except where explicitly stated otherwise.

We show that the main predictions of the basic models remain largely unchanged, though with some subtle changes.

First, consider the case without EPSW. In this case, in any core outcome, almost all $B-$group workers are hired by firm 2, with the wage of the worker of productivity v falling in $[\max\{0, v-\lambda\}, v]$; each $A-$group worker can be hired by either firm, and a worker with productivity v receives wage equal to v. Any such profile, with the restriction that the wages for $B-$group workers is nondecreasing in productivity, constitutes a core outcome.

[42] Per the discussion of the last paragraph, one can also interpret the model as cases in which both firms have bias, but one of the firms incurs larger disutility from hiring a $B-$group worker than the other.

Next, we analyze core outcomes when there is a group-based EPSW.[43] Our first observation is that group-based segregation continues to hold as in our base model, and in addition, $B-$group workers are hired by firm 2 due to firm 1's biased preferences.

PROPOSITION **12.** *In any core outcome under group-based EPSW, firms completely segregate. Specifically, firm 1 hires almost all $A-$group workers and firm 2 hires almost all $B-$group workers (formally, $f_A^1(v)=f_A(v)$ for almost all $v\in[0,1]$ and $f_B^2(v)=f_B(v)$ for almost all $v\in[0,1]$).*

Proof. Suppose not for contradiction, and let O be a core outcome which violates the conditions in the statement of the proposition. Then by the same arguments as in the "non-bias" base model, each firm $i\in\{1,2\}$ sets common wages $w_i(v):=w_i$ for all v such that $f^i(v)>0$, where $w_1\neq w_2$ and $w_1,w_2\in[0,1)$. Note that for all i, all g and almost all v, it is the case that either $f_g^i(v)=f_g(v)$ or $f_g^i(v)=0$. In what follows, for $g\in\{A,B\}$ let $S_i^g\subset[0,1]$ represent the set of worker productivities for which $f_g^i(v)=f_g(v)$. Then it must be that

$$\pi_1^O=\beta\int_{S_1^A}(v-w_1)f_A(v)dv+\int_{S_1^B}(v-\lambda-w_1)f_B(v)dv,$$
$$\pi_2^O=\beta\int_{S_2^A}(v-w_2)f_A(v)dv+\int_{S_2^B}(v-w_2)f_B(v)dv.$$

For each worker employed in the proposed core outcome ofproductivity v and group $g\in\{A,B\}$, let $w^g(v)$ represent the wage received in the core outcome. Then the total surplus from employed $A-$ and $B-$group workers without accounting for disutility due to taste-based bias is, respectively:

$$\pi_A:=\beta\int_{S_1^A\cup S_2^A}(v-w^A(v))f_A(v)dv,$$
$$\pi_B:=\int_{S_1^B\cup S_2^B}(v-w^B(v))f_B(v)dv.$$

The previous profit conditions mechanically lead to the following "adding up" requirement:

$$\pi_1^O+\pi_2^O\leq\pi_A+\pi_B. \tag{61}$$

Since O is a core outcome, it must be that no firm i can block O via an outcome $\tilde{O}_i$ such that for some g and almost all v, $\tilde{f}_g^i(v)=0$, that is, O must prevent firms from segregating. This implies the following inequalities

$$\begin{aligned}\pi_2^O&\geq\pi_A,\\ \pi_2^O&\geq\pi_B,\\ \pi_1^O&\geq\pi_A.\end{aligned} \tag{62}$$

It must be that $\pi_1^O+\pi_2^O=\pi_A+\pi_B$: From (62) we have $\pi_1^O\geq\pi_A$ and $\pi_2^O\geq\pi_B$ implying $\pi_1^O+\pi_2^O\geq\pi_A+\pi_B$, and from (61) have that $\pi_1^O+\pi_2^O\leq\pi_A+\pi_B$. Because firm 1 suffers a

[43]We do not present results on core outcomes with non-group-based EPSW, but the main intuitions from Item 3 carry over to the case with biased preferences, albeit the full characterization of all core outcomes is somewhat more complicated.

disutility from hiring $B-$group workers, satisfying this equality implies that firm 2 hires almost all $B-$group workers, and in order to satisfy (62) (specifically $\pi_1^O \geq \pi_A$) it must be that firm 2 hires almost no $A-$group workers. □

Following the previous result, we assume that every core outcome exhibits full segregation by group. Therefore, we assume in any core outcome O, without loss of generality, that firm 1 hires all $A-$group workers (specifically $f_A^1(v)=f_A(v)$ for all v) and firm 2 hires all $B-$group workers (specifically $f_B^2(v)=f_B(v)$ for all v). Similarly, for $i\in\{1,2\}$ let $w_i(v)$ represents the wages paid by firm i to the hired group of workers. By Remark 3, it suffices to consider $w_i(\cdot):[0,1]\to[0,1]$ for each $i\in\{1,2\}$ in any core outcome. As before, we refer to $AW_A^O - AW_B^O$ as the *wage gap* in outcome O.

Now we present analogues of Parts 1 and 2 of Proposition 3.[44]

PROPOSITION **13.**

1. *Suppose there is a group-based EPSW. Then there exist a continuum of non-equivalent core outcomes.*

2. *Let $\beta>1$. For any wage gap that results from a core outcome without EPSW, there exists a core outcome under group-based EPSW with the same wage gap. For any wage gap that results from a core outcome under group-based EPSW but not without EPSW, that wage gap is strictly larger than any wage gap that can result from a core outcome without EPSW.*[45] *There exist a continuum of core outcomes with group-based EPSW with a wage gap strictly larger than any wage gap in a core outcome without EPSW.*

Proof of Part 1. We argue that any core outcome O in our base model (i.e. in the absence of biased preferences) continues to be a core outcome in the case with biased preferences. To see this point, observe that

1. Noting that firm 1 hires almost all $A-$group workers and firm 2 hires almost all $B-$group workers in any core outcome, it must be the case that each firm i's profit π_i^O is unchanged by the presence of biased preferences. Similarly, any candidate blocking outcome for firm 2, $\tilde{O}_2$, will yield the same profit as in the game without biased preferences. However, due to biased preferences, any candidate blocking outcome for firm 1, $\tilde{O}_1$, will yield weakly lower profit than in the game without biased preferences. Therefore, the set of core outcomes under biased preferences is weakly larger than the set of core outcomes without biased preferences.

2. The proposed wage profile satisfies the nondecreasingness requirement because it is part of a wage profile in a core outcome without biased preferences.

This observation, combined with part 1 of Proposition 3, shows the desired result. □

Proof of Part 2. First consider the following outcome $\breve{O}$, which is a core outcome without EPSW:

$$\breve{f}_A^1(v):=f_A(v) \text{ for all } v\in V, \qquad \breve{w}_1(v):=v \text{ for all } v\in V,$$
$$\breve{f}_B^2(v):=f_B(v) \text{ for all } v\in V, \qquad \breve{w}_2(v):=\max\{0,v-\lambda\} \text{ for all } v\in V,$$

[44]We do not offer an analogue of Part 3 of Proposition 3. The reason is that, because the two firms' profits do not need to be equal to each other in the presence of biased preferences, there is no single number that represents the firms' common profit on which the proposition's statement is based on.

[45]In particular, the set of wage caps supported across core outcomes under group-based EPSW dominates the counterpart without EPSW in strong set order.

and let the wage gap in outcome $\breve{O}$ be denoted $\breve{G}$. Note that $\breve{O}$ is also an outcome under group-based EPSW, and because group-based EPSW places more restrictions on allowable blocking outcomes, it is also the case that $\breve{O}$ is a core outcome under group-based EPSW.

LEMMA **10.** *The set of possible wage gaps across all core outcomes without EPSW is equal to* $[\mathbb{E}_A[v]-\mathbb{E}_B[v],\breve{G}]$.

Proof. As argued earlier, in any core outcome, almost all $B-$group workers are hired by firm 2, with the wage of the worker of productivity v falling in $[\max\{0,v-\lambda\},v]$, each $A-$group worker can be hired by either firm, and the worker with productivity v receives the wage of v; any such profile, with the restriction that the wage for $B-$group workers is nondecreasing constitutes a core outcome. Thus, the wage gap $AW_A^O-AW_B^O\in[\mathbb{E}_A[v]-\mathbb{E}_B[v],\breve{G}]$ for any core outcome O.

Now, consider the following class of core outcomes $O^{\bar{v}}$ parameterized by $\bar{v}\in[0,1]$:

$$f_A^1(v):=f_A(v) \text{ for all } v\in V, \qquad w_1(v):=v \text{ for all } v\in V,$$
$$f_B^2(v):=f_B(v) \text{ for all } v\in V, \qquad w_2^{\bar{v}}(v):=\begin{cases}\max\{0,v-\lambda\} & \text{if } v\in[0,\bar{v}]\\ v & \text{otherwise.}\end{cases}$$

Note that, as argued earlier, any profile in this class indeed constitutes a core outcome. We observe that the wage gap is $\mathbb{E}_A[v]-\mathbb{E}_B[v]$ for $\bar{v}=0$ and $\breve{G}$ for $\bar{v}=1$, and the wage gap is a continuous function of $\bar{v}$. Therefore, by the intermediate value theorem, any wage gap in $[\mathbb{E}_A[v]-\mathbb{E}_B[v],\breve{G}]$ is achieved in a core outcome. □

Recall that, as explained earlier in this proof, any core outcome in the case without biased preferences continues to be a core outcome in the case with biased preferences. This implies that for any wage gap that results from a core outcome without EPSW, there exists a core outcome under group-based EPSW with the same wage gap.

Next, we will show that for any wage gap that results from a core outcome under group-based EPSW but not without EPSW, that wage gap is strictly larger than any wage gap that can result from a core outcome without EPSW. Because the set of wage gaps across all core outcomes without EPSW is given as an interval $[\mathbb{E}_A[v]-\mathbb{E}_B[v],\breve{G}]$ by Claim 10, it suffices to show that the wage gap at any core outcome with group-based EPSW is at least $\mathbb{E}_A[v]-\mathbb{E}_B[v]$.

To show this, consider a core outcome O under a group-based EPSW. Recall from Proposition 12 that firm 1 hires almost all $A-$group workers while firm 2 hires almost all $B-$group workers in O. Therefore, $\pi_2^O=\mathbb{E}_B(v)-AW_B^O$, and it can achieve profit arbitrarily close to $\beta[\mathbb{E}_A(v)-AW_A^O]$ by poaching all $A-$group workers while firing all $B-$group workers. Thus, the requirement that firm 2 cannot block O requires

$$\beta[\mathbb{E}_A(v)-AW_A^O]\le\mathbb{E}_B(v)-AW_B^O.$$

Rearranging terms, we obtain

$$AW_A^O-AW_B^O\ge\mathbb{E}_A(v)-\mathbb{E}_B(v)+(\beta-1)(\mathbb{E}_A(v)-AW_A^O). \tag{63}$$

By individual rationality, $w_1(v)\le v$ for almost all v which implies that $\mathbb{E}_A(v)\ge AW_A^O$. This inequality, (63), and the assumption that $\beta>1$ imply $AW_A^O-AW_B^O\ge\mathbb{E}_A(v)-\mathbb{E}_B(v)$, as desired.

Finally, we will show that there exist a continuum of core outcomes under group-based EPSW each with a wage gap strictly larger than the wage gap in any core outcome without EPSW. To do so, recall that the outcome $\breve{O}$ is the core outcome without EPSW that results in the largest wage gap $\breve{G}$.

Now consider outcome $O^{\bar{v}_1,\bar{v}_2}$ parameterized by $\bar{v}_1,\bar{v}_2 \in [0,1]$ such that

$$f_A^1(v) := f_A(v) \text{ for all } v \in V, \qquad w_1^{\bar{v}_1}(v) = \begin{cases} \breve{w}_1(v), \text{ for } v \leq \bar{v}_1 \\ \breve{w}_1(\bar{v}_1), \text{ for } v > \bar{v}_1 \end{cases},$$

$$f_B^2(v) := f_B(v) \text{ for all } v \in V, \qquad w_2^{\bar{v}_2}(v) = \begin{cases} \breve{w}_2(v), \text{ for } v \leq \bar{v}_2 \\ \breve{w}_2(\bar{v}_2), \text{ for } v > \bar{v}_2 \end{cases}.$$

Given $\bar{v}_1$, set $\bar{v}_2$ such that

$$\beta \int_{\bar{v}_1}^{1} [\breve{w}_1(v) - w_1^{\bar{v}_1}(v)] f_A(v) dv = \int_{\bar{v}_2}^{1} [\breve{w}_2(v) - w_2^{\bar{v}_2}(v)] f_B(v) dv. \tag{64}$$

Note that for any $\bar{v}_1 < 1$ that is sufficiently close to 1, there exists $\bar{v}_2$ that satisfies (64), and the corresponding outcome $O^{\bar{v}_1,\bar{v}_2}$ is in the core.[46] The wage gap in outcome $O^{\bar{v}_1,\bar{v}_2}$ is

$$\begin{aligned}
AW_A^{O^{\bar{v}_1,\bar{v}_2}} - AW_B^{O^{\bar{v}_1,\bar{v}_2}} :=& \int_0^1 w_1^{\bar{v}_1}(v) f_A(v) dv - \int_0^1 w_2^{\bar{v}_2}(v) f_B(v) dv \\
=& \int_0^{\bar{v}_1} \breve{w}_1(v) f_A(v) dv + \int_{\bar{v}_1}^1 w_1^{\bar{v}_1}(v) f_A(v) dv - \left[\int_0^{\bar{v}_2} \breve{w}_2(v) f_B(v) dv + \int_{\bar{v}_2}^1 w_2^{\bar{v}_2}(v) f_B(v) dv \right] \\
=& \int_0^1 \breve{w}_1(v) f_A(v) dv - \int_{\bar{v}_1}^1 [\breve{w}_1(v) - w_1^{\bar{v}_1}(v)] f_A(v) dv - \left[\int_0^1 \breve{w}_2(v) f_B(v) dv - \int_{\bar{v}_2}^1 [\breve{w}_2(v) - w_2^{\bar{v}_2}(v)] f_B(v) dv \right] \\
=& \breve{G} - \int_{\bar{v}_1}^1 [\breve{w}_1(v) - w_1^{\bar{v}_1}(v)] f_A(v) dv + \int_{\bar{v}_2}^1 [\breve{w}_2(v) - w_2^{\bar{v}_2}(v)] f_B(v) dv \\
>& \breve{G} - \beta \int_{\bar{v}_1}^1 [\breve{w}_1(v) - w_1^{\bar{v}_1}(v)] f_A(v) dv + \int_{\bar{v}_2}^1 [\breve{w}_2(v) - w_2^{\bar{v}_2}(v)] f_B(v) dv \\
=& \breve{G},
\end{aligned}$$

as desired, where the second equality follows from definitions of $w_1^{\bar{v}_1}$ and $w_2^{\bar{v}_2}$ (specifically, $w_1^{\bar{v}_1}(v) = \breve{w}_1(v)$ for $v \leq \bar{v}_1$ and $w_2^{\bar{v}_2}(v) = \breve{w}_2(v)$ for $v \leq \bar{v}_2$), the third equality from simple manipulations of terms, the fourth equality from the definition of $\breve{G}$, the inequality from $\beta > 1$ and $\breve{w}_1(v) - w_1^{\bar{v}_1}(v) > 0$ for all $v > \bar{v}_1$, and the last equality from (64). □

The following analogue of Proposition 4 continues to hold under biased preferences, with the proof largely unchanged.

Proposition **14.** *Suppose there is a group-based EPSW and fix* λ, F_A, *and* F_B *arbitrarily. For any* $\delta > 0$ *there exists* $\beta^* \in [1,\infty)$ *such that in the market* (β, F_A, F_B) *with any* $\beta > \beta^*$, $w_1(v) > v - \delta$ *for all* v *in any core outcome.*

[46] We sketch the proof of the claim that $O^{\bar{v}_1,\bar{v}_2}$ is a core outcome: First, the wage functions $w_1^{\bar{v}_1}$ and $w_2^{\bar{v}_2}$ are non-decreasing by construction. Second, note that the profits of firms 1 and 2 under $w_1^{\bar{v}_1}$ and $w_2^{\bar{v}_2}$ are larger than those under $\breve{w}_1$ and $\breve{w}_2$ by the same amount. This observation and the fact that $\breve{O}$ is a core outcome, and therefore there is no blocking outcome for either firm imply that neither firm profits by firing all of its existing employees and poaching the employees who are currently hired by the other firm. Therefore, the only remaining class of blocking outcomes to consider are those in which one firm desegregates by hiring (a positive measure of) workers of both groups, and therefore pays a common wage to all workers it hires. To see that no firm profits by desegregating, note that compared to $\breve{w}_1$ and $\breve{w}_2$, we are flattening the wages only at the top, namely for $[\bar{v}_1,1]$ and $[\bar{v}_2,1]$. Hence, for any $\bar{v}_1$ and $\bar{v}_2$ that are sufficiently close to 1, there are no such blocking outcomes. Finally, note that $w_1^{\bar{v}_1}(v) \leq \breve{w}_1(v)$ for all v and $w_2^{\bar{v}_2}(v) \leq \breve{w}_2(v)$ for all v implies that $w_1^{\bar{v}_1}(v) \leq v$ for all v and $w_2^{\bar{v}_2}(v) \leq v$, satisfying Individual Rationality. This completes the sketch of the proof.

Finally, an analogue of Proposition 5 holds with the proof unchanged, noting that any core outcome without biased preferences in which firm 1 hires almost no $B-$group workers continues to be a core outcome with biased preferences.

PROPOSITION **15.** *Suppose there is a group-based EPSW and fix* λ, F_A, *and* F_B *arbitrarily. Let* $w_2(\cdot)$ *be an arbitrary wage function such that* $w_2(v) \leq v$ *for all* $v \in [0,1)$. *There exists* $\beta^* \in [0,\infty)$ *such that in the market* (β, F_A, F_B) *with any* $\beta > \beta^*$, *there exists a core outcome in which the wage schedule of* $B-$*group workers is given by* $w_2(\cdot)$.

B.4 Other Types of Heterogeneity

In addition to difference in firms' bias toward minority studied in the last subsection, one could study different kinds of heterogeneity. As we will see below, however, there does not appear to be much one could unambiguously establish in models with a general form of heterogeneity.

To be more specific, consider a model in which the firms are different in their productivity. Specifically, a match between firm 1 and a worker of productivity v creates a per-worker surplus of cv where $c > 1$ is a constant, while firm 2's per-worker surplus is v as before.

Suppose that there is no EPSW. Then, in any core outcome almost all workers are hired by firm 1 and the wage for a worker with productivity v is in $[v, cv]$. Together with the restriction that wages are nondecreasing in v, any such outcome constitutes a core outcome.

Now, suppose that there is a group-based EPSW. Consider the case with a large c. Then there is a core outcome such that, for some $w \in (0,1)$, firm 2 hires all workers with productivity in $[0,w]$ at a common wage 0, while firm 1 hires all workers in $[w,1]$ at a common wage w. In this core outcome, the proportion of $A-$group workers in the workforce at each firm could be higher or lower than the one at firm 1 without EPSW depending on the distribution of worker types. Given that the proportion of $A-$ and $B-$group workers at firm 1 without EPSW is exactly identical to the population average (while firm 2 hires no worker), there is a sense in which job segregation is weakly higher under group-based EPSW. Beyond that, however, there appear to be few general predictions, if any. Specifically, consider wages in core outcomes. The average wages of $A-$ and $B-$ group workers is exactly equal to each other if f_A and f_B are identical, so the wage gap under group-based EPSW could be strictly smaller than the gap at a core outcome without EPSW (note that some core outcomes without EPSW feature positive wage gap). Meanwhile, for some productivity distributions, a core outcome under group-based EPSW can feature a positive wage gap, and the wage gap under group-based EPSW can be strictly larger than that at a core outcome without EPSW. Generally, we cannot make a sharp comparison of wage gaps with and without group-based EPSW, e.g., the "strong set order" comparison does not generally hold.

Those observations offer a sense in which a kind of "anything goes" result holds, and that no definite conclusion is possible in a model with general forms of heterogeneity. The only robust prediction we find is that employment weakly decreases with EPSW.

C Non-cooperative Game Formulation

In this appendix, we describe a non-cooperative game played by firms and workers. We show that the subgame perfect Nash equilibrium outcomes of this game are isomorphic to the core of the game analyzed in the main text.

The set of players are composed of two firms 1,2 and a continuum of workers. The set of workers are given by $\{A,B\} \times [0,1]^2$. A worker is identified by a tuple $(g, v, ß) \in \{A,B\} \times [0,1]^2$, where g is the group that the worker belongs to, v is her productivity, and ß is an index. For each g, we assume that there is measure μ_g that is given as a product measure of μ_g^{p} and μ_g^{w}. More specifically, let μ_g^{p} be the Lebesgue measure on Lebesgue σ-algebra $\mathcal{B}^{\mathrm{p}}$ on [0,1], representing the measure of productivity.[47] Let μ_g^{w} be a measure on a σ-algebra $\mathcal{B}^{\mathrm{w}}$ on [0,1], representing the measure of workers. We assume that μ_g^{w} is non-atomic. The density function associated with μ_g is given by $f_g(v,ß)=f_g^{\mathrm{p}}(v)\times f_g^{\mathrm{w}}(ß)$, where $f_g^{\mathrm{p}}(v)$ is associated with measure μ_g^{p} and represents the density of $g-$group workers with productivity v while $f_g^{\mathrm{w}}(ß)$ is associated with measure μ_g^{w} and represents the density of workers whose indices are ß.

The game proceeds as follows. In the first period, each firm i simultaneously announces measurable sets of workers $§_i^A$ and $§_i^B$ of groups A and B, respectively, to which it makes job offers, as well as a measurable function w_i^g on $§_i^g$ for each $g\in\{A,B\}$ where $w_i^g(v,ß)$ is the wage that the firm makes to worker $(g,v,ß)$. We assume that $w_i^g(v,ß)\geq w_i^g(v',ß')$ if $v\geq v'$. Each worker observes the identity of the firm that made an offer to her (if any) and the associated wage offered to her and chooses to accept one of the offers or stay unassigned and receive the wage of zero. Then, each firm i is matched to the workers from $§_i^A\cup§_i^B$ who accepted its offers and pays wages to the hired workers according to the offer it made to them.

We assume that each worker is only interested in monetary transfer, so the worker's payoff is equal to the wage paid to her if she accepts an offer from a firm and zero otherwise. Next, we describe the firm's payoff. First consider the case where there is no EPSW. Let $\tilde{§}_i^A$ and $\tilde{§}_i^B$ be the sets of workers from groups A and B, respectively, who accepted firm i's offer. If $\tilde{§}_i^A$ and $\tilde{§}_i^B$ are both measurable, then the firm i obtains the payoff of

$$\beta\int_{\tilde{§}_i^A}[v-w_i^A(v,ß)]d\mu_A+\int_{\tilde{§}_i^B}[v-w_i^B(v,ß)]d\mu_B.$$

If at least one of $\tilde{§}_i^A$ and $\tilde{§}_i^B$ is nonmeasurable, then the firm's payoff is -1.

When there is a group-based EPSW, we modify the firm's payoff such that it receives a payoff of -1 if there are sets of workers with positive measure $\tilde{§}_i^g\subseteq g\times[0,1]^2$ and $\tilde{§}_i^{g'}\subseteq g'\times[0,1]^2$ hired by it such that $g\neq g'$ and $w_i^g(v,ß)\neq w_i^{g'}(v',ß')$ for all $(g,v,ß)\in\tilde{§}_i^g$ and $(g',v',ß')\in\tilde{§}_i^{g'}$. We define a non-group-based EPSW in an analogous manner.

Given an action profile, the associated *outcome* is defined as $O:=(O_1,O_2)$, $O_i=\{(f_g^i(v),w_i^g(v))\}_{v\in[0,1],g=A,B}$ for $i=1,2$, where $f_g^i(v)$ is the density of workers of group g with productivity v who are hired by firm i in equilibrium, and $w_i^g(v)$ be the wage paid to those workers if $f_g^i(v)>0$ and zero otherwise.

Our solution concept is the subgame perfect Nash equilibrium.

PROPOSITION **16.** *The set of subgame perfect Nash equilibrium outcomes of the noncooperative game without EPSW, with group-based EPSW, and with non-group-based EPSW, respectively, coincides with the core of the cooperative game without EPSW, with group-based EPSW, and with non-group-based EPSW, respectively.*

Proof. In this proof, we only consider the case without EPSW: The proof for the case with (group-based and non-group-based) EPSW is analogous and thus omitted.

We first state the following mathematical result, which is a known modification of Sierpiński's theorem on non-atomic measures.[48]

[47] Formally, we define a Borel measure $\tilde{\mu}_g^{\mathrm{p}}$ such that $\tilde{\mu}_g^{\mathrm{p}}([0,x])=F_g(x)$ for all $x\in[0,1]$, which exists and is unique (Royden and Fitzpatrick, 2010, Proposition 25, Section 20.3). Let μ_g^{p} be the unique measure defined on the Lebesgue measurable sets and coincides with $\tilde{\mu}_g^{\mathrm{p}}$ on Borel measurable sets: such μ_g^{p} exists and is unique due to the Caratheodory Extension Theorem and the Hahn Extension Theorem (see Stokey and Lucas, 1989, Theorems 7.3 and 7.2').

[48] See Sierpiński (1922) for the original result, and see `https://en.wikipedia.org/wiki/Atom\_(measure\_theory)` for the modified result we state.

LEMMA **11.** *Suppose that μ_g^w is a non-atomic measure. Let $c=\mu_g^w([0,1])$. Then, there exists a function $\phi:[0,c]\to\mathcal{B}^w$ such that*

$$\mu_g^w(\phi(t))=t \text{ for all } t\in[0,c], \text{ and} \tag{65}$$

$$\phi(t)\subseteq\phi(t') \text{ for all } t,t'\in[0,c] \text{ with } t\le t'. \tag{66}$$

Let $X=[0,1]\times[0,1]$. For $Y\subseteq X$ and $v\in[0,1]$, let $Y_v=\{t\in[0,1]:(v,t)\in Y\}$. Let μ_g denote the product measure induced from μ_g^{p} and μ_g^{w} with the corresponding σ-algebra $\mathcal{B}$. By the product measure theorem, there exists a unique product measure induced from the two measures. Theorem 7.14 of Stokey and Lucas (1989) implies that for any $Y\in\mathcal{B}$ and $v\in[0,1]$, it holds that $Y_v\in\mathcal{B}^{\mathrm{w}}$.

We say that $h:[0,1]\to\mathbb{R}$ is *$\mathcal{B}^p$-measurable* if

$$\{v\in[0,1]:h(v)\le a\}\in\mathcal{B}^{\mathrm{p}} \text{ for all } a\in\mathbb{R}.$$

For $A\subseteq[0,1]$, let $\chi_A:[0,1]\to\{0,1\}$ denote the *indicator function* for A, i.e.,

$$\chi_A(v)=\begin{cases}1 & \text{if } v\in A,\\ 0 & \text{otherwise.}\end{cases}$$

We say that a function mapping $[0,1]$ to $\mathbb{R}$ is *simple* if its range is a finite set. The following useful result is stated as Theorem 7.5 of Stokey and Lucas (1989, p. 180).

LEMMA **12.** *Suppose that $h:[0,1]\to\mathbb{R}_{\ge0}$ is $\mathcal{B}^p$-measurable. Then, there exists a sequence of $\mathcal{B}^p$-measurable simple functions $(h^n)_{n=1}^\infty$ such that*

$$0\le h^n(v)\le h^{n+1}(v)\le h(v) \text{ for all } v\in[0,1] \text{ and } n=1,2,..., \tag{67}$$

$$h^n(v)\to h(v) \text{ (as } n\to\infty) \text{ for all } v\in[0,1]. \tag{68}$$

Now we are ready to state and prove the following result.

LEMMA **13.** *Let $h:[0,1]\to\mathbb{R}_{\ge0}$ be a $\mathcal{B}^p$-measurable function such that $h(v)\le c$ for all $v\in[0,1]$. Then, there exists a subset $Y^*\subseteq X$ such that $Y^*\in\mathcal{B}$ and*

$$\mu_g^w(Y_v^*)=h(v) \text{ for all } v\in[0,1].$$

Proof. By Lemma 12, there exists a sequence of $\mathcal{B}^{\mathrm{p}}$-measurable simple functions $(h^n)_{n=1}^\infty$ that satisfies (67) and (68). For each $n=1,2,...$, since h^n is a simple function, its range consists of a finite number of reals; let $k(n)$ denote the number. Then, there exists a sequence of reals $(a^{n,r})_{r=1}^{k(n)}$ and a sequence of mutually disjoint $\mathcal{B}^{\mathrm{p}}$-measurable sets $(A^{n,r})_{r=1}^{k(n)}$ such that

$$h^n=\sum_{r=1}^{k(n)}a^{n,r}\cdot\chi_{A^{n,r}}.$$

By Lemma 11, there exists a function ϕ that satisfies (65)) and (66). For each $n=1,2,...$, and $r=1,2,...,k(n)$, by (67) and $h(v)\le c$ for all $v\in[0,1]$, it holds that $a^{n,r}\le c$. We define

$$\bar{A}^{n,r}=\phi(a^{n,r}) \text{ for all } n=1,2,..., \text{ and } r=1,2,...,k(n).$$

We define

$$Y^*=\cup_{n=1}^\infty\cup_{r=1}^{k(n)}(A^{n,r}\times\bar{A}^{n,r}).$$

It holds that $A^{n,r}\in\mathcal{B}^{\mathrm{p}}$ (because h^n is $\mathcal{B}^{\mathrm{p}}$-measurable) and $\bar{A}^{n,r}=\phi(a^{n,r})\in\mathcal{B}^{\mathrm{w}}$. Since μ_h is a product measure, $A^{n,r}\times\bar{A}^{n,r}\in\mathcal{B}$. Since Y^* is obtained by taking union of measurable sets countably many times, we have $Y^*\in\mathcal{B}$.

Fix an arbitrary $v\in[0,1]$. Recall that, for any $n=1,2,...$, the subsets $(A^{n,r})_{r=1}^{k(n)}$ are mutually disjoint. Thus, there exists a unique sequence $(A^{n,r(n)})_{n=1}^{\infty}$ such that

$$v\in A^{n,r(n)} \text{ for all } n=1,2,.... \tag{69}$$

By (67), we have $a^{n,r(n)}=h^n(v)\leq h^{n+1}(v)=a^{n+1,r(n+1)}$ for all $n=1,2,....$ Together with (66),

$$\bar{A}^{n,r(n)}=\phi(a^{n,r(n)})\subseteq\phi(a^{n+1,r(n+1)})=\bar{A}^{n+1,r(n+1)} \text{ for all } n=1,2,.... \tag{70}$$

We obtain

$$\begin{aligned}\mu_g^{\mathrm{w}}(Y_v^*)&=\mu_g^{\mathrm{w}}(\cup_{n=1}^{\infty}\bar{A}^{n,r(n)})\\&=\lim_{n\to\infty}\mu_g^{\mathrm{w}}(\bar{A}^{n,r(n)})\\&=\lim_{n\to\infty}a^{n,r(n)}\\&=\lim_{n\to\infty}h^n(v)\\&=h(v),\end{aligned}$$

where the first equality follows from (69), the second equality follows from (70) and the monotone convergence theorem of a measure, the third equality follows from (65), and the fourth equality follows (69), and the fifth equality follows from (68). □

Now we will present the proof of Proposition 16. We first prove that for any core outcome of the cooperative game $O:=\{(f_g^i(v),w_i^g(v))\}_{i\in\{1,2\},v\in[0,1],g=A,B}$, there exists a subgame perfect Nash equilibrium in the noncooperative game whose outcome is O. To show this, note by Proposition 1 that, for almost all i,g and v, we have $f_g^1(v)+f_g^2(v)=f_g(v)$ and $w_i^g(v)=v$ if $f_g^i(v)>0$ in any outcome in the core.

For each g, let $Y_g^*\subseteq\{g\}\times[0,1]^2$ be a measurable set of workers containing, for each v, $f_g^1(v)$ share of workers–note that such sets exist thanks to Lemma 13. Consider the following strategy profile in the non-cooperative game:

1. Both firms make job offers to all workers $(g,v,t)\in\{g\}\times[0,1]^2$ where the wage offer for each worker is equal to her productivity v.

2. All workers accept an offer from the firm whose wage offer is the highest and weakly positive, if any. If they receive the same wage offers from both firms, then the workers in set $Y_A^*\cup Y_B^*$ accept an offer of firm 1 while all other workers accept an offer from firm 2.

It is straightforward to verify that the above strategy profile results in outcome O. To see that this is a subgame perfect Nash equilibrium, first observe that by definition of the workers' strategies, each worker is clearly maximizing their payoffs against all possible strategies of other players. For the firms, there is no other strategy that results in a strictly higher payoff than the payoff of zero under the prescribed strategy profile because all workers are offered a wage that is equal to their own productivity from both firms under the prescribed strategy profile. These observations show that the specified strategy profile constitutes a subgame perfect Nash equilibrium, as desired.

We next prove that any subgame perfect Nash equilibrium outcome of the noncooperative game is in the core of the cooperative game. To show this, assume that the outcome $O := \{(f_g^i(v), w_i^g(v))\}_{v\in[0,1], i=1,2, g=A,B}$ is an outcome associated with a strategy profile of the noncooperative game and is not in the core of the cooperative game. Then we constuct an outcome $\tilde{O}_j = \{(\tilde{f}_g^j(v), \tilde{w}_j^g(v))\}_{v\in[0,1], g=A,B}$ such that the associated strategy profile of the noncooperative game yields a profitable deviation for at least one of the firms. We take $(\tilde{f}_A^j(v), \tilde{w}_j^A(v)) = (f_A^j(v), w_j^A(v))$ (or $\tilde{O}_{-j} = \{(\tilde{f}_g^{-j}(v), \tilde{w}_{-j}^g(v))\}_{v\in[0,1], g=A,B}$ where $(\tilde{f}_A^{-j}(v), \tilde{w}_{-j}^A(v)) = (f_A^{-j}(v), w_{-j}^A(v))$), i.e. we do not change the outcome for $A-$group workers for either firm. The argument in which the deviation involves workers in group $g=A$ is analogous, where terms related to the firm's profit must be multiplied by β. To show a profitable deviation, we consider exhaustive cases as in Lemma 5.

First, suppose there exist a firm j and a subset of productivities $V \subset [0,1]$ with positive measure such that $w_j^g(v) > v$ for all $v \in V$. Then $\tilde{O}_j$ where for all v:

$$\tilde{f}_g^j(v) := \begin{cases} f_g^j(v) & \text{if } v \notin V, \\ 0 & \text{if } v \in V. \end{cases} \qquad \tilde{w}_j^g(v) := \begin{cases} w_j^g(v) & \text{if } v \notin V, \\ 0 & \text{if } v \in V. \end{cases}$$

blocks O as j's profit increases and Condition 4 of the definition of block is satisfied for all $v \in V$ (i.e. the workers in V are fired) and for all $v \in [0,1] \setminus V$ Condition 2 of the definition of block is satisfied (i.e. there is no change in the hiring or wages of workers in $[0,1] \setminus V$). By construction, $\tilde{w}_j^g(v)$ satisfies our monotonicity condition. In the non-cooperative game, consider a deviation of firm j such that it makes an offer to the same set of workers as in the present strategy profile while offering wage of $\min\{w_j^g(v), v\}$ to workers with productivity v (while not changing offers to workers in the other group–this feature is kept for other cases and thus we omit mentioning this for other cases). This will be a profitable deviation, so the said strategy profile is not a subgame perfect Nash equilibrium.

Second, suppose there exist a firm j and a subset of productivities V with positive measure such that $f_g^1(v) + f_g^2(v) < f_g(v)$ and $w_j^g(v) < v$ for all $v \in V$. Then $\tilde{O}_j$ where for all v:

$$\tilde{f}_g^j(v) := f_g(v) - f_g^{-j}(v). \qquad \tilde{w}_j^g(v) := \begin{cases} 0 & \text{if } \tilde{f}_g^j(v) = 0, \\ \sup_{v' \le v} w_j^g(v') & \text{otherwise.} \end{cases}$$

blocks O as firm j's profit increases as some previously unemployed workers are hired at a wage strictly less than their productivity while all existing workers at j continue to be employed at the same wage as before, and Condition 2 of the definition of a block is satisfied for all $v \in [0,1]$ (i.e. no worker receives a wage cut and no workers are poached from firm $-j$). By construction, $\tilde{w}_j^g(v)$ satisfies our monotonicity condition. In the non-cooperative game, consider a deviation of firm j such that it makes an offer to a set of workers $[\tilde{\S}_{-j}^g]^c$, where $\tilde{\S}_{-j}^g$ is the set of the workers that firm $-j$ hires at the original strategy profile and S^c represents the complement of set S; and the wage offer is $\tilde{w}_j^g(v)$ if the worker's productivity is v. It is clear that this deviation results in firm j's outcome $\tilde{O}_j$, making j strictly better off. Therefore, the original strategy profile is not a subgame perfect Nash equilibrium.

Third, suppose there exists a firm j and a subset of productivities V with positive measure such that $f_g^1(v) + f_g^2(v) < f_g(v)$ and $w_j^g(v) = v$ for all $v \in V$. Then, there exists $\varepsilon > 0$ and V' with positive measure such that $f_g^1(v) + f_g^2(v) < f_g(v) - \varepsilon$ and $w_j^g(v) = v$ for all $v \in V'$.[49] Now, arbitrarily fix

[49]The proof is given in Footnote 23.

$p<1$ such that $\bar{f}_g(1-p)\leq p\varepsilon$ and $\frac{1}{2}p^2\varepsilon>\bar{f}_g(1-p)$ (note that those inequalities are satisfied by any sufficiently large $p<1$). By Halmos (1974, Theorem A, Page 68), there exists an interval $I:=[\underline{v},\bar{v}]\subseteq[0,1]$ such that $\mu(V'\cap I)>p\mu(I)$, where $\mu(\cdot)$ is the Lebesgue measure. Then $\tilde{O}_j$ where for all v:

$$\tilde{f}_g^j(v):=\begin{cases} f_g(v)-f_g^j(v)-f_g^{-j}(v) & \text{if } v\in I,\\ f_g^j(v) & \text{otherwise.}\end{cases} \qquad \tilde{w}_j^g(v):=\begin{cases} 0 & \text{if } \tilde{f}_g^j(v)=0,\\ \sup\limits_{v'<\underline{v}} w_j^g(v') & \text{if } v\in I \text{ and } \tilde{f}_g^j(v)>0,\\ w_j^g(v) & \text{otherwise.}\end{cases}$$

blocks O. To see this, note Condition 4 of the definition of block is satisfied for all $v\in I$ (firm j fires all existing workers in set I and hires unemployed workers of the same productivity), and Condition 2 of the definition of block is satisfied for all $v\notin I$ (no worker receives a wage cut and are poached from firm $-j$). By construction, $\tilde{w}_j^g(v)$ satisfies our monotonicity condition.

To see that firm j's profit increases, let $\delta:=\underline{v}-\sup_{v\leq\underline{v}}w_j^g(v)$. Note that the firm makes an additional profit of at least

$$\delta p\mu(I)\varepsilon+\frac{1}{2}(p\mu(I))^2\varepsilon,$$

from hiring workers from set $V'\cap I$ while firing existing workers from $V'\cap I$ causes no loss (because those workers were hired at wages equal to their productivities), and the loss from losing workers from $I\setminus V'$ is bounded from above by $\bar{f}_g[(1-p)\mu(I)\times(\delta+\mu(I))]=\bar{f}_g(1-p)\delta\mu(I)+\bar{f}_g(1-p)\mu(I)^2$. Because p satisfies $\bar{f}_g(1-p)\leq p\varepsilon$ and $\frac{1}{2}p^2\varepsilon>\bar{f}_g(1-p)$ by assumption, the total change of firm j's payoff is strictly positive, as desired. In the non-cooperative game, consider a deviation of firm j such that it makes an offer to a set of workers $([\{g\}\times([0,1]\setminus I)\times[0,1]]\cap\S_j^g)\cup([\{g\}\times I\times[0,1]]\cap[\tilde{\S}_j^g\cup\tilde{\S}_{-j}^g]^c)$; and the wage offer is $\tilde{w}_j^g(v)$ if the worker's productivity is v. It is clear that this deviation results in firm j's outcome $\tilde{O}_j$, and the set of workers hired is a measurable set.[50] Therefore, j is made strictly better off. Therefore, the original strategy profile is not a subgame perfect Nash equilibrium.

Other cases, as enumerated in the proof of Proposition 1, can be treated in a similar manner. □

D Empirical appendix

This appendix presents additional details and results related to our empirical analysis of EPSW.

D.1 Designation of treatment status

We define a treated firm as a firm that employs at least 10 long-term workers at the time of EPSW announcement. In this section, we discuss several reasons lending validity to this choice.

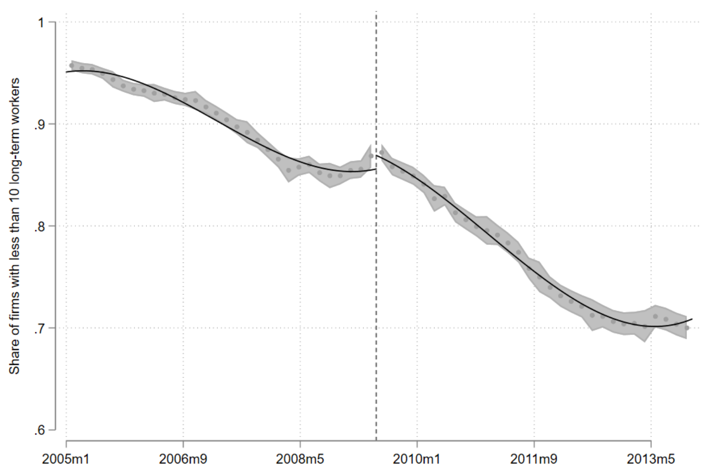
First, as firm size is endogenous, a potential concern is that manipulations in size at the time of EPSW announcement could affect our analysis. As discussed by McCrary (2008), a discontinuity in the share of firms with fewer than 10 long-term workers at the time of announcement suggests that firms may strategically alter their workforce quickly to avoid EPSW, which would mean our designation of treatment would not be valid. Figure 8 investigates this concern by plotting the share of firms in our sample with strictly fewer than 10 long-term workers across time, and overlays a separate best-fit polynomial for the time period before versus after policy announcement. As can

[50] To see this, note that I is measurable by assumption, and taking a countable number of union, intersection, and complement of measurable sets results in a measurable set.

be seen visually, there is a small increase in the share of firms with fewer than 10 long-term workers around the time of policy announcement, but there is no statistically significant discontinuity. The share of firms with fewer than 10 long-term workers averaged over the 6 months leading up to announcement (December 2008–May 2008) is 0.857, and the share of firms with fewer than 10 long-term workers averaged over the 6 months including and following announcement (June 2009–November 2009) is 0.861 (the p-value of the difference between the means is 0.197).

Second, would an alternative time, such as policy enactment instead of policy announcement, be more appropriate to denote the "post" period? We believe not. As seen in Figure 8, there is one notable time interval over which the share of firms with strictly fewer than 10 long-term workers increases, and this is centered around policy announcement. This indicates EPSW announcement likely led to anticipatory firm responses. Note that firm size responses are consistent with firms attempting to avoid the bite of the policy, suggesting policy announcement was salient to firms (but as discussed in the previous paragraph, the lack of discontinuity around the announcement date allows us to proceed with our difference-in-difference analysis). No such change is discernible around the time of policy enactment.

Figure 8: Share of Firms with fewer than 10 long-term workers over time

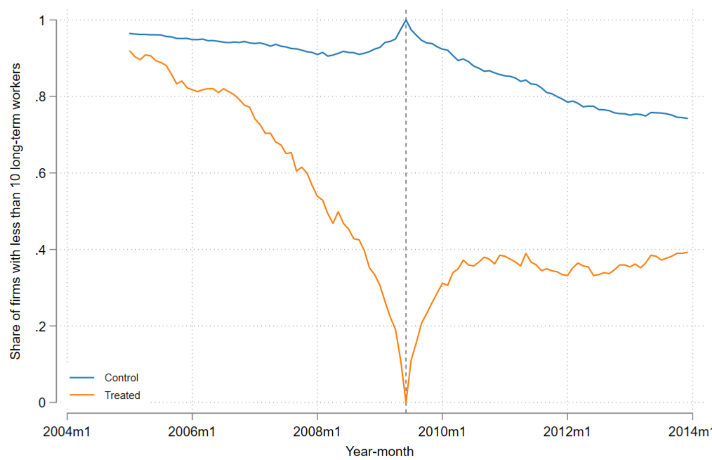


Notes: We display binned means (across time) for the share of firms with fewer than 10 long-term workers in our balanced sample of firms that had between 6 and 13 workers in June 2009, using 30 bins of equal length both before and after June 2009. The fitted line is a 3rd-degree polynomial, fitted on each side of the cutoff. The shaded gray region represents 95% confidence bands for the local means, computed as in Cattaneo and Farrell (2013). Plots were implemented using rdplot from the rdrobust STATA package (See `https://rdpackages.github.io/rdrobust/`).

Third, a concern may be that our definition of treatment using time of announcement may not affect the probability that a firm is bound by EPSW in subsequent months. Figure 9 replicates Figure 8 but generates separate series by firm treatment status. While there is a mechanical mean reversion due to our definition of treatment (i.e. control firms all have fewer than 10 long-term workers at EPSW announcement, and treated firms all have at least 10 long-term workers at EPSW announcement), our treatment variable is positively correlated with a firm being affected by EPSW in future time periods. In all months after announcement, strictly fewer than 25% of firms in our control group are bound by EPSW while strictly more than 60% of firms in our treatment group are bound by EPSW.

Figure 9 also suggests that the magnitudes of our estimates are likely conservative, because some control firms are bound by EPSW and some treated firms are not bound by EPSW in every time period after announcement. An alternative empirical approach would be to instrument for these shares across the two groups. However, such an analysis would require additional assumptions on how a firm's specific history of long-term workers translates into policy bite.[51] Our approach, while conservative, avoids such ad hoc assumptions.

Figure 9: Share of firms affected by the policy over time and by treatment status



Notes: This figure includes all firm in our balanced sample of firms that employ between 6 and 13 workers in June 2009. It displays the monthly share of firms that employ fewer than 10 long-term workers by treatment status. Treatment is defined as having at least 10 long-term workers in June 2009.

Fourth, one may worry that there is an alternative reason for the effects that we observe. We are unaware of any contemporaneous labor-market policy that differentially affects firms in our treatment and control groups. However, it is possible that other labor-market forces could be differentially affecting firms with more long-term workers. To assuage this concern, we run several "placebo" tests. Specifically, we rerun our analyses in (5) and (7) but instead suppose the treatment cutoff for number of long-term workers at a firm varies from 10. If the true cause for the effects we observe in our baseline specification is the policy itself, we should expect little to no effects around these placebo cutoffs for firms with the same actual treatment status according to the real policy. For each cutoff c we consider, we construct a sample of firms that have between $[c-4,c+3]$ workers at the time of EPSW announcement, excluding firms with fewer than 10 long-term workers at announcement as they are not treated by the actual policy. For each cutoff c, firms are considered treated if and only if they employ at least c long-term workers at announcement. All other details are as in our baseline specification. Results are presented below in Tables 5 and 6. The alternative cutoffs we consider do not lead to statistically significant effects, and point estimates are often close to zero or reversed in sign from our baseline estimates, suggesting that the effects we observe in our analysis are not due to other factors that differentially affect firms with different numbers of long-term workers.

[51] For example, consider a firm j that employs strictly fewer than 10 long-term workers at some time t, but at least 10 long-term workers for time periods $t-10,...,t-1$. Consider another firm i that employs strictly fewer than 10 long-term workers in times $t-10,...,t-1$, but at least 10 long-term workers in time period t. It is not

Table 5: Effect of EPSW on Segregation–Placebos

	(I) Baseline - 6 to 13	(II) 11 to 18	(III) 16 to 23	(IV) 21 to 28	(V) 26 to 33
($\hat{\beta}^{seg}$) Post × Treated	0.0455***	-0.0098	0.0254	0.0173	0.0017
	(0.0171)	(0.0253)	(0.0325)	(0.0302)	(0.0412)
Number of Firms	2,638	599	438	313	220
Fixed effects					
Firm	Yes	Yes	Yes	Yes	Yes
Month × industry × county	Yes	Yes	Yes	Yes	Yes
Firm-month level controls	Yes	Yes	Yes	Yes	Yes

Notes: In this table we display estimated coefficients for the difference-in-differences regression described in (5). Column I presents our baseline specification for our balanced sample with firms that employed between 6 to 13 workers at June 2009, and corresponds to column I of Table 2. Subsequent columns are identical except they consider placebo cutoffs in the number of long-term workers $c \in \{15,20,25,30\}$. For each cutoff c we consider, we construct a sample of firms that have between $[c-4, c+3]$ workers at the time of EPSW announcement, excluding firms with fewer than 10-long term workers at announcement which are treated by the actual policy. For each cutoff c we consider, firms are considered treated if they employ at least c long-term workers at EPSW announcement, and are considered control otherwise. Throughout, standard errors in parentheses are two-way clustered at the firm and month levels. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

D.2 Alternative firm comparison groups

In this section, we re-estimate our baseline specifications with different firm comparison groups, which yields time trends that are more aggregated.

Table 7 presents estimates from (5). The specifications in columns I–V differ in the firm comparison groups $k(j)$, and column VI additionally includes controls. Across specifications, we find a 2.6–4.6 percentage point increase in segregation due to EPSW, from a baseline of 31% of firms that were fully segregated at EPSW announcement. These findings are statistically significant at conventional levels.

Table 8 presents estimates from (5) with the "near segregation" outcome variable as in Table 3. The specifications in columns I–V differ in the firm comparison groups $k(j)$, and column VI additionally includes controls. Across specifications, we find a 3.7–5.1 percentage point decrease in near segregation due to EPSW. These findings are statistically significant at conventional levels.

For completeness, we present estimates from (7) with time-varying fixed effects at more aggregated levels than the industry-county. Recalling that our analysis designates local labor markets as being male or female majority based on the share of male workers in a particular industry-county, we note that the time trends of these additional specifications are somewhat misaligned with this earlier designation. Table 9 presents estimates from (7). The specifications in columns I–V differ in the firms included in comparison group $k(ij)$, and column VI additionally includes controls. In each specification, increases (in favor of men) in the gender wage gap are statistically significant at conventional levels *in male majority labor markets.* Across specifications, our results indicate a 2.9–3.9 percentage point increase in the gender wage gap in treated firms following EPSW in these labor markets. However, there is a negative differential effect of EPSW on the wage gap in female majority labor markets. Across specifications, we find a 3.6–9.3 percentage point lower differential effect of EPSW on the wage gap in female majority labor markets. This effect is statistically significant at conventional levels except in our simplest specification where all

obvious, given potential wage rigidities, which of these two firms is more affected by EPSW in period t.

Table 6: Effect of EPSW on Gender Wage Gap, by Majority Worker Group–Placebos

	(I) Baseline - 6 to 13	(II) 11 to 18	(III) 16 to 23	(IV) 21 to 28	(V) 26 to 33
($\hat{\beta}^{Mgap}$) Treated × Male × Post	0.0378***	0.0028	0.0094	-0.0314	-0.0060
	(0.0143)	(0.0194)	(0.0193)	(0.0202)	(0.0168)
($\hat{\beta}^{Fgap}$) Treated × Male × Post	-0.0902***	0.0277	0.0063	-0.0078	0.0010
× Female Majority Labor Market	(0.0233)	(0.0399)	(0.0308)	(0.0286)	(0.0281)
($\hat{\beta}^{Mgap}$+$\hat{\beta}^{Fgap}$) Effect in Female	-0.0524**	0.0305	0.0156	-0.0392	-0.0050
Majority Labor Market	(0.0217)	(0.0427)	(0.0281)	(0.0245)	(0.0250)
Number of Firms	3,168	843	684	535	378
Number of Observations	3,333,272	1,361,869	1,487,488	1,401,396	1,202,162
Fixed effects					
Firm	Yes	Yes	Yes	Yes	Yes
Worker	Yes	Yes	Yes	Yes	Yes
Worker age	Yes	Yes	Yes	Yes	Yes
Month × industry × county × tertiary education × contract type × worker age	Yes	Yes	Yes	Yes	Yes
Firm-month level controls	Yes	Yes	Yes	Yes	Yes
Worker-month level controls	Yes	Yes	Yes	Yes	Yes

Notes: In this table we display estimated coefficients for the difference-in-differences regression described in (7). Column I presents our baseline specification for our balanced sample with firms that employed between 6 to 13 workers at June 2009, and corresponds to column I of Table 4. Subsequent columns are identical except they consider placebo cutoffs in the number of long-term workers $c \in \{15,20,25,30\}$. For each cutoff c we consider, we construct a sample of firms that have between $[c-4, c+3]$ workers at the time of EPSW announcement, excluding firms with fewer than 10-long term workers at announcement which are treated by the actual policy. For each cutoff c we consider, firms are considered treated if they employ at least c long-term workers at EPSW announcement, and are considered control otherwise. Throughout, standard errors in parentheses are two-way clustered at the firm and month levels. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

firms are in the same comparison group. The overall effect of EPSW on the wage gap in female majority labor markets is the sum of these two estimated coefficients. In all specifications, the point estimate is negative implying that EPSW decreases (in favor of women) the gender wage gap in female majority labor markets. However, we are somewhat underpowered due to the fact that relatively few local labor markets are majority female (see Table 1). The effect of EPSW on the wage gap in majority female labor markets is statistically significant at conventional levels in columns V and VI when we allow time fixed effects to vary at the level of the local labor market (i.e. industry by county) over which we define female versus male majority.

Table 7: Effect of EPSW on Segregation–Alternative Time Trends

	(I)	(II)	(III)	(IV)	(V)	(VI)
($\hat{\beta}^{seg}$) Post × Treated	0.0264*	0.0306*	0.0343**	0.0366**	0.0368**	0.0455***
	(0.0151)	(0.0155)	(0.0150)	(0.0154)	(0.0171)	(0.0171)
Mean Pre-Treatment	0.0056	0.0005	0.0032	-0.0008	-0.0004	-0.0044
	(0.0121)	(0.0123)	(0.0121)	(0.0123)	(0.0141)	(0.0140)
No Pre-Trend p-value	0.5596	0.7177	0.6991	0.7052	0.6262	0.3237
Number of Firms	3,201	3,200	3,148	3,147	2,638	2,638
Number of Observations	345,708	345,600	339,984	339,876	284,904	284,904
Fixed effects						
Firm	Yes	Yes	Yes	Yes	Yes	Yes
Month	Yes	No	No	No	No	No
Month × industry	No	Yes	No	Yes	No	No
Month × county	No	No	Yes	Yes	No	No
Month × industry × county	No	No	No	No	Yes	Yes
Firm-month level controls	No	No	No	No	No	Yes

Notes: This table displays estimated coefficient $\hat{\beta}^{seg}$ for the difference-in-differences regression described in (5). The unit of the panel is the firm-month and the dependent variable is a binary variable that indicates whether all workers at the firm in question are of a single gender in a given month. In June 2009, the share of firms in our sample with all workers of a single gender was 31.21%. Each column presents time-varying fixed effects corresponding to a different comparison group of firms $k(j)$ for each firm j. In column I, all firms are included in the comparison group. In column II firms with the same industry code are included in the comparison group. In column III firms in the same geographic county are included in the comparison group. In column IV, there are two comparison groups: firms in the same geographic county and firms with the same industry code. In column V, the comparison group includes firms that are both in the same county and have the same industry code. Column VI (our baseline specification in column I of Table 2) adds the following firm-month controls to the model in column V: the share of workers younger than the median age in the industry-region, the share of workers with tertiary education, and the share of workers that have long-term contracts. The mean pre-treatment effect is the mean of $\hat{\beta}^{seg}_{\tau}$ for $\tau \in \{2005,2006,2007\}$ calculated from (6), and the no pre-trend p-value is derived from a joint F-test that $\beta^{seg}_{\tau} = 0$ for all $\tau \in \{2005,2006,2007\}$. Throughout, standard errors in parentheses are two-way clustered at the firm and month levels. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 8: Effect of EPSW on Near Segregation–Alternative Time Trends

	(I)	(II)	(III)	(IV)	(V)	(VI)
($\hat{\beta}^{nearseg}$) Post × Treated	-0.0371**	-0.0398**	-0.0403**	-0.0413**	-0.0418**	-0.0505***
	(0.0167)	(0.0169)	(0.0165)	(0.0166)	(0.0180)	(0.0179)
Mean Pre-Treatment	-0.0162	-0.0144	-0.0145	-0.0136	-0.0197	-0.0155
	(0.0131)	(0.0130)	(0.0133)	(0.0132)	(0.0147)	(0.0148)
No Pre-Trend p-value	0.0860	0.1878	0.2883	0.4108	0.2502	0.2335
Number of Firms	3,201	3,200	3,148	3,147	2,638	2,638
Number of Observations	345,708	345,600	339,984	339,876	284,904	284,904
Fixed effects						
Firm	Yes	Yes	Yes	Yes	Yes	Yes
Month	Yes	No	No	No	No	No
Month × industry	No	Yes	No	Yes	No	No
Month × county	No	No	Yes	Yes	No	No
Month × industry × county	No	No	No	No	Yes	Yes
Firm-month level controls	No	No	No	No	No	Yes

Notes: This table displays estimated coefficient $\hat{\beta}^{nearseg}$ for the difference-in-differences regression described in (5), where the outcome variable is "near" segregation, i.e. the dependent variable equals 1 if and only if the share of the majority gender of workers at firm j at time t is an element of $[.8,1)$. Each column presents time-varying fixed effects corresponding to a different comparison group of firms $k(j)$ for each firm j. In column I, all firms are included in the comparison group. In column II firms with the same industry code are included in the comparison group. In column III firms in the same geographic county are included in the comparison group. In column IV, there are two comparison groups: firms in the same geographic county and firms with the same industry code. In column V, the comparison group includes firms that are both in the same county and have the same industry code. Column VI (our baseline specification in column I of Table 3) adds the following firm-month controls to the model in column V: the share of workers younger than the median age in the industry-region, the share of workers with tertiary education, and the share of workers that have long-term contracts. The mean pre-treatment effect is the mean of $\hat{\beta}_{\tau}^{nearseg}$ for $\tau \in \{2005,2006,2007\}$ calculated from (6), and the no pre-trend p-value is derived from a joint F-test that $\beta_{\tau}^{nearseg}=0$ for all $\tau \in \{2005,2006,2007\}$. Throughout, standard errors in parentheses are two-way clustered at the firm and month levels. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 9: Effect of EPSW on Gender Wage Gap, by Majority Worker Group–Alternative Time Trends

	(I)	(II)	(III)	(IV)	(V)	(VI)
($\hat{\beta}^{Mgap}$) Treated × Male × Post	0.0299*	0.0349**	0.0291**	0.0330**	0.0387***	0.0378***
	(0.0158)	(0.0154)	(0.0146)	(0.0144)	(0.0144)	(0.0143)
($\hat{\beta}^{Fgap}$) Treated × Male × Post	-0.0356	-0.0424*	-0.0493**	-0.0536**	-0.0934***	-0.0902***
× Female Majority Labor Market	(0.0238)	(0.0238)	(0.0223)	(0.0225)	(0.0234)	(0.0233)
($\hat{\beta}^{Mgap}+\hat{\beta}^{Fgap}$) Effect in Female	-0.0057	-0.0075	-0.0202	-0.0207	-0.0547**	-0.0524**
Majority Labor Market	(0.0220)	(0.0221)	(0.0206)	(0.0208)	(0.0216)	(0.0217)
Mean Pre-Treatment	0.0023	-0.0014	-0.0012	-0.0055	-0.0058	-0.0069
(Male Majority Labor Market)	(0.0148)	(0.0141)	(0.0139)	(0.0136)	(0.0129)	(0.0126)
No Pre-Trend p-value	0.9886	0.9137	0.9998	0.9694	0.8361	0.8285
(Male Majority Labor Market)						
Mean Pre-Treatment	0.0011	-0.0000	0.0047	0.0039	0.0104	0.0125
(Female Majority Labor Market)	(0.0247)	(0.0249)	(0.0238)	(0.0238)	(0.0254)	(0.0253)
No Pre-Trend p-value	0.9783	0.9764	0.9069	0.9290	0.8480	0.8427
(Female Majority Labor Markets)						
Number of Firms	3,169	3,169	3,169	3,169	3,168	3,168
Number of Observations	3,477,849	3,477,292	3,455,213	3,454,621	3,333,272	3,333,272
Fixed effects						
Firm	Yes	Yes	Yes	Yes	Yes	Yes
Worker	Yes	Yes	Yes	Yes	Yes	Yes
Worker age	Yes	Yes	Yes	Yes	Yes	Yes
Month × tertiary education × contract type × worker age	Yes	No	No	No	No	No
Month × industry × tertiary education × contract type × worker age	No	Yes	No	Yes	No	No
Month × county × tertiary education × contract type × worker age	No	No	Yes	Yes	No	No
Month × industry × county × tertiary education × contract type × worker age	No	No	No	No	Yes	Yes
Firm-month level controls	No	No	No	No	No	Yes
Worker-firm-month level controls	No	No	No	No	No	Yes

Notes: This table displays estimated coefficients for the regression described in (7). In particular, we present estimates of $\hat{\beta}^{Mgap}$ and $\hat{\beta}^{Fgap}$. The unit of the panel is the worker-firm-month and the dependent variable is the natural logarithm of monthly earnings. Among firms in our sample that employ both male and female workers at EPSW announcement, the within-firm wage gap in favor of men, averaged across firms, is 35.8%. Each column presents a time-varying fixed effect for a different comparison group of firms $k(ij)$ for each worker i and each firm j. All columns include worker comparison groups defined by equivalence across three binary dimensions at time t at firm j: an indicator for tertiary education, an indicator for long-term versus fixed-term contract, and an indicator for being above median age in the particular industry-region in which firm j operates. Columns differentially include firm comparison groups. In column I, all firms are included in the comparison group. In column II firms with the same industry code are included in the comparison group. In column III firms in the same geographic county are included in the comparison group. In column IV, there are two comparison groups: firms in the same geographic county and firms with the same industry code. In column V, the comparison group includes firms that are both in the same county and have the same industry code. Column VI (our baseline specification in column I of Table 4) adds the firm-month level controls (the share of workers younger than the median age in the industry-region, the share of workers with tertiary education, and the share of workers that have long-term contracts) and worker-firm-month level controls (number of months at the firm and an indicator for reaching the earnings truncation threshold) to the model in column V. The mean pre-treatment effects are the mean of $\hat{\beta}_\tau^{Mgap}$ and $\hat{\beta}_\tau^{Mgap}+\hat{\beta}_\tau^{Fgap}$, respectively, for $\tau \in \{2005,2006,2007\}$ calculated from (8), and the no pre-trends p-values are derived from joint F-tests that $\beta_\tau^{Mgap}=0$ and $\beta_\tau^{Mgap}+\beta_\tau^{Fgap}=0$, respectively, for all $\tau \in \{2005,2006,2007\}$. Throughout, standard errors in parentheses are two-way clustered at the firm and month levels. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

D.3 Alternative empirical specifications

Section 5 of the paper presents results from several alternative empirical specifications and alternative samples as robustness checks. Specifically, results from each of these robustness specifications is presented as a column in each of Tables 2, 3, and 4. In this section, we describe each of these robustness specifications.

Removing firm fixed effects

The results from this specification are presented in column II ("No firm FEs") of the aforementioned tables. The sample of firms and workers used is the same as in our main analysis, but we replace the specifications in (5), (6), (7), and (8) with the following, respectively:

$$full_{jt} = \alpha_{k(j)t} + \delta above10_j + \beta^{seg}(above10_j \times post_t) + X_{jt}\Lambda + \epsilon_{jt}$$

$$full_{jt} = \alpha_{k(j)t} + \delta above10_j + \sum_{\tau \in \mathcal{T}} \beta^{seg}_{\tau} D_{jt} + X_{jt}\Lambda + \epsilon_{jt}$$

$$\begin{aligned}
\ln w_{ijt} = \ & \alpha_i + \omega_{it} + \alpha_{k(ij)t} + \delta above10_j + \gamma_1(above10_j \times post_t) + \psi_1(above10_j \times post_t \times femalemaj_{jt}) + \\
& \gamma_2(male_i \times post_t) + \psi_2(male_i \times post_t \times femalemaj_{jt}) + \\
& \gamma_3(above10_j \times male_i) + \psi_3(above10_j \times male_i \times femalemaj_{jt}) + \\
& \beta^{Mgap}(above10_j \times male_i \times post_t) + \beta^{Fgap}(above10_j \times male_i \times post_t \times femalemaj_{jt}) + X_{ijt}\Lambda + \epsilon_{ijt}
\end{aligned}$$

$$\begin{aligned}
\ln w_{ijt} = \ & \alpha_i + \omega_{it} + \alpha_{k(ij)t} + \delta above10_j + \sum_{\tau \in \mathcal{T}} \gamma_{1\tau}(above10_j \times year_\tau) + \sum_{\tau \in \mathcal{T}} \psi_{1\tau}(above10_j \times femalemaj_{jt} \times year_\tau) \\
& + \sum_{\tau \in \mathcal{T}} \gamma_{2\tau}(male_i \times year_\tau) + \sum_{\tau \in \mathcal{T}} \psi_{2\tau}(male_i \times femalemaj_{jt} \times year_\tau) \\
& + \gamma_3(above10_j \times male_i) + \psi_3(above10_j \times male_i \times femalemaj_{jt}) \\
& + \sum_{\tau \in \mathcal{T}} \beta^{Mgap}_{\tau} D^M_{ijt} + \sum_{\tau \in \mathcal{T}} \beta^{Fgap}_{\tau} D^F_{ijt} \\
& + X_{ijt}\Lambda + \epsilon_{ijt}
\end{aligned}$$

Removing controls

The results from this specification are presented in column III ("No controls") of the aforementioned tables. The sample of firms and workers used is the same as in our main analysis, but we remove the vector of firm-month levels controls X_{jt} from (5) and (6), and we remove the vector of worker-firm-month level controls X_{ijt} from (7) and (8).

Unbalanced panel of firms

The results from this specification are presented in column IV ("Unbalanced sample") of the aforementioned tables. The sample of firms (and employed workers) is defined as all firms that are present in the market at EPSW announcement. That is, it includes all firms present in our main analysis, but also those that enter the sample strictly later than January 2005 or exit the sample prior to December 2013. This leads to a total of 6,124 firms.

To control for potential cohort effects for firm exit times, we include exit-date fixed effects interacted with existing time-verying fixed effects, that is, let $\alpha'_{k(j)t}$ ($\alpha'_{k(ij)t}$) be defined as the interaction of $\alpha_{k(j)t}$ ($\alpha_{k(ij)t}$) with an indicator variable that takes value 1 if firm j last appears in our sample at time period t, and zero otherwise. We replace $\alpha_{k(j)t}$ with $\alpha'_{k(j)t}$ in (5) and (6), and we replace $\alpha_{k(ij)t}$ with $\alpha'_{k(ij)t}$ in (7) and (8).

Firm size restrictions

We present four different samples of firms, defined by number of workers at EPSW announcement. Recall that our baseline sample includes firms with between 6 and 13 workers at announcement, accounting respectively for percentiles 39 and 78 of the firm size distribution at announcement, resulting in a total of 3,201 firms. Column V presents a "doughnut hole" sample which excludes all firms with either 9 or 10 workers at announcement from our baseline sample, resulting in a total of 2,789 firms. This sample is to account for the mechanical increase in likelihood that the excluded firms' treatment status does not match whether they are bound by EPSW at any given point in time. Column VI presents a narrower sample which includes firms with between 7 and 12 workers at announcement and excludes all others, accounting for percentiles 48 and 75 respectively in the firm size distribution at announcement, resulting in a total of 2,424 firms. Column VII presents a wider sample which includes firms with between 5 and 14 workers at announcement and excludes all others, accounting for percentiles 28 and 80 respectively in the firm size distribution at announcement, resulting in a total of 3,898 firms.

Our results reanalyze the specifications in (5), (6), (7), and (8) with these alternative samples.

D.4 Effect of EPSW on the overall gender wage gap

In Section 5.6, we estimate the effect of EPSW on the wage gap in male majority versus female majority labor markets. Here, we estimate the effect of EPSW across all labor markets, which can be interpreted as the average effect of EPSW on the total wage gap in Chile.

We consider a panel in which an observation is a worker-firm-month. We estimate triple difference models of the following form:

$$\begin{aligned} \ln w_{ijt} = {} & \alpha_i + \omega_{it} + \alpha_j + \alpha_{k(ij)t} + \gamma_1(above10_j \times post_t) + \gamma_2(male_i \times post_t) \\ & + \gamma_3(above10_j \times male_i) + \beta^{gap}(above10_j \times male_i \times post_t) + X_{ijt}\Lambda + \epsilon_{ijt} \end{aligned} \tag{71}$$

where X_{ijt} is a vector of firm-month level controls and worker-firm-month level controls. The firm-month level controls are the share of workers younger than the median age at the industry-region, share of workers with tertiary education, and the share of workers that have long-term contracts. The worker-firm-month level controls are the number of months at the firm and an indicator for whether the worker's earnings reach the top-coding threshold. $\alpha_{k(ij)t}$ are time fixed effects for workers in set $k(ij)$, where $k(ij)$ is a set of workers including i and a comparison group of workers to i employed at a comparison group of firms to j. Workers comparison groups are defined by equivalence across three binary dimensions at time t at firm j: an indicator for tertiary education, an indicator for long-term versus fixed-term contract, and an indicator for being above median age in the particular industry-region in which firm j operates. Across specifications, we consider finer and finer firm comparison groups defined by firm industry and region. Our coefficient of interest is β^{gap} and we interpret it as the effect of the policy on the (percentage) wage gap between male and female workers. Positive values indicate a relative increase in male wages.

To understand more about the dynamic effects of EPSW, we estimate the following triple difference model year by year, where we omit the year before policy announcement as the reference period, so that the set of years included is $\mathcal{T}=\{2005,2006,2007,2009,...,2013\}$. By construction, each year (indexed by τ) corresponds to twelve time periods (indexed by t).

$$\begin{aligned} \ln w_{ijt} = {} & \alpha_i+\omega_{it}+\alpha_j+\alpha_{k(ij)t}+\sum_{\tau\in\mathcal{T}}\gamma_{1\tau}(above10_j\times year_\tau) \\ & +\sum_{\tau\in\mathcal{T}}\gamma_{2\tau}(male_i\times year_\tau)+\gamma_3(above10_j\times male_i) \\ & +\sum_{\tau\in\mathcal{T}}\beta_\tau^{gap}D_{ijt}+X_{ijt}\Lambda+\epsilon_{ijt} \end{aligned} \tag{72}$$

where D_{ijt} is an indicator that equals 1 in time period t if firm j employs at least 10 long-term workers at the time of policy announcement and i is male, and zero otherwise. β_τ^{gap} is the average difference in log wages between men and women in treated versus control firms in year τ (relative to 2008).

Our identifying assumption is that parallel trends hold between treated and control firms, that is, $\mathbb{E}[\epsilon_{ijt}\cdot D_{ijt}]=0$ for all t (Olden and Møen, 2022). This strategy builds in a partial falsification test, in that we expect coefficient estimates of β_τ^{gap} to be zero for $\tau<2009$.

Tables 10 and 11 present estimates from (71). Table 10 presents specifications detailed in Appendix D.3 and Table 11 presents specifications with various firm comparison groups $k(j)$. In all specifications EPSW increases the gender wage gap (in favor of men), however, the results of some specifications are not statistically significant at conventional levels. Additionally, we present estimates from (72) to support our parallel trends assumption. The mean pre-treatment estimates are small and statistically insignificant, and we cannot reject an F-test that all pre-treatment coefficients are jointly zero.

Figure 10 displays the estimated coefficients of interest from (72). Prior to EPSW, the coefficient of interest is statistically indistinguishable from 0 in all periods. In the first year of EPSW, the wage gap rises by 2.02 percentage points in the treated group compared to the control group (p-value = 0.102) and rises to 2.52 percentage points (p-value = 0.172) by year five of EPSW.

D.5 Details on data samples

We use administrative data from the Chilean unemployment insurance system from January 2005 to December 2013. In our data, an observation is a worker-firm-month. We observe two stratified (by firm size) random samples—a 1% sample and a 3% sample—of firms, and for each sampled firm we observe the entire monthly working history of every worker that was every employed by the sampled firm, regardless of whether the worker remains at the sampled firm or not. Therefore, we observe some workers during time periods they are employed by other, non-sampled firms. We do not directly observe which firms are sampled. For that reason, our dataset includes "incidental firms" for which we do not necessarily observe the entire workforce at any given moment in time. This happens, for example, if a worker from a sampled firm switched to a non-sampled firm.

Because we do not observe the entire workforce for these incidental firms, we do not observe the size (number of workers) for these firms. This naturally leads to a potential concern with our size-based empirical strategy. To address this potential concern, we attempt to filter out the

Table 10: Effect of EPSW on Gender Wage Gap

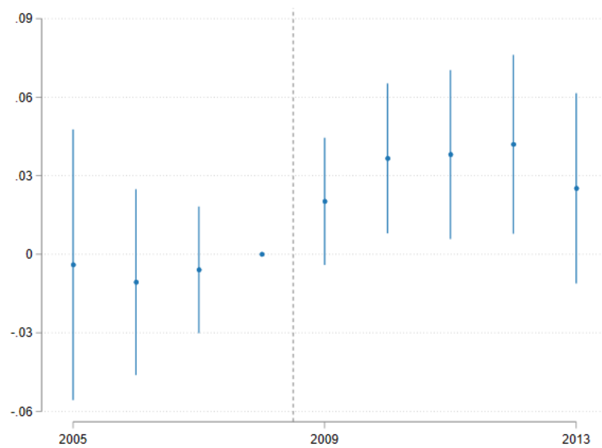
	(I) Baseline	(II) No firm FEs	(III) No controls	(IV) Unbalanced sample	(V) Doughnut hole	(VI) Narrower band	(VII) Wider band
($\hat{\beta}^{gap}$) Treated × Male × Post	0.0257*	0.0236*	0.0264*	0.0140	0.0319*	0.0170	0.0079
	(0.0133)	(0.0128)	(0.0134)	(0.0106)	(0.0179)	(0.0142)	(0.0113)
Mean Pre-Treatment	-0.0051	-0.0025	-0.0048	-0.0146	-0.0019	-0.0200	-0.0132
	(0.0124)	(0.0118)	(0.0126)	(0.0107)	(0.0164)	(0.0142)	(0.0116)
No Pre-Trend p-value	0.9041	0.8934	0.8984	0.3330	0.8190	0.5739	0.5602
Number of Firms	3,201	3,201	3,201	6,120	2,789	2,424	3,898
Number of Observations	3,405,855	3,405,855	3,405,855	5,427,234	2,902,147	2,687,403	4,051,757
Fixed effects							
Firm	Yes	No	Yes	Yes	Yes	Yes	Yes
Worker	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Worker age	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Month × industry × county × tertiary education × contract type × worker age	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Firm-month level controls	Yes	Yes	No	Yes	Yes	Yes	Yes
Worker-month level controls	Yes	Yes	No	Yes	Yes	Yes	Yes

Notes: The unit of analysis is the worker-firm-month and the dependent variable is the natural logarithm of the worker's wage at the firm in a given month. Column I presents estimated coefficients $\hat{\beta}^{gap}$ for our baseline regression specification presented in (71). Time-varying fixed effects are defined as the intersection of: the firm's industry, the firm's county, an indicator for worker tertiary education, an indicator for worker contract type, an indicator for a worker being above median age in the industry-region. Firm-month controls included are: the share of workers younger than the median age in the industry-region, the share of workers with tertiary education, and the share of workers that have long-term contracts. Worker-firm-month levels controls included are the number of months at the firm and an indicator for reaching the earnings truncation threshold. The mean pre-treatment effect is the mean of $\hat{\beta}^{gap}_{\tau}$ for $\tau \in \{2005, 2006, 2007\}$ calculated from (72), and the no pre-trend p-value is derived from a joint F-test that $\beta^{gap}_{\tau} = 0$ for all $\tau \in \{2005, 2006, 2007\}$. Columns II-VII present the analogous information for alternative sample selections and empirical specifications as described in Appendix D.3. Throughout, standard errors in parentheses are two-way clustered at the firm and month levels. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

incidental firms. Our filtering approach is built on the notion that, due to the sampling procedure, we anticipate incidental firms to have high variance in the number (and presence) of workers across time periods.

Our filtering procedure does the following. 1) For each of the 1% and 3% samples, we compute for each firm the first and the last month it is observed. We drop firms that employ no workers for some months in between these two dates in any of the two samples, i.e. if the firm has "holes" in its employment history. 2) For firms in our sample, we compute the monthly average number of workers in each firm in our data and we drop firms that have an average of fewer than 4 in any of the two samples. 3) For each of the firms present in both the 1% and 3% samples, we compute the average number of workers of each firm across time, and we drop firms that have different average numbers of workers in the two samples. Descriptive statistics for the set of firms and workers left after the filtering are presented in column I of Table 1.

Figure 10: Dynamic Path of EPSW's Effect on Gender Wage Gap



Notes: This figure displays estimated coefficients $\hat{\beta}_{\tau}^{gap}$ for the regression described in (72). The specification corresponds to that in column I of Table 10. Bars depict 95% confidence intervals.

Table 11: Effect of EPSW on Gender Wage Gap–Alternative Time Trends

	(I)	(II)	(III)	(IV)	(V)	(VI)
($\hat{\beta}^{gap}$) Treated × Male × Post	0.0269*	0.0298**	0.0234*	0.0262**	0.0264*	0.0257*
	(0.0146)	(0.0143)	(0.0133)	(0.0131)	(0.0134)	(0.0133)
Mean Pre-Treatment	-0.0024	-0.0055	-0.0027	-0.0066	-0.0048	-0.0051
	(0.0139)	(0.0134)	(0.0131)	(0.0128)	(0.0126)	(0.0124)
No Pre-Trend p-value	0.9759	0.8678	0.9842	0.9107	0.8984	0.9041
Number of Firms	3,201	3,201	3,201	3,201	3,201	3,201
Number of Observations	3,559,172	3,558,635	3,535,936	3,535,362	3,405,855	3,405,855
Fixed effects						
Firm	Yes	Yes	Yes	Yes	Yes	Yes
Worker	Yes	Yes	Yes	Yes	Yes	Yes
Worker age	Yes	Yes	Yes	Yes	Yes	Yes
Month × tertiary education × contract type × worker age	Yes	No	No	No	No	No
Month × industry × tertiary education × contract type × worker age	No	Yes	No	Yes	No	No
Month × county × tertiary education × contract type × worker age	No	No	Yes	Yes	No	No
Month × industry × county × tertiary education × contract type × worker age	No	No	No	No	Yes	Yes
Firm-month level controls	No	No	No	No	No	Yes
Worker-month level controls	No	No	No	No	No	Yes

Notes: This table displays estimated coefficients for the regression described in (71). In particular, we present estimates of $\hat{\beta}^{gap}$. The unit of the panel is the worker-firm-month and the dependent variable is the natural logarithm of monthly earnings. Among firms in our sample that employ both male and female workers at EPSW announcement, the within-firm wage gap in favor of men, averaged across firms, is 35.8%. Each column presents a time-varying fixed effect for a different comparison group of firms $k(ij)$ for each worker i and each firm j. All columns include worker comparison groups defined by equivalence across three binary dimensions at time t at firm j: an indicator for tertiary education, an indicator for long-term versus fixed-term contract, and an indicator for being above median age in the particular industry-region in which firm j operates. Columns differentially include firm comparison groups. In column I, all firms are included in the comparison group. In column II firms with the same industry code are included in the comparison group. In column III firms in the same geographic county are included in the comparison group. In column IV, there are two comparison groups: firms in the same geographic county and firms with the same industry code. In column V, the comparison group includes firms that are both in the same county and have the same industry code. Column VI adds the firm-month level controls (the share of workers younger than the median age in the industry-region, the share of workers with tertiary education, and the share of workers that have long-term contracts) and worker-firm-month level controls (number of months at the firm and an indicator for reaching the earnings truncation threshold) to the model in column V. The mean pre-treatment effect is the mean of $\hat{\beta}^{gap}_{\tau}$ for $\tau \in \{2005,2006,2007\}$ calculated from (72), and the no pre-trend p-value is derived from a joint F-test that $\beta^{gap}_{\tau}=0$ for all $\tau \in \{2005,2006,2007\}$. Throughout, standard errors in parentheses are two-way clustered at the firm and month levels. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.