Lecture 4: Beyond Affirmative Action: Targeted Priority Reserve Policies

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Introduction

In Lectures 1-3 we studied various school choice mechanisms with

1. quota/reserve-based affirmative action policies;
2. a new type of affirmative action policy which treats students asymmetrically in terms of their reported preferences.

In this last lecture we

- return to reserve-based policies;
- examine preferential treatment policies and their objectives more generally (instead of focusing only on affirmative action policies);
- provide characterizations of mechanisms with a focus on stability/fairness axioms which reflect different policy objectives.

Overview

Based on **Pápai and Sayedahmed (2022)** [working paper]:

"Targeted Priority Reserve Policies"

- Priority Reserves: Theoretical Background
- Entity Selection and Precedence Order
- Representation vs. Effective Preferential Treatment
- DA-TPR (Targeted Priority Reserves) Mechanism
- Stability Axioms and Results
- Comparing DA-TPR to DA-R
- The Class of DA-SPR (Sequential Priority Reserves) Mechanisms
- Split DA-SPR Mechanisms

Priority Reserves: Theoretical Background

Priority Reserves

- Reserved positions for a group that is to be prioritized, i.e., receive preferential treatment.
- The group may be
  - underrepresented (e.g., some racial minorities)
  - in need of protection (e.g., refugees in emergency zones)
  - prioritized by law (e.g., children living within school walk zone)

What Are the General Aims of Priority Reserves?

- Provide equal opportunity for the underprivileged.
- Give priority to members of groups who have suffered discrimination and thereby right past wrongs.
- Promote diversity.
- Provide help for groups that are in need.
- Implement mandated priority rights.

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- **Not necessarily affirmative action!**

Given the broader context, here we have *"entities"* instead of "schools" and call school choice functions *"entity selection rules."*

Model with Priority Reserves

- Finite set of agents N.
  - Priority group: N^+
  - Non-priority group: $N^0 = N \setminus N^+$
  - $N = N^+ \cup N^0$
- Finite set of entities E. For each entity $e \in E$, the overall capacity is q_e.
  - Reserved positions for agents in the priority group: q_e^r
  - Unreserved positions: q_e^u
  - $q_e = q_e^r + q_e^u$
- For all $i \in N$, strict preferences P_i over $E \cup \{i\}$.
- For all $e \in E$, a strict ranking $\succ_e$ over N.

DA with an Entity Selection Rule: General Definition

- **Step 1:**

  Each agent applies to her top-ranked entity. Each entity tentatively assigns its positions according to its selection rule. Any remaining applicants are rejected.

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- **Step t:**

  Each agent who was rejected in the previous step applies to her next-ranked entity. Each entity considers the agents who are tentatively assigned to the entity, if any, together with its new applicants (the *"applicant pool"*), and tentatively assigns its positions according to its selection rule. Any remaining applicants are rejected.

DA with an Entity Selection Rule

The algorithm terminates when each agent is either tentatively assigned to an entity or has been rejected by each entity that is acceptable to the agent, and thus the agent remains unassigned.

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DA-R is a DA matching mechanism with an entity/school selection rule which aims to implement Affirmative Action with a minority reserve policy in the school choice context.

Hafalir, Yenmez and Yildirim (2013): DA-R Mechanism

First we revisit the preferential treatment policy proposed by Hafalir et al. (2013):

DA-R (DA with Minority Reserves) mechanism

(already introduced and studied in Lecture 1 and Lecture 3).

- **Minority reserve seats:** schools give higher priority to minority students up to the reserved number of seats at each school.
- It is a Deferred Acceptance (DA) algorithm with an entity selection rule.

DA-R Mechanism (Hafalir et al., 2013)

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DA-R entity/school selection rule:

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DA-R entity/school selection rule:

(1a): First each school accepts minority applicants only according to the school's ranking, up to the number of reserved seats.

(1b): Then each school accepts applicants according to their ranking from all remaining applicants, up to its total capacity (including the already accepted minority applicants). The rest of the applicants, if any remain, are rejected by the school.

All acceptances are **tentative.**

The DA-R Mechanism

Step k: Each student who was rejected in step $k-1$ applies to her next-ranked acceptable school (if any remains).

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Each school considers the new applicants together with students matched to the school tentatively in step $k-1$.

The entity/school selection rule is the same from this applicant pool as in Step 1.

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The algorithm terminates when there are no more rejections. The tentative matching in the last step becomes the final matching.

Entity Selection and Precedence Order

DA-R Entity/School Selection Example

Let school c's applicant pool (new applicants and already tentatively matched) be $\{a_1, a_2, a_3, i_1, i_2\}$ in a given step.
Minority students: i_1, i_2.
Minority reserve for school c : $r_c = 1$
Overall capacity for school c : $q_c = 3$
School c's ranking:

$$a_1 \succ_c a_2 \succ_c a_3 \succ_c i_2 \succ_c i_1$$

DA-R accepts i_2 for the one minority reserve position, and then to fill the overall capacity a_1 and a_2 are selected.

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Now consider the following ranking for school c:

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DA-R accepts i_2 for the one minority reserve position, and then to fill the overall capacity a_1 and a_2 are selected.

Note that **the minority reserve policy makes no difference here,** since minority student i_2 takes up the only minority reserve position, and i_2 did not need the preferential treatment.

However, minority student i_1 could make use of a minority reserve position at school c.

Reversing the Entity Selection Precedence Order: the DA-TPR Entity Selection

DA-R: Reserved positions first, then unreserved positions.

E.g. $(\boldsymbol{r}, \boldsymbol{r}, \boldsymbol{r}, u, u, u, u, u)$

We study the **DA with Targeted Priority Reserves (DA-TPR)** mechanism (in the simple model with a partition of the set of agents into prioritized and non-prioritized agents).

DA-TPR: Unreserved positions first, then reserved positions.

E.g. $(u, u, u, u, u, \boldsymbol{r}, \boldsymbol{r}, \boldsymbol{r})$

Comparing DA-TPR and DA-R Entity Selections

Let entity e's applicant pool be $\{a_1, a_2, a_3, i_1, i_2\}$ in a given step.

Priority agents: i_1, i_2

Priority reserve for entity e: $q_e^r = 1$

Overall capacity for entity e: $q_e^u = 3$

Entity e's ranking:

$$a_1 \succ_e i_2 \succ_e a_2 \succ_e i_1 \succ_e a_3.$$

Comparing DA-TPR and DA-R Entity Selections

DA-R:

1: i_2 is accepted for the one **reserved** position

2: a_1 and a_2 are accepted for the two **unreserved** positions

Priority agent i_2, who is accepted on his own right in the DA-TPR, takes the reserved position in the DA-R.

$$a_1 \succ_e i_2 \succ_e a_2 \succ_e i_1 \succ_e a_3.$$

$\Rightarrow$ Only one priority agent is selected by e.

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DA-TPR:

1: a_1 and i_2 are accepted for the two **unreserved** positions

2: i_1 is accepted for the one **reserved** position

Priority agent i_2 is accepted on his own right, and priority agent i_1 is targeted for a priority reserve position.

$$a_1 \succ_e i_2 \succ_e a_2 \succ_e i_1 \succ_e a_3.$$

$\Rightarrow$ Two priority agents are selected by e.

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Changing the Precedence Order

When a preferential treatment policy turns out to select already highly-ranked prioritized agents or more generally does not have the desired impact, it is already understood in practice and has been studied in the literature in practical school assignment contexts that **changing the (so-called) "precedence order"** may be helpful.

For example, Chicago's exam schools started using a reverse precedence order by filling the merit seats first in order to help disadvantaged applicants.

Such issues are examined in more complex models than ours in school choice design by

1. Dur, Kominers, Pathak and Sönmez (2018): "Reserve Design: Unintended Consequences and the Demise of Boston's Walk Zones" (Dur et al., 2018)
2. Dur, Pathak and Sönmez (2020): "Explicit vs. Statistical Targeting in Affirmative Action: Theory and Evidence from Chicago's Exam Schools" (Dur et al., 2020)

Our Contribution

- We carry out an analysis in the **simple model** of Hafalir et al. (2013) introduced by Kojima (2012), as opposed to looking at more complex models with multiple agent types.

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- The **use of stability concepts**, which are central to our analysis.
- We articulate that **different priority reserve policies represent different goals:** representation versus targeted (effective) preferential treatment; we believe that this is an important conceptual difference that is not well recognized.

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- **Theoretical analysis** of normative and incentive properties of **different mechanisms with priority reserve policies in a unified framework** (albeit in a simple model).

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- **Theoretical analysis** of normative and incentive properties of **different mechanisms with priority reserve policies in a unified framework** (albeit in a simple model).
- We provide **characterizations of mechanisms.**

Representation vs. Effective Preferential Treatment

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If **representation** of the priority group is the main goal (i.e., diversity) then the DA-R policy works well.

If the primary goal is *not* representation but **effective preferential treatment**, then DA-R's entity selection rule is less appropriate, especially when not all agents in the priority group need preferential treatment.

DA-TPR (Targeted Priority Reserves) Mechanism

DA-TPR: Targeted Priority Reserves

We study alternative policies to DA-R which prioritize agents using reserves, with the goal of **providing help up to the reserve quota without diversity constraints.**

The DA-TPR mechanism **effectively helps to prioritize agents** in the priority group who may not be matched otherwise to entities that are highly ranked by them.

DA-TPR: Definition

Step 1: Each agent applies to her top-ranked entity. Each entity $e \in E$ selects among all the applicants according to the following substeps:

(1a): Choose the highest-ranked agents in N up to the number of unreserved positions q_e^u.

(1b): From the remaining agents, choose the highest-ranked agents in N^+ up to the number of targeted reserve positions q_e^r.

(1c): If any open positions remain, choose the highest-ranked agents from the remaining ones to have up to q_e in total selected. (note: in substep (1c) all selected agents, if any, must come from N^0)

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In general:

Step k: Each agent i who was rejected in step $k-1$ applies to her next-choice entity. Each entity e considers the new applicants *and* the agents who were admitted tentatively at step $k-1$ (the "applicant pool").

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The algorithm terminates when there are no more rejections, and the tentative matching in the last step becomes the final matching.

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DA-TPR (Targeted Priority Reserves)

- **The DA-TPR targets exactly those agents in the priority group who are in need** of a reserved position, recognizing that not all agents in the priority group need preferential treatment.
- In contrast to the DA-R policy, **DA-TPR's policy allows for a higher number of priority agents than the reserve quota set aside for them** (without forfeiting the priority rights of agents who need them).
- "Strong" priority group applicants can get accepted by entities on their own right, **without using up reserved positions.**

Targeted Priority Reserves

- The number of set-aside positions should be determined by **how many priority group agents are to be helped**, not by how many should actually be matched to an entity.
- Appropriate if the priority reserve quota may be exceeded by the number of priority group agents matched to an entity, while providing protection up to the reserve quota.
- May promote diversity but does not aim for representation.
- Not appropriate if a proportional representation is the goal, such as with visible minorities.
- Use only if underrepresentation (or overrepresentation) is not an issue (i.e., not approriate for diversity purposes, although it may increase diversity).

Potential Applications

- **Refugee settlement:** refugee families in war zones or with other emergency priorities.
- **Scholarship allocation:** students from economically challenged backgrounds but not necessarily minorities.
- **Centralized job markets:** applicants with disabilities.

Etc., any situation where a disadvantaged group or a group with a critical need deserves preferential treatment, but some members of the group may not need the advantage provided by prioritization.

DA-TPR vs. DA-R

Important to note:

- The DA-TPR is not the same as the DA-R with an increased number of reserved positions.
- The DA-TPR is not simply an "intensified" DA-R policy. The two policies operate on different principles and serve different goals.
- The number of reserved seats may be adjusted according to the goal (representation vs. effective treatment), and thus switching from DA-R to DA-TPR need not mean "more affirmative action."

Stability Axioms and Results

Our Main Stability Axiom: Protection-Stability

Definition (Protection-Stability)

A matching μ is **protection-stable** at P:

1. For all $i \in N$, $\mu(i)\ R_i\ i$.

   (each agent is matched to an acceptable entity or the agent remains unmatched)

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2. If there are $i \in N$ and $e \in E$ such that $e\ P_i\ \mu(i)$, then $|\mu(e)| = q_e$;

   (if agent i strictly prefers entity e to her assignment then entity e is filled to capacity)
   and

Our Main Stability Axiom: Protection-Stability

Definition (continued)

2.a if $i \in N^+$ then

- for all $j \in \mu(e)$, $j \succ_e i$

  (all the agents matched to e are ranked higher by e than agent i)

Our Main Stability Axiom: Protection-Stability

Definition (continued)

2.a if $i \in N^+$ then

- for all $j \in \mu(e)$, $j \succ_e i$

  (all the agents matched to e are ranked higher by e than agent i)
- $|\bar{N}_e^+(\mu_e, \succ_e)| \geq q_e^r$,

  where $\bar{N}_e^+(\mu_e, \succ_e) \equiv \{j \in \mu_e \cap N^+ : \text{for all } l \in \mu_e \cap N^0, l \succ_e j\}$

  (all reserved positions at e are filled with targeted priority agents who would not have been selected by e without the reserve policy)

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  (all reserved positions at e are filled with targeted priority agents who would not have been selected by e without the reserve policy)

This is to enforce the priority reserve policy.

Our Main Stability Axiom: Protection-Stability

Definition (continued)

2.b if $i \in N^0$ then

- for all $j \in \mu(e) \cap N^0$, $j \succ_e i$;

  (all non-priority agents matched to e are ranked higher by e than agent i)

Our Main Stability Axiom: Protection-Stability

Definition (continued)

2.b if $i \in N^0$ then

- for all $j \in \mu(e) \cap N^0$, $j \succ_e i$;

  (all non-priority agents matched to e are ranked higher by e than agent i)

- $|\{h \in \mu(e) \cap N^+ : i \succ_e h\}| \leq q_e^r$

  (the number of priority agents matched to e who are ranked lower by entity e than i does not exceed the number of reserved positions)

Our Main Stability Axiom: Protection-Stability

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- for all $j \in \mu(e) \cap N^0$, $j \succ_e i$;

  (all non-priority agents matched to e are ranked higher by e than agent i)
- $|\{h \in \mu(e) \cap N^+ : i \succ_e h\}| \leq q_e^r$

  (the number of priority agents matched to e who are ranked lower by entity e than i does not exceed the number of reserved positions)

This is to ensure the rights of non-priority agents.

Results on the DA-TPR

Proposition 1: Protection-stability and optimality

The DA-TPR is protection-stable and it is agent-optimal with respect to the set of protection-stable matchings.

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The DA-TPR is protection-stable and it is agent-optimal with respect to the set of protection-stable matchings.

Proposition 2: Weak group-strategyproofness

The DA-TPR is weakly group-strategyproof.

Note: DA-R is also weakly group-strategyproof, as shown by Hafalir et al. (2013).

Results on the DA-TPR

Proposition 1: Protection-stability and optimality

The DA-TPR is protection-stable and it is agent-optimal with respect to the set of protection-stable matchings.

Proposition 2: Weak group-strategyproofness

The DA-TPR is weakly group-strategyproof.

Note: DA-R is also weakly group-strategyproof, as shown by Hafalir et al. (2013).

Theorem 1: DA-TPR characterization

Protection-stability and strategyproofness characterize the DA-TPR.

Properties of the DA-R

We prove a similar characterization to Theorem 1 for the DA-R: the stability condition of Hafalir et al. (2013), what we call **representation-stability**, and strategyproofness characterize the DA-R.

A similar result to Proposition 1 applies to the DA-R using representation-stability, which was proved by Hafalir et al. (2013).

Representation-Stability

Definition (Representation-Stability)

A matching μ is **representation-stable** at P:

1. For all $i \in N$, $\mu(i)\ R_i\ i$.
2. If there is $i \in N$ and $e \in E$ such that $e\ P_i\ \mu(i)$, then $|\mu(e)| = q_e$ and
   - 2.a if $i \in N^+$ then, for all $j \in \mu(e)$, $j \succ_e i$ and $|\mu(e) \cap N^+| \geq q_e^r$;
   - 2.b if $i \in N^0$ then, for all $j \in \mu(e) \cap N^0$, $j \succ_e i$ and if $|\mu(e) \cap N^+| > q_e^r$ then for all $j \in \mu_e$, $j \succ_e i$.

2.a: rights for priority agents (weaker than in protection-stability)

2.b: rights for non-priority agents (stronger than in protection-stability)

Protection-Stability (for Comparison)

Definition (Protection-Stability)

A matching μ is **protection-stable** at P:

1. For all $i \in N$, $\mu(i)\ R_i\ i$.
2. If there is $i \in N$ and $e \in E$ such that $e\ P_i\ \mu(i)$, then $|\mu(e)| = q_e$ and
   - 2.a if $i \in N^+$ then for all $j \in \mu(e)$, $j \succ_e i$ and $|\bar{N}_e^+(\mu_e, \succ_e)| \geq q_e^r$, where $\bar{N}_e^+(\mu_e, \succ_e) \equiv \{j \in \mu_e \cap N^+ : \text{for all } l \in \mu_e \cap N^0,\ l \succ_e j\}$.
   - 2.b if $i \in N^0$ then, for all $j \in \mu(e) \cap N^0$, $j \succ_e i$ and $|\{h \in \mu(e) \cap N^+ : i \succ_e h\}| \leq q_e^r$.

2.a: rights for priority agents (stronger than in representation- stability)

2.b: rights for non-priority agents (weaker than in representation- stability)

Characterization of the DA-R

Theorem 2: DA-R characterization

Representation-stability and strategyproofness characterize the DA-R.

Comparing DA-TPR to DA-R

Example: DA-R vs. DA-TPR Comparison

Example

Let $N^+ = \{i_1, \ldots, i_5\}$ and $N^0 = \{a_1, a_2, a_3\}$.

Let $E = \{e_1, \ldots, e_4\}$ with capacities $q = (2, 1, 2, 3)$ and priority reserves $q^r = (1, 0, 1, 0)$.

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}	P_{a_1}	P_{a_2}	P_{a_3}
$\boxed{\underline{e_1}}$	$\boxed{\underline{e_3}}$	$\boxed{e_1}$	e_1	$\boxed{e_3}$	e_3	$\underline{e_1}$	$\underline{e_3}$
e_3		e_2	$\underline{e_2}$	$\underline{e_4}$	$\boxed{\underline{e_4}}$	$\boxed{e_2}$	$\boxed{e_4}$
		$\underline{e_4}$	$\boxed{e_4}$				

$\succ_{e_1}$	$\succ_{e_2}$	$\succ_{e_3}$	$\succ_{e_4}$
i_1	i_1	i_2	$\vdots$
a_1	a_1	a_3	
a_2	a_2	i_5	
i_3	i_4	a_1	
i_4	i_3		

The DA-R matching is underlined, the DA-TPR matching is in squares.

Priority agents i_3 and i_5 are better off under the DA-PTR, but priority agent i_4 is worse off.

Therefore, the two mechanisms are not Pareto-comparable for N^+ agents.

Example: DA-R vs. DA-TPR Comparison

Example (continued)

DA-R

Step	e_1	e_2	e_3	e_4
1	$\boxed{i_1}$ i_3 i_4 $\boxed{a_2}$	-	$\boxed{i_2}$ i_5 a_1 $\boxed{a_3}$	-
2		i_3 $\boxed{i_4}$		$\boxed{i_5}$ $\boxed{a_1}$
3				$\boxed{i_3}$

DA-TPR

Step	e_1	e_2	e_3	e_4
1	$\boxed{i_1}$ $\boxed{i_3}$ i_4 a_2	-	$\boxed{i_2}$ $\boxed{i_5}$ a_1 a_3	-
2		i_4 $\boxed{a_2}$		$\boxed{a_1}$ $\boxed{a_3}$
3				$\boxed{i_4}$

DA-TPR vs. DA-R

Observation

There is no Pareto-dominance relationship for the priority group between the DA-R and the DA-TPR.

Pareto-dominance applies at the entity-selection level, but does not necessarily carry over to the entire matching mechanism due to rejection chains in the Deferred Acceptance algorithm.

DA-TPR vs. DA-R

Theorem 3: DA-TPR – DA-R Comparison under Homogeneous Profiles

If

1. all agent preferences P_i over entities are the same
2. all entity rankings $\succ_e$ over agents are the same

then the DA-TPR weakly Pareto-dominates the DA-R mechanism for the priority group:

$$\text{for all } i \in N^+,\ \mu_i^{\text{DA-TPR}}(P, \succ)\ R_i\ \mu_i^{\text{DA-R}}(P, \succ).$$

Intuition: The two matching mechanisms are comparable since homogeneous profiles have no "noise". Benchmark case.

The Class of DA-SPR (Sequential Priority Reserves) Mechanisms

The Class of DA-SPR Mechanisms

Dur et al. (2020) has inspired us to extend our analysis to a class of mechanisms which includes both the DA-R and the DA-TPR.

DA with Sequential Priority Reserves (DA-SPR) mechanisms:
(studied at the entity level by Dur et al. (2020))

Any arbitrary sequence of the reserved and unreserved positions may be used as the entity selection rule by each entity.
E.g., (u, r, r, u, u, r, u).

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Extreme members of DA-SPR mechanisms:

DA-R: all reserved positions come first for each entity

DA-TPR: all unreserved positions come first for each entity

Weak Reserve-Stability

Definition (Weak Reserve-Stability)

A matching μ is **weakly reserve-stable** at P:

1. For all $i \in N$, $\mu(i)\ R_i\ i$.
2. If there are $i \in N$ and $e \in E$ such that $e\ P_i\ \mu(i)$, then $|\mu(e)| = q_e$ and
   - 2.a if $i \in N^+$ then, for all $j \in \mu(e)$, $j \succ_e i$, and $|\mu(e) \cap N^+| \geq q_e^r$;
   - 2.b if $i \in N^0$ then, for all $j \in \mu(e) \cap N^0$, $j \succ_e i$ and $|\{h \in \mu(e) \cap N^+ : i \succ_e h\}| \leq q_e^r$.

Note: Weak reserve-stability takes the weaker stability condition from both representation-stability (rights for priority agents: 2.a) and protection-stability (rights for non-priority agents: 2.b).

Thus, it is weaker than both representation-stability and protection-stability.

Main Characterization

Theorem 4: Characterization of DA-SPR mechanisms

A matching mechanism is weakly reserve-stable and strategyproof if and only if it is a DA-SPR mechanism.

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A matching mechanism is weakly reserve-stable and strategyproof if and only if it is a DA-SPR mechanism.

Note: DA-SPR mechanisms are also weakly group-strategyproof and constrained efficient within the class of weakly reserve-stable mechanisms.

Declaration-Proofness

Definition

A matching mechanism is **susceptible to non-priority declaration** if there exist a preference profile and a set of priority agents $T \subset N^+$ such that if $N^+ \setminus T$ were considered the priority group then no priority agent would be worse off and at least one priority agent would be better off.

Definition (Declaration-proofness)

A matching mechanism is **declaration-proof** if it is not susceptible to non-priority declaration.

Intuition: The priority group cannot weakly benefit at any preference profile by "declaring" some of their members non-priority agents.

Declaration-Proofness and the DA-TPR

Theorem 5: Alternative Characterization of the DA-TPR

The only declaration-proof mechanism among the DA-SPR mechanisms is the DA-TPR.

Thus, by Theorem 4, the only weakly reserve-stable, strategyproof, and declaration-proof mechanism is the DA-TPR.

Criticism of DA-SPR Mechanisms in General

DA-SPR mechanisms in general are not intuitive or easy to interpret. For example, consider the sequence

$$(u, r, r, u, u, r, u, u, u)$$

What does this mean? What is the priority reserve policy?

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What does this mean? What is the priority reserve policy?

Two arbitrary sequences for the same entity cannot always be compared in terms of their impact on selecting priority agents (hence the "comparative statics" results of Dur et al. (2020) with only two adjacent positions switched).

For example, (u, r, r, u) and (r, u, u, r) are not comparable as two different selection rules for entity e with two reserved positions and two unreserved positions.

Criticism of DA-SPR Mechanisms: An Example

Let the applicant pool be $\{a_1, a_2, a_3, i_1, i_2, i_3\}$, with priority agents i_1, i_2, i_3.

If the entity ranking is $\succ_e$: $(i_1, a_1, a_2, i_2, a_3, i_3)$ then the selections are

i_1	i_2	i_3	a_1
u	r	r	u

and

i_1	a_1	a_2	i_2
r	u	u	r

Criticism of DA-SPR Mechanisms: An Example

However, if the entity ranking is $\hat{\succ}_e : (a_1, i_1, i_2, a_3, a_2, i_3)$ then the selections are

a_1	i_1	i_2	a_3
u	r	r	u

and

i_1	a_1	i_2	i_3
r	u	u	r

For the first entity ranking, $\succ_e$, using one sequence selects more priority agents than the other, while **it is the reverse for the other entity ranking, $\hat{\succ}_e$.**

Comparing Sequences in DA-SPR Entity Selection

$\boldsymbol{c_r(s)}$ **:** the list of the *cumulative* reserved position count in the sequence of positions s for some entity e.

For example, for two sequences s_e and s'_e of some entity e with 2 reserved positions and 3 unreserved positions:

- the cumulative count for $s_e = (u, r, u, u, r)$ is $c_r(s_e) = (0, 1, 1, 1, 2)$
- the cumulative count for $s'_e = (u, r, r, u, u)$ is $c_r(s'_e) = (0, 1, 2, 2, 2)$

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- the cumulative count for $s'_e = (u, r, r, u, u)$ is $c_r(s'_e) = (0, 1, 2, 2, 2)$

Definition: More targeted selection sequences

If $c_r(s_e) \leq c_r(s'_e)$ then the selection sequences s_e and s'_e are comparable and we say that s_e is **more targeted** than s'_e.

Definition: More targeted DA-SPR mechanisms

If for all $e \in E$, $c_r(s_e) \leq c_r(s'_e)$ then **mechanism** f is **more targeted** than mechanism f'.

Split DA-SPR Mechanisms

Transparency in Targeting: Split DA-SPR Mechanisms

A set of priority sequences that are comparable to each other and can easily be understood in terms of their policy:

Split DA-SPR mechanisms.

Each mechanism splits the reserve seats available to an entity between non-targeted positions up front and targeted reserved positions at the end of the sequence.

E.g.: $(\boldsymbol{r}, \boldsymbol{r}, \boldsymbol{r}, u, u, u, \boldsymbol{r}, \boldsymbol{r})$

- reserved positions at the **beginning** are **not targeted**
- reserved positions at the **end** are **targeted**

The Class of Split DA-SPR Mechanisms

Reserved positions are either at the beginning or at the end of the sequence in the entity selection rule for each entity.

The targeted reserved positions are at the end, while all the other (non-targeted) reserved positions are at the beginning.

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The targeted reserved positions are at the end, while all the other (non-targeted) reserved positions are at the beginning.

Extreme members:

DA-R: all reserved positions are at the beginning - none are targeted

DA-TPR: all reserved positions are at the end - all are targeted

Extensions of DA-TPR – DA-R comparisons

- All Split DA-SPR entity selection rules are comparable.
- The more targeted reserved positions there are, the more targeted the entity selection rule becomes.
- If all entity selection rules are weakly more targeted in one Split DA-SPR mechanism than in another, then the former is **more targeted.**

Extensions of DA-TPR – DA-R comparisons

- All Split DA-SPR entity selection rules are comparable.
- The more targeted reserved positions there are, the more targeted the entity selection rule becomes.
- If all entity selection rules are weakly more targeted in one Split DA-SPR mechanism than in another, then the former is **more targeted.**

Comparison Result

The benchmark case that compares the DA-TPR to the DA-R extends to comparisons of more targeted versus less targeted mechanisms: when profiles are homogeneous, a more targeted mechanism leads to a weak Pareto-improvement for priority agents compared to a less targeted mechanism.

Summary and Policy Implications I

- Different priority reserve policies should be evaluated in terms of their different objectives: representation (diversity) versus effective preferential treatment.
- DA-R is most appropriate if representation is the only goal.
- DA-TPR is most appropriate if effective preferential treatment is the goal.
- The two different objectives imply a different way of determining the number of reserved positions.

Summary and Policy Implications II

- The class of DA-SPR mechanisms includes the DA-R as the least targeted priority reserve policy, and the DA-TPR as the most targeted priority reserve policy.
- This class is characterized by strategyproofness and weak reserve-stability.
- Not all DA-SPR policies make sense, but there is a range of policies, the Split DA-SPR mechanisms, which provide an intuitive compromise.
- Split DA-SPR mechanisms are transparent in terms of their impact and allow for flexibility when choosing the extent of targeting for a priority reserve policy.

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