Job Matching with Constraints, Subsidy and Taxation

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Myths among Policymakers

- What you enforce is what is destined to appear.
- What you forbid is what is destined to disappear.
- What you subsidize is what will happen more often.
- What you penalize/tax is what will happen less often.

All will be well and end well!

Problem with a Subsidy Policy

- Large Farm Subsidy: per-unit-area subsidy for operating multiple land plots, if the total area exceeds a threshold
- It didn't end well: rentiers demand higher rents ⇒ less renting, less economy of scale

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首 页 >> 今日谈 >> 种粮大户补贴，看得到拿不到 >> 阅读

种粮大户的补贴，为何看得到拿不到

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为了鼓励农业新型经营主体发展，近年来，国家和地方开始对其进行财政补贴。在重庆，补贴标准按经营规模分为每亩 160 元、230 元两档。但在基层操作中，不少散户变身"田老板"，种粮大户要向其"返还"部分补贴款，一些区县大户实得补贴每亩在 90 元左右，个别地方甚至不到 10 元。

Economic Problems with Complements

- Holdup Problem: owners of complements demanding high compensations and thus holding up efficient assembly of resources
  - Recall example: Large Farm Subsidy
  - $\Rightarrow$ Complementarity among land plots $\uparrow$
  - In rural China, it is reported that land owners can capture high rents due to stronger bargaining positions
- Exposure problem: agents desiring multiple complements but hesitating to make initial offers in fear of facing high costs for later trades
  - renters become reluctant to enter the market

References: Chandra and Wong (2016); Ehrlich and Overman (2020); Chen et al. (2021); Slattery and Zidar (2020); Milgrom (2004); Hazlett and Muñoz (2009)

Transfer Policies in Markets for Indivisible Resources

- Markets for Workers
  - income tax
  - subsidy for a firm to hire many workers
  - subsidy for hiring many disadvantaged, local, or R&D workers, ...
- Markets for Goods
  - subsidy for preferred bidders, e.g., "weak bidders" in radio spectrum auctions
  - tax for excessive consumption, e.g., penalizing owning multiple homes
  - Large Farm Subsidy

References: Chandra and Wong (2016); Ehrlich and Overman (2020); Chen et al. (2021); Slattery and Zidar (2020); Milgrom (2004); Hazlett and Muñoz (2009)

Constraints in Markets for Indivisible Resources

- Markets for Goods
  - affordable housing policies $\Rightarrow$ ceiling constraints: a family can own no more than n houses
  - environmental policies $\Rightarrow$ proportionality constraints: a taxi company's fleet must be at least $x\%$ electronic
- Markets for Workers
  - public goods provision $\Rightarrow$ floor constraints: a rural school must have at least n teachers
  - affirmative action $\Rightarrow$ type-specific floor/ceiling constraints: a company's board has to have at least n minorities
  - legal punishment $\Rightarrow$ never-hiring constraints: a firm forbidden to hire someone

Research Agenda: to study effects of market interventions

- Objective: Study how policy interventions may reshape demands, market outcomes, and mechanism performances
- Method: Augment job matching model of Kelso and Crawford (1982), which embeds object assignment and auction models (e.g., Gul and Stacchetti, 2000)
- Focus: (Gross) Substitutes Condition (on each hospital's demand, requiring a set of demanded doctors still be demanded after a rise of others' salaries)
  - Sufficient and necessary (in a sense of maximal domain) for competitive equilibria existence and nonempty core (stability)
  - Crucial for nice equilibria structure and incentive properties: lattice theorem; rural hospitals theorem; law of aggregate demand; group-incentive-compatibility; pseudo-equilibria being competitive equilibria; Vickrey outcomes residing in cores
  - Crucial for mechanism performances: deferred acceptance, multi-object auctions

Research Question

Question Which policy interventions preserve the substitutes condition (and thus all the nice properties it entails)?

- Analogous question in discrete convex analysis: which mathematical operations preserve $\mathrm{M}^\natural$-concavity?

The Model for Transfer Policies (Job Matching with Subsidy and Taxation, REStud, forthcoming)

- Set of doctors D
  - *salary schedule* $\mathbf{s} = (s_d)_{d \in D} \in \mathbb{R}^D$
- One hospital
  - Kojima, Sun and Yu (2021, "Job Market Interventions") study a model with multiple hospitals and establish equilibrium existence, lattice structure, and comparative statics
- A government which designs transfer policies

The Model: Revenue and Transfer Functions

- $R(A) \in \mathbb{R}$: hospital's revenue when matched to $A \subseteq D$.
- $T(A) \in \mathbb{R}$: hospital's transfer from the government when matched to A.
- $V(A; \mathbf{s}, R + T) = R(A) + T(A) - \sum_{d \in A} s_d$: hospital's *profit*
  - maximal profit function:
    $\Pi(\mathbf{s}; R + T) = \max\{V(A; \mathbf{s}, R + T) : A \subset D\}$
  - demand correspondence:
    $X(\mathbf{s}; R + T) = \{A \subset D : V(A; \mathbf{s}, R + T) = \Pi(\mathbf{s}; T)\}$
    - Each $A \in X(\mathbf{s}; R + T)$ is called a demand set

The Substitutes Condition

Definition 1 (Substitutes Condition).

$X(\,\cdot\,; R+T)$ satisfies the substitutes condition if for any two salary schedules $\mathbf{s}$ and $\mathbf{s}'$ with $\mathbf{s}' \geq \mathbf{s}$, and any $A \in X(\mathbf{s}; R+T)$, there exists $A' \in X(\mathbf{s}'; R+T)$ such that $\{d \in A : s_d = s'_d\} \subset A'$.

Always Preserving the Substitutes Condition

Definition 2 (Always Preserving the Substitutes Condition).

A transfer function T always preserves the substitutes condition if whenever a demand correspondence $X(\,\cdot\,;R)$ satisfies the substitutes condition, $X(\,\cdot\,;R+T)$ satisfies it.

Important Classes of Transfer Functions

- T is additively separable if $T(A) = \sum_{d \in A} T(\{d\})$ for every $A \subset D$.

- T is cardinal if there exists a function $f : \{0, 1, \ldots, |D|\} \to \mathbb{R}$ such that $T(A) = f(|A|)$ for every $A \subset D$.
- T is cardinally concave if it is cardinal and the associated function f is concave.
  - A cardinal function is cardinally concave if and only if the marginal transfer from an additional doctor is non-increasing.

Main Theorem for Simple Transfer Policies

Theorem 1.

T always preserves the substitutes condition

$$\Updownarrow$$

T=(additively separable function) + (cardinally concave function).

Discussion 1

- Recall the theorem:
  Always preserving subst. $\Leftrightarrow$ additively separable + cardinally concave.
- Cardinally concave transfer policies can be used to address rural public goods shortages (Roth, 1986; Kojima, 2012)
- Affirmative action can be carried out only in the form of individual subsidy/taxation.

Discussion 2

The necessity part of the Theorem follows from a stronger result:

Proposition 1.

T preserves the substitutes condition for all binary unit-demand revenue functions

$\Downarrow$

T=(additively separable function) + (cardinally concave function).

- Binary unit-demand: there exist $\alpha > 0$, $d, d' \in D$ with $R(A) = \alpha \min\{1, |A \cap \{d, d'\}|\}$.
- Even a small subclass of revenue functions already give lots of restrictions.

Complex Transfer Function: Examples

- In many cases, government transfers depend on salaries
  - Income taxes
  - "Credit for Increasing Research Activities"
    - For wages paid to R&D workers
    - Their total salary may count toward thresholds for greater tax breaks (Chen et al., 2021, Table 1)
  - In sports: luxury taxes (Kaplan, 2004; Coates and Frick, 2012).

Complex Transfer Function: Model

- A complex function $\Upsilon : 2^D \times \mathbb{R}^D \to \mathbb{R}$ is called an complex transfer function if:
  - Government transfer is only related to the salaries of its own employees
    - $\Upsilon(A, \mathbf{s}) = \Upsilon(A, \mathbf{s}')$ for all $A \subset D$ and $\mathbf{s}, \mathbf{s}' \in \mathbb{R}^D$ with $\mathbf{s}|_A = \mathbf{s}'|_A$.
  - Overall hiring cost is strictly increasing in the salaries of hired doctors
    - Define the hiring cost function associated with Υ as $H^{\Upsilon} : 2^D \times R^D \to \mathbb{R}$ such that $H^{\Upsilon}(A, \mathbf{s}) = \sum_{d \in A} s_d - \Upsilon(A, \mathbf{s})$ for all $A \subset D$ and $\mathbf{s} \in \mathbb{R}^D$; it is required that for every $A \subset D$, $H^{\Upsilon}(A, \cdot\,)$ is strictly increasing in each s_d with $d \in A$

Main Theorem for Complex Transfer Functions

- Υ is C-additively separable if there is a family of functions $\{f_d : \mathbb{R} \to \mathbb{R}\}_{d \in D}$ such that for each $A \subset D$ and $\mathbf{s} \in \mathbb{R}^D$, $\Upsilon(A, \mathbf{s}) = \sum_{d \in D} f_d(s_d)$
- Υ is C-cardinally concave if there exists a cardinally concave transfer function T such that for all $A \subsetneq D$ and $\mathbf{s} \in \mathbb{R}^D$, $\Upsilon(A, \mathbf{s}) = T(A)$; $\Upsilon(D, \mathbf{s})$ as a function of $\mathbf{s} \in \mathbb{R}^D$ is weakly increasing; for each $\mathbf{s} \in \mathbb{R}^D$, $\Upsilon(D, \mathbf{s}) \leq T(D)$ and the transfer function $T^{\mathbf{s}} := \Upsilon(\,\cdot\,, \mathbf{s})$ is cardinally concave

Theorem 2.

Υ *always preserves the substitutes condition*

$$\Updownarrow$$

Υ *=(C-additively separable complex transfer function)*
+ (C-cardinally concave complex function).

Subclasses of Revenue/Transfer Functions

- $\mathcal{P}$ is Partition of D; $P \in \mathcal{P}$ is referred to as a group
- R is group separable if there exists a family of functions $\{R_P\}_{P\in\mathcal{P}}$ such that every R_P satisfies the substitutes condition on 2^P, and for every $A \subset D$, $R(A) = \sum_{P\in\mathcal{P}} R_P(A \cap P)$
- $\tau : 2^D \to \mathbb{Z}^{\mathcal{P}}$ with $\tau(A)(P) = |A \cap P|$: "vectorization"
- R is group concave if it's of the form $R(A) = S(\tau(A))$ and substitutable.
  - Same as function S satisfying $\mathrm{M}^\natural$-concavity (Murota, 2003).
- Can be defined for transfer functions too.

Group Separable Revenue Functions

Theorem 3.

T preserves the substitutes condition for all group separable revenue functions

$$\Updownarrow$$

T=(additively separable function) + (group concave function).

- Compare with "all substitutes" case:
  T=(additively separable) + (cardinally concave).
- Certain affirmative action policies are allowed, e.g., decreasing marginal subsidy for minority.
- Necessity part can be obtained for a smaller class of revenue functions.

Group Concave Revenue Functions

Theorem 4.

Assume $|\mathcal{P}| > 2$. T preserves the substitutes condition for all group concave revenue functions

$$\Updownarrow$$

$T=$*(group separable function)* $+$ *(cardinally concave function).*

- Compare with the "all substitutes" case: $T=$(additively separable) $+$ (cardinally concave)
- Certain affirmative action policies are allowed.
- Necessity part can be obtained for a smaller class of revenue functions.

Reestablishing the Substitutes Condition

Definition 3.

A transfer function T (or a complex transfer function Υ) **reestablishes the substitutes condition** for a revenue function R if $R + T$ (or $R + \Upsilon$) satisfies the condition.

- Designers may not know R, and even if they do, they may not be able to customize policies for different firms
- Are there transfer policies which reestablish the substitutes condition for a wide variety of revenue functions?
- So focus on: additively separable policies & cardinally concave ones
- But the former doesn't help $\Rightarrow$ focus on the latter

Some notation

- Consider: the firm is constrained to pick a set out of a nonempty collection $\mathcal{F} \subset 2^D$, called its feasibility collection as in KSY
- Define: the maximal profit function Π and demand correspondence X such that for each salary schedule $\mathbf{s}$,

$$\begin{aligned} \Pi(\mathbf{s}; R+T, \mathcal{F}) &= \max\{V(A; \mathbf{s}, R+T) : A \in \mathcal{F}\}; \\ X(\mathbf{s}; R+T, \mathcal{F}) &= \{A \in \mathcal{F} : V(A; \mathbf{s}, R+T) = \Pi(\mathbf{s}; R+T, \mathcal{F})\}. \end{aligned}$$

- The substitutes condition and its preservation are still well-defined
- Given $m \in [0, M]_{\mathbb{Z}}$, the feasibility collection $\mathcal{D}_m := \{A \subset D : |A| = m\}$ is defined by an exact constraint, requiring the firm to hire exactly m workers.

Two New and Weaker Substitutes Conditions

Definition 4.

A revenue function R satisfies the exactly-constrained substitutes condition if given any $\mathbf{s}, \mathbf{s}' \in \mathbb{R}^D$ with $\mathbf{s}' \geq \mathbf{s}$, $m \in [1, M]_{\mathbb{Z}}$, and $A \in X(\mathbf{s}; R, \mathcal{D}_m)$, there exist $A' \in X(\mathbf{s}'; R, \mathcal{D}_m)$ satisfying $\{d \in A : s_d = s'_d\} \subset A'$.

Definition 5.

A revenue function R satisfies the chain-constrained substitutes condition if given any $\mathbf{s} \in \mathbb{R}^D$, $m \in [1, M]_{\mathbb{Z}}$, and $A \in X(\mathbf{s}; R, \mathcal{D}_m)$, there exists $A' \in X(\mathbf{s}; R, \mathcal{D}_{m-1})$ satisfying $A' \subset A$.

Main Theorem

Theorem 5.

For a revenue function R, $\exists$ a cardinally concave T such that $R + T$ satisfies the substitutes condition

$$\Updownarrow$$

R simultaneously satisfies

- *the exactly-constrained substitutes condition*
- *the chain-constrained substitutes condition*

- This is a surprisingly small class of revenue functions for which the substitutes condition can be reestablished by cardinally concave policies

The Model for Constraints (Job Matching under Constraints, AER 2020)

- The hospital is constrained to choose among $\mathcal{F} \subset 2^D$
  - called feasibility collection of h
  - its elements called feasible sets
  - $\mathcal{F} = \mathcal{F}^0 \cap \mathcal{F}^g$: $\mathcal{F}^0$ is self-imposed; $\mathcal{F}^g$ is government-imposed

- **Example 1**: For $A \subset D$ and integers $0 \leq f \leq c \leq |A|$, the feasibility collection

$$\mathcal{D}^A_{[f,c]} := \{B \subset D : f \leq |B \cap A| \leq c\}$$

is defined by an interval constraint on A

  - $\mathcal{D}^A_{[f,|A|]}$ is defined by a floor constraint on A
  - $\mathcal{D}^A_{[0,c]}$ is defined by a ceiling constraint on A
  - $\mathcal{D}_{[f,c]} := \mathcal{D}^D_{[f,c]}$ is defined by an interval constraint (on D)

The Model for Constraints

- **Example 2**: For $\overline{D}, \underline{D} \subset D$, $\overline{D} \cap \underline{D} = \emptyset$, integers $0 \le f \le c \le |D \setminus (\overline{D} \cup \underline{D})|$, the feasibility collection

$$\{A \subset D : \overline{D} \subset A,\ \underline{D} \cap A = \emptyset,\ \text{and } f \le |A \setminus \overline{D}| \le c\}$$
$$=\{A \subset D : \overline{D} \subset A\} \cap \{A \subset D : \underline{D} \cap A = \emptyset\} \cap \mathcal{D}_{[f,c]}^{D\setminus(\overline{D}\cup\underline{D})}$$

is defined by a generalized interval constraint.

  - $\{A \subset D : \overline{D} \subset A\}$ is defined by a always-hiring constraint on $\overline{D}$
  - $\{A \subset D : \underline{D} \cap A = \emptyset\}$ is defined by a never-hiring on $\underline{D}$
  - $\mathcal{D}_{[f,c]}^{D\setminus(\overline{D}\cup\underline{D})}$ is defined by an interval constraint (on $D \setminus (\overline{D} \cup \underline{D})$)

The Substitutes Condition

- *profit*: $V(A;\mathbf{s}) = R(A) - \sum_{d\in A} s_d$
- demand correspondence:
  $X(\mathbf{s};\mathcal{F}) = \{A \in \mathcal{F} : V(A;\mathbf{s}) \geq V(A';\mathbf{s})$ for every $A' \in \mathcal{F}\}$
- $X(\mathbf{s};\mathcal{F}^0)$ is innate; $X(\,\cdot\,;\mathcal{F}^0 \cap \mathcal{F}^g)$ is compelled

Definition 6 (Substitutes Condition).

Demand correspondence $X(\,\cdot\,;\mathcal{F})$ satisfies the substitutes condition

if

for any two salary schedules $\mathbf{s}$ and $\mathbf{s}'$ with $\mathbf{s}' \geq \mathbf{s}$, and any $A \in X(\mathbf{s};\mathcal{F})$, there exists $A' \in X(\mathbf{s}';\mathcal{F})$ such that $\{d \in A : s_d = s'_d\} \subset A'$.

Always Preserving Substitutes

Definition 7 (Always preserving the Substitutes Condition).

A feasibility collection $\mathcal{F}^g$ always preserves the substitutes condition

if

for every R and $\mathcal{F}^0$ with $\mathcal{F}^0 \cap \mathcal{F}^g \neq \emptyset$ such that $X(\,\cdot\,;\mathcal{F}^0)$ satisfies the substitutes condition, $X(\,\cdot\,;\mathcal{F}^0 \cap \mathcal{F}^g)$ satisfies the condition as well.

Always Preserving the Substitutes Condition

Theorem 1.

A feasibility collection always preserves the substitutes condition if and only if it is defined by a generalized interval constraint.

Generalized Polyhedral Constraint (given partition $\mathcal{P}$)

- A feasibility collection $\mathcal{F}$ is defined by a generalized polyhedral constraint if there is a supermodular function $\mu : 2^{\mathcal{P}} \to \{0, 1, \ldots, |\hat{\chi}(\mathcal{F})|\}$ and a submodular function $\rho : 2^{\mathcal{P}} \to \{0, 1, \ldots, |\hat{\chi}(\mathcal{F})|\}$ such that for any $\mathcal{Q}, \mathcal{Q}' \subset \mathcal{P}$, $\mu(\mathcal{Q}) - \mu(\mathcal{Q} \setminus \mathcal{Q}') \leq \rho(\mathcal{Q}') - \rho(\mathcal{Q}' \setminus \mathcal{Q})$, and

$$\mathcal{F} = \{A \subset D : \overline{\chi}(\mathcal{F}) \subset A,\ \underline{\chi}(\mathcal{F}) \cap A = \emptyset,$$
$$\text{and } \mu(\mathcal{Q}) \leq |A \cap (\cup \mathcal{Q}) \cap \hat{\chi}(\mathcal{F})| \leq \rho(\mathcal{Q}) \text{ for every } \mathcal{Q} \subset \mathcal{P}\}.$$

  - $\mathcal{Q}$ may contain multiple groups, and $\mu(\mathcal{Q})$ and $\rho(\mathcal{Q})$ respectively dictate the floor and ceiling on the set of real-decision doctors in $\cup \mathcal{Q}$.
  - The equation above simply states that $\mathcal{F}$ is defined by a family of (potentially degenerate) generalized interval constraints on unions of groups.

Preserving the Substitutes Condition for Group Separable Revenue Functions

Theorem 2.

A feasibility collection preserves the substitutes condition for group separable revenue functions if and only if it is defined by a generalized polyhedral constraint.

Conclusion

- Policy interventions are ubiquitous in markets for indivisible resources
- Two classes of economic policies
  - transfer policies (providing financial incentive)
  - constraints (mandatory)
  - $\rightarrow$ systematically investigating which policy interventions preserve the substitutes condition
- Two mathematical operations in discrete convex analysis
  - summation of two discrete functions
  - domain restriction of a discrete function
  - $\rightarrow$ systematically investigating which operations preserve $\mathrm{M}^\natural$-concavity
- A lot of extensions...

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