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Adjustment Dynamics for Human Players

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Abstract
I point out that there is a fundamental difficulty associated with the formal study of dynamic adjustment processes towards a Nash equilibrium in the context of social and economic problems, i.e., for human players. This difficulty has created an unfortunate dichotomy of researchers and has hindered progress in this area of research. I suggest, with a couple of examples, that a promising way to overcome this problem is to strengthen the empirical side of research on adjustment dynamics.

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The concept of Nash equilibrium plays a key role in game theory, and therefore it is vital to understand how players come to play a Nash equilibrium. There are several ways to answer this question, but a prominent possibility is that a Nash equilibrium emerges via a dynamic adjustment process. If players accumulate experience in the same game or similar games, they learn what others do and which strategies fare well, and in the end (if the process ever converges) they would end up playing a Nash equilibrium.

The history of the research on adjustment dynamics dates back to as early as 1949 when Brown proposed fictitious play as an algorithm to find a Nash equilibrium (Brown (1949)). This area of research had a big impact around 1970 when biologists started to employ game theoretic ideas to study evolutionary dynamics and its convergent point, a stable Nash equilibrium called an ESS (Maynard Smith (1974)). This inspired economists to formally analyze adjustment dynamics in the late 1980s and evolutionary game theory then became a hot research topic. Various deterministic and stochastic dynamics were proposed and their properties examined. In the 1990s, the behavioral approach to dynamic adjustment processes was born, where the focus is to come up with parsimonious models that closely reproduce human subjects' adjustment behavior in lab experiments. Around the year 2000, game theoretic ideas were actively introduced in computer science and engineering, where adjustment dynamics and learning algorithms play a key role.

Rather than providing a comprehensive overview of those developments, here I focus on one conceptual issue that I view as crucial to advance this area of research: How should we formulate adjustment dynamics?[1] This is not an issue in biology because the adjustment dynamics there are undoubtedly driven by natural selection and mutations. In computer science and engineering, the answer to the question is also uncontroversial because adjustment dynamics and learning rules are directly designed and programmed by the researchers. In contrast, in social and economic problems, *researchers have not yet reached a consensus on how adjustment dynamics should be formulated*. I will point out the fundamental difficulty associated with this issue and then explain how the difficulty has badly hindered the progress of research in this area. Finally, I will suggest a possible way to overcome the difficulty.

Let me make my point by means of a simple example. A well-established stylized fact is that people do play mixed strategy Nash equilibria in zero-sum 2-person games where the outcome is binary (win or lose). Instances include a card game (O'Neil (1987)), tennis serves (Walker and Wooders (2001)) and penalty kicks in soccer games (Palacios-Huerta (2003)). As there is no theoretical result that shows the Nash equilibrium in those games is derived via rational reasoning, if human players behave according to a Nash equilibrium strategy, it must have been achieved as a result of dynamic adjustment.

So, what does the existing research on dynamics tell us? A general, unifying condition for convergence to an equilibrium in such games has not yet been obtained, but it is known that best response dynamics and fictitious play do converge[2]. Now focus on a particular case of Matching Pennies (Table 1). Figure 1 depicts the associated state space.

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[1] Bill Sandholm's ambitious textbook (2010) provides a comprehensive overview of the field, encompassing both deterministic and stochastic dynamics.

[2] Sandholm (2010) and (2015) show that a general condition on dynamics, called positive correlation, plus various additional conditions guarantee the convergence of dynamic adjustment processes towards a Nash equilibrium in zero-sum 2-person games.

A point in this diagram may represent the current distribution of strategies in player 1'spopulation and in player 2's, when we consider the best response dynamics. A point in the diagram can also be interpreted as the beliefs of players in the fictitious play model, which represent the past empirical frequencies of strategies of a pair of players who repeatedly play this game.

1 \ 2	H	T
H	1, -1	-1, 1
T	-1, 1	1, -1

Table 1
Matching Pennies

Figure 1 shows the best reply profile for each subset of the state space. For example, in the upper left square, the best reply profile is (H, H), and if the current state is x, the state under the best response dynamics and fictitious play gradually moves towards (H, H), the upper right corner of the state space, as indicated by the arrow in the diagram. Following this kind of argument, we can see that the state eventually converges to the midpoint, which is the mixed strategy Nash equilibrium.

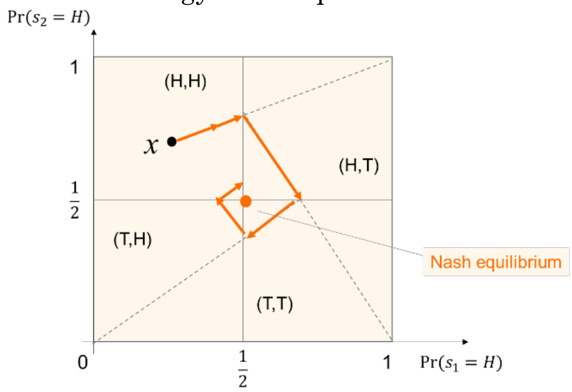


Figure 1

So far, so good. But if we view fictitious play as a learning process of players, it raises the following concern. When the current state is x at time t where t is large, both players are going to play H for a rather long time. This is implausible, because sooner or later player 2 must realize that the opponent continues to play H and therefore would like to switch to T (so that she can win). In other words, player 2 is highly likely to deviate from fictitious play if the opponent mechanically sticks to the fictitious play adjustment process.

Let me explain what is happening here. Human players are intelligent, unlike animals (in biological applications) or machines (in engineering applications). Therefore, if players are mechanically following certain adjustment rules, sooner or later one might detect the opponent's adjustment rule and switch to the best or a better reply. In summary, adjustment dynamics are subject to potential instability if players are human. This is a *valid concern about ad-hoc adjustment rules* (rules that are not optimal given what others do) imposed on human behavior, and such a concern never arises in the biological or engineering applications.

The fundamental difficulty associated with this observation is that it is *logically impossible to wipe out this concern on a purely theoretical basis*. The only way to completely eliminate the concern is to look at a profile of adjustment processes that are free from the potential instability, i.e., the processes where switching to another adjustment rule is unprofitable for each player. In other words, the only way to eliminate the concern is to assume a Nash equilibrium in the dynamic adjustment game. This contradicts, however, the whole purpose of studying adjustment dynamics. To explain how a Nash equilibrium of a given game emerges over time, one should not assume a Nash equilibrium in a more complicated game of dynamic adjustment.

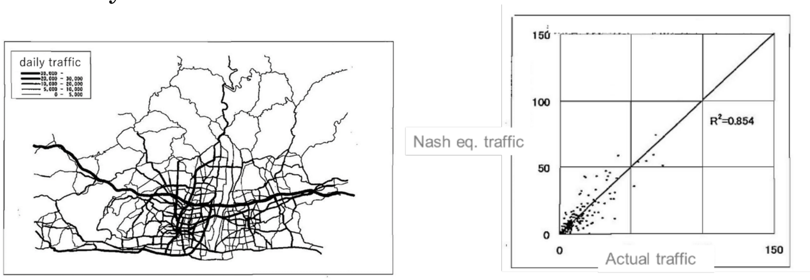
In summary, the fundamental difficulty is that *adjustment processes are subject to potential instability if players are human* and that it is impossible to eliminate this concern on a theoretical basis. This difficulty has created an unfortunate dichotomy of researchers that has badly hindered the progress of research in this area. On the one hand, there are researchers on dynamics who pay due attention to the importance of dynamic adjustment processes but do not fully address the valid concern about the potential instability issue. I am under the impression that some researchers, especially in disciplines other than economics, such as physics, biology, engineering, and sociology, when they come to analyze *human* adjustment processes for social problems, might not fully realize the importance of the valid concern about the potential instability. On the other hand, we have the skeptics. A group of researchers, probably including a non-negligible fraction of economists, shy away from or even dismiss the research on adjustment dynamics because of the concern about the ad-hocness and potential instability of adjustment processes. For a long time, economists have fought against various ad-hoc approaches and have achieved commendable success by adopting a logically consistent, non-ad-hoc methodology based on the unifying principle of rationality. Anything ad-hoc is regarded as faulty, and not without good reason. In my opinion, the flip side of the coin of this success story is that one of the most fundamental questions in game theory, that of how a Nash equilibrium emerges via an adjustment process, has not received its due attention in the field of economics.

How can we overcome the difficulty and the unfortunate dichotomy of researchers? First, the best way to overcome the skepticism about the research on adjustment dynamics is to present convincing empirical evidence. Second, the key question of how dynamic adjustment processes for human players should be formulated can only be answered empirically. In summary, what we need to do is to strengthen the empirical side of research on adjustment dynamics.

In what follows, I will present some promising observations about what empirical study can offer. Let me start with some convincing empirical evidence.

Convergence:

A leading “success story” of dynamic adjustment would be traffic allocation. Drivers try to go from their origins to destinations as fast as they can, and their strategies specify which route to choose. How fast one can go depends on how congested one’s route is. This “congestion game” has a highly non-trivial Nash equilibrium, but empirical research has shown that the actual traffic allocation is reasonably close to the Nash equilibrium. Panel (a) of Figure 2 shows the Nash equilibrium traffic allocation around a mid-sized city in Japan (thicker lines represent more traffic), and Panel (b) compares the actual versus Nash equilibrium traffic allocation (each dot in this diagram represents a segment of the routes). As you can see, the Nash equilibrium prediction fares reasonably well. It is *a priori* obvious that the Nash allocation is not achieved by ex-ante super rational reasoning of the drivers in this city, and that it must have been achieved via their trial-and-error dynamic adjustment of which route to choose. It is by no means obvious if such a process ever converges, but the theoretical research reveals that the congestion game is a potential game, and dynamic adjustment does converge to the Nash equilibrium under a fairly weak condition.[3] And finally, this concerns a significant problem in our society.



(a) (b)

Figure 2
Traffic allocation
Hamamatsu city, Japan, Doboku Gakkai (2003)

Non-convergence and Cycles:

Anderson, Plott, Shimomura and Granat (2004) analyzed multi-market price adjustment in lab experiments. There are three goods, X, Y and money, and players have utility functions over the consumption of those three goods. They come with initial holdings of those goods and exchange the goods in markets. The benchmark theoretical dynamics is given by the textbook model of price adjustment, where the price of good X or Y increases (decreases) when there is an excess demand (supply). In other words, the rate of change in the price (dp/dt) in each market is proportional to the excess demand. The authors chose special utility functions and initial endowments for which the theoretical price adjustment process is cyclical, as depicted in Panel (a) of Figure 3. The shape of the cycle depends on the relative speed of price adjustment in markets for X and Y, and the figure presents two cases. In the lab, subjects traded their goods under basically the same rules as in the New York Stock Exchange. They can post any selling or buying order (such as "selling two units of good X at price 10") at any moment in time, and they can also take any outstanding selling or buying orders at any time. The dynamics of the average trading prices are shown in Panel (b) of Figure 2, and they closely follow the theoretical prediction: price trajectories cycle and the direction of movement is counterclockwise.

For another specification of utility functions and endowments, the theoretical price adjustment converges to the equilibrium, as is shown in Panel (c) of Figure 3. And again, the lab data, shown in Panel (d), are amazingly close to the theoretical prediction. This is not just a coincidence. Goeree and Lindsay (2016) replicated the counterclockwise cyclical movement in their lab experiments.

[3] Research in civil engineering by Wardrop (1952) provided the key insights into the traffic allocation game. Sandholm (2001) shows that any "positively correlated dynamic", whose condition is much weaker than "switching to a better reply", converges to the Nash equilibrium.

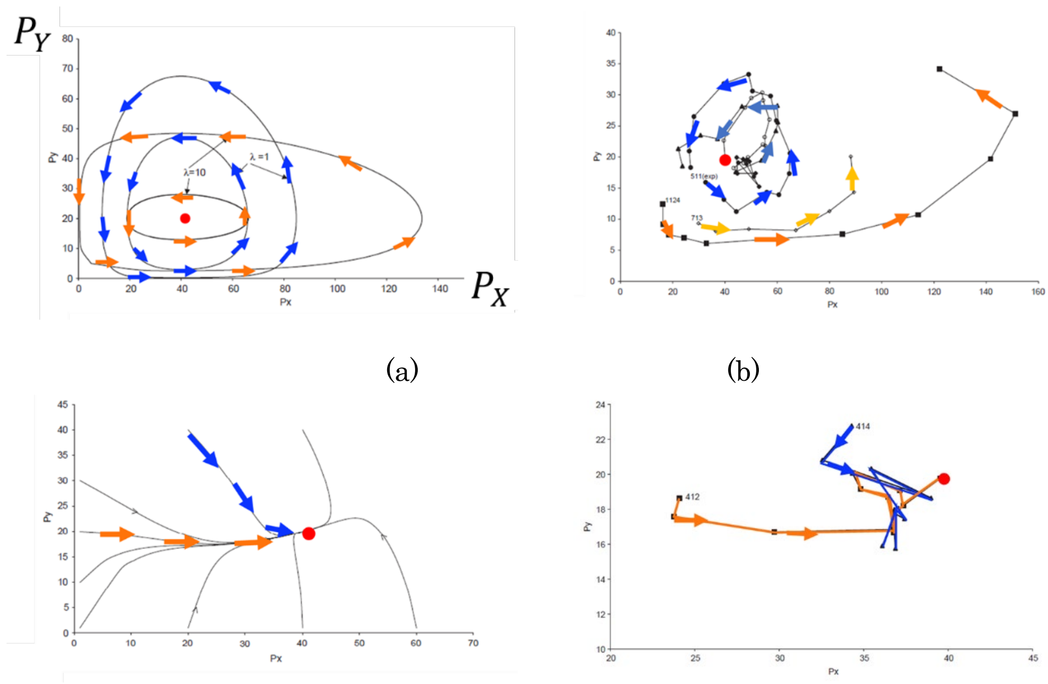


Figure 3
Multi-market Price Adjustment
Anderson et al. (2004)

Equilibrium Selection - Risk Dominance:

This work is in the field of biology, but I hope similar findings can be obtained for human adjustment processes. When there are multiple Nash equilibria, one can sometimes predict which one arises by analyzing dynamic adjustment processes. For example, one of the prominent findings in evolutionary game theory in economics is that a risk-dominant equilibrium emerges under certain conditions. Consider a game with two strategies A and B, where "everyone plays A" and "everyone plays B" are both Nash equilibria. A is called a risk-dominant strategy if A is the unique best reply when at least 1/2 of the opponents play A. Selection of risk dominant equilibrium as the unique outcome of adjustment dynamics was first shown by Kandori, Mailath and Rob (1993) under a slight possibility of repeated random choice in the long run, and the same selection result when players have local interaction was proved by Ellison (1993) . Figure 4 illustrates a simple case, where players are located on a line. The circled player, who is currently choosing B, interacts with her two neighbors. Since half of her opponents play A and A is risk-dominant, she would like to switch to A in the next round. As the figure shows, this creates a "domino effect" and eventually everyone ends up playing A, as long as at least one player initially chooses A.

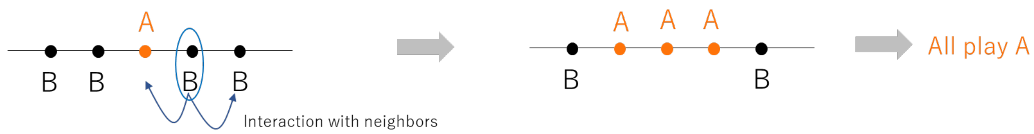
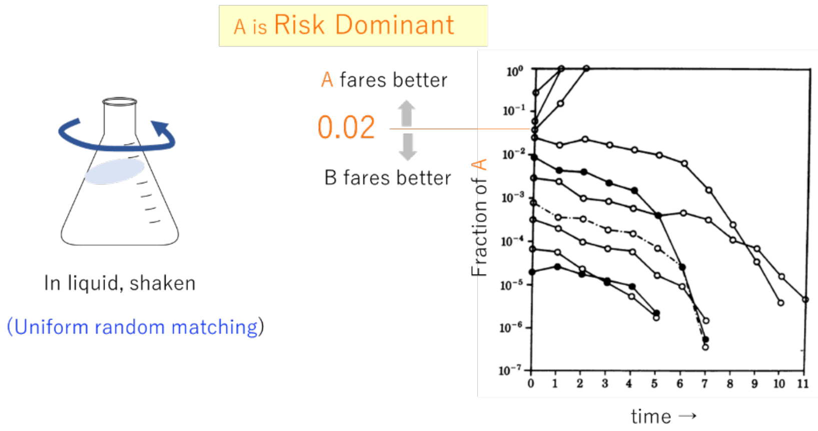
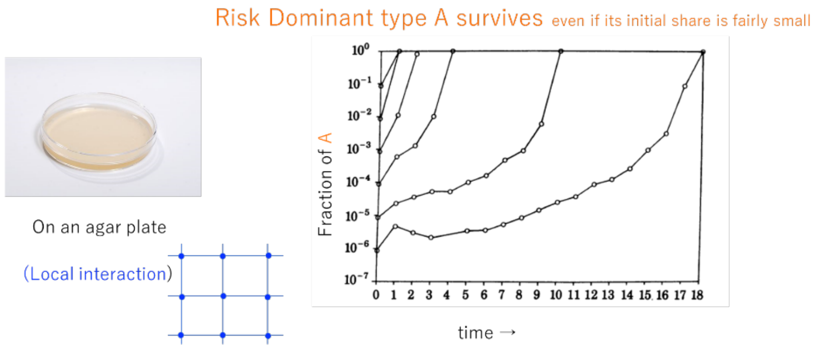


Figure 4

Chao and Levin (1981) can be regarded as striking empirical evidence of Ellison's result. They analyzed a type of bacteria (*Escherichia coli*) that comes in two types, A and B. A produces a toxic substance, and B is sensitive to the toxin. Because producing the toxin is costly, A has a smaller reproduction rate, but the toxin can interfere with the reproductive success of type B. When those two types are cultured in liquid that is constantly shaken, the situation is characterized by uniform random matching. In this case, the outcome depends on the initial condition, as shown in Panel (a) of Figure 5. It shows that A fares better (has more offspring) when the fraction of A is more than 2%. This means that A is (very strongly) risk dominant. In contrast, if the two types are grown on agar plates, the situation is characterized as (two-dimensional) local interaction. Panel (b) of Figure 5 shows that in that case, the risk dominant A type survives even if its initial share is fairly small, as is predicted by the local interaction model.[4]



(a)



(b)

Figure 5
Chao et al. (1981)
Image of agar plate in (b): Lilly_M, CC BY-SA 3.0 <https://creativecommons.org/licenses/by-sa/3.0>, via Wikimedia Commons

Now I turn to some preliminary empirical findings about the nature of adjustment dynamics for human players. Here I report some laboratory experimental results I and my collaborators have obtained about a 2-person zero-sum game with binary outcomes.

[4] Iwasa, Nakamaru and Levin (1998) provides an alternative model of dynamics on the agar plates.

As I mentioned, O'Neil (1987) discovered that subjects' behavior in his lab experiment about a rather complicated card game is very close to the mixed Nash equilibrium. Panel (a) of Table 6 shows the payoff table of the card game, and in the unique mixed strategy equilibrium of this game, each player chooses K (King card) with probability 0.4 and each number card (1, 2, and 3) with probability 0.2. The equilibrium winning rate is 0.4 for the row player and 0.6 for the column player. I was able to replicate O'Neil's finding in a large data set I have collected through my online game theory course[5]. In the data set, 2,589 pairs played the game for 30 rounds. In Panel (b), the second row shows the winning rates in my data set. The subjects' behavior is pretty close to the Nash equilibrium, and by inspecting the payoff table one can see that it is highly unlikely that the subjects managed to compute the equilibrium. They must have learned, in some way, to play the mixed strategy equilibrium. To discover their adjustment behavior, we estimated a leading learning model in behavioral economics called EWA (Camerer and Ho (1999)), which is basically a convex combination of fictitious play and reinforcement learning. The third row of Panel (b) of Table 6 reports the implied winning rates of the estimated EWA model, and it shows that the subjects' behavior is reasonably well replicated by the estimated model.

1 \ 2	K	1	2	3
K	1, -1	-1, 1	-1, 1	-1, 1
1	-1, 1	-1, 1	1, -1	1, -1
2	-1, 1	1, -1	-1, 1	1, -1
3	-1, 1	1, -1	1, -1	-1, 1

(a)
O'Neil game

	Player 1	Player 2
Nash equilibirum	**0.4**	**0.6**
Human subjects	0.41	0.59
EWA	0.39	0.61
EWA vs **BR-EWA**	0.24	**0.76**
Human vs **BR-EWA**	0.85	**0.15**

(b)
Winning rates

Table 6

We then computed the dynamic best reply to the estimated EWA strategy in the 30-round repetition of the card game. The fourth row of Panel (b) shows the implied winning rates where player 1 uses the estimated EWA strategy while player 2 adopts the dynamic best reply to the estimated EWA. As you can see, if the opponent is using EWA, one can gain substantially by switching from EWA to the best reply. The gain is substantial (winning rate increases from 0.61 to 0.76), and therefore the concern about the ad-hocness of the adjustment process is a real issue here. Finally, I conducted an experiment where subjects played against a computer program that plays the dynamic best reply to the estimated EWA. The result is reported in the bottom row of Panel (b), and it is revealing. Against a computer programmed to play the best reply to EWA, human players no longer used the estimated EWA-like behavior and acted very differently so as to exploit the program. The computer program, which is supposed to outsmart human players, fared miserably.

Those results indicate the following. First, human players do have some capacity to detect the opponent's behavior pattern and adjust their own behavior accordingly. Second, human players often stick to adjustment processes even though there is a substantial gain from switching to a better reply.

Call a profile of adjustment processes *compatible* if no one wants to deviate to another adjustment rule. As we have seen, fictitious play for Matching Pennies is apparently

[5] *Welcome to Game Theory* at Coursera https://www.coursera.org/learn/game-theory-introduction.

incompatible, and the EWA profile seems compatible. In contrast, the profile (EWA, Best reply to EWA) turned out to be incompatible.

Given these observations, I would suggest the following agenda of potentially promising research directions. This would help us to address the concern about the ad-hocness of adjustment processes and to dissolve the unfortunate dichotomy of researchers. My suggestion is to bridge the gap between the top-down theoretical approach on dynamic adjustment, which has been a dominant mode of research, and the recent bottom-up research by behavioral economists. First, we need more empirical research to discover the nature of actual adjustment behavior in various social and economic problems. Logically speaking, this is the only way to address the valid concern about ad-hocness. Empirics should encompass not only laboratory experimental data but also field data on significant economic or social issues as was suggested by the traffic allocation example. Once we have a better idea about the nature of dynamic adjustment processes actually adopted by human players, we could go about theorizing why a particular mode of adjustment behavior is prevalent (despite its potential instability). This amounts to identifying which processes are compatible and understanding why. Human adjustment is not completely mechanical, and players think strategically to some extent. Formulating such a mental process and coming up with models that fit the empirical data can be achieved through active feedback between bottom-up empirical research and top-down theoretical research.

Let me summarize. Adjustment dynamics are subject to potential instability if players are human. Because of that, some researchers are skeptical about the research on adjustment dynamics for social and economic problems. To convince the skeptics about the importance of adjustment dynamics and find out what the right adjustment dynamics for human players are, we need to strengthen the empirical side of research on adjustment dynamics. By doing so, I hope we can take ourselves one step closer to the answer to one of the most fundamental questions in game theory: how players come to play a Nash equilibrium.

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