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Auctions with Ethical Concerns

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Abstract

We investigate the implementation of social choice rules (SCRs) in dominant strategy, where the central planner evaluates social welfare ethically rather than financially by excluding monetary transfers from the welfare evaluation, prohibiting redistribution, and putting heterogenous welfare weights on agents' willingness to pay in a state-contingent manner. With such ethical concerns in mind, we show that side-payment devices play a significant role in incentivizing agents to be honest. Focusing on multiunit auctions with a single-unit demand, we consider the case in which the central planner faces a situation where multiple conflicting ethical criteria with their own advantages coexist and cannot be aggregated into a single criterion. We demonstrate a new side-payment rule design that successfully implements reasonable SCRs induced by the method of procedure explored by Matsushima (2021). We clarify when and how to use subsidies and set-asides as incentive and fairness devices, depending on the state.

JEL Classification Codes: D44, D62, D63, D71, D82

Keywords: Conflicting Criteria, Ethical Social Choice, Multiunit Auctions, Single-Crossing, Strategy-Proofness.

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1. Introduction

This study investigates the case in which a central planner attempts to implement a desirable allocation; that is, the value of a social choice rule (SCR), in a state-contingent manner. Multiple selfish agents are involved in this allocation problem. To determine a reasonable SCR, the central planner seeks to maximize social welfare by comprehensively considering agents' willingness to pay as well as various social and ethical factors such as externalities, community values, life, dignity, and the global environment. The central planner does not regard social welfare as the sum of agents' willingness to pay and understands that the in-kind merit gained from the allocation cannot be replaced by the achievement of their willingness to pay with money, thereby placing importance on evaluating social welfare ethically rather than financially. The central planner evaluates social welfare without integrating money and prohibits the resale (redistribution) of the allocation.

With such ethical concerns in mind, this study considers the role of side payments in agents' incentives. We assume that the central planner is ex-ante unaware of which state occurs, while agents have information about the state as their respective types. The central planner attempts to elicit information from them by constructing a side-payment rule as an incentive device. This study aims to clarify how side payments function when implementing ethically desirable SCRs in the dominant strategy. We show that in a wide class of multiunit auction problems with a single-unit demand, the central planner can implement ethically desirable SCRs in the dominant strategy.

Typically, when the central planner determines an SCR with ethical concerns, there are multiple conflicting ethical criteria for this determination, each of them has its own advantage, and they cannot be aggregated into a single artificially created criterion reasonably. Hence, the central planner must compromise between these criteria without aggregating them. For example, in the problem of allocating scarce resources to businesses, the central planner intends to preferentially allocate to businesses that create little environmental load. However, multiple conflicting criteria coexist on various issues such as how to define environmental load, how to balance environmental load and profitability, and how to consider the welfare of future generations. The central planner cannot find a way to combine them into a single criterion to satisfy everyone; they[3] must decide on an SCR that respects individual criteria as much as possible. For example, consider the debate during the COVID-19 pandemic as to who should be preferentially assigned scarce resources such as ventilators. To avoid neglecting any valuable criterion for triaging patients, Pathak et al. (2020) proposed the reserve system, which belongs to the class of SCRs that this study discusses intensively. In the early days of the COVID-19 pandemic, the Japanese government provided all citizens with sanitary masks in-kind. This policy intended to address the reality that the willingness of the poor to pay was below the market price of a mask.

Focusing on multiunit assignment problems with a single-unit demand, Matsushima (2021) carefully explained the importance of building a new theory of social choice with multiple ethical concerns, and characterized SCRs that emphasize a consistent respect for individual criteria across assignment problems associated with various participants and units. For determining ethically desirable SCRs, Matsushima demonstrated *the method of procedure* as a definite alternative to the aggregation method. The central planner predetermines a priority order over criteria (i.e., procedure), according to which, they sequentially assign the commodity to the top-ranked agent at the corresponding criterion, thereby determining the value of the SCR as the set of all agents assigned to the commodities through these steps. Matsushima axiomatized the class of all SCRs that can be induced by the method of procedure from the viewpoint of inter-problem regularities. Thus, Matsushima showed that the method of procedure is dominant for determining reasonable SCRs whenever it is impossible to emphasize a consistent respect for individual agents, not individual criteria, across problems.

As a follow-up to Matsushima (2021), this study investigates a multiunit auction with a single-unit demand. In addition to Matsushima (2021), we consider the role of side payments. We define the welfare evaluation for each agent as the willingness to pay multiplied by the welfare weight. Social welfare is expressed as the sum of these welfare evaluations. The vector of welfare weights differs between the criteria. Allowing the vector of welfare weights to depend on the state, we assume the single-crossing condition over welfare evaluations. The single-crossing condition permits each agent's type to

[3] To avoid gendered language, this study uses "they" instead of she or he.

include information about the other agents' welfare weights, implying interdependent values for the welfare evaluation.

The previous literature on auction and mechanism design has never considered such ethical concerns, focusing instead on surplus or revenue maximization regardless of whether it is constrained. The technical contribution of this study is the development of knowledge cultivated in the literature such as the Vickrey-Clarke-Groves (VCG) mechanisms so that it can be applied to the implementation of ethically desirable SCRs.

We first consider the case of a single criterion, and show that a modification of the uniform-price auction with price discrimination successfully implements the ethically desirable SCR in the dominant strategy. This auction format is closely related to the generalized VCG mechanism that Crémer and McLean (1985, 1988) explored in single-unit auctions but with a substantial difference. This study assumes private values for the willingness to pay but assumes interdependent values for the welfare evaluation. Conversely, Crémer and McLean (1985, 1988) assumed interdependent values for the willingness to pay. Owing to this difference, this study can prove its possibility results using the dominant strategy as the solution concept, whereas Crémer and McLean (1985, 1988) used ex post equilibrium, which is a weaker concept than dominant strategy.

Next, as the main part of this study, we consider the case of multiple conflicting criteria. In this case, the difficulty of implementation is that an SCR induced by the method of procedure cannot be considered as the result of maximizing a single objective function, making it impossible to apply the VCG mechanism with only a slight modification.

However, we overcome this difficulty by demonstrating a new side-payment rule design, which makes any SCR induced by the method of procedure implementable in the dominant strategy. Virtually, depending on each agent, we prepare as many markets as there are criteria. For each market, we set the quantity of the commodity to be sold, which is less than or equal to the total units in question. We also set the price for each market as a variant of the uniform price defined in the same manner as in the single-criterion case. Each agent can purchase the commodity in any market. However, importantly, any agent has to pay only the lowest price in all markets irrespective of which market they purchase. According to this virtual scenario, we design a new side-payment rule, and show that this rule successfully incentivizes agents to be honest, thereby implementing the SCR.

Previous studies in auction theory, such as those of Ayres and Cramton (1996), Pai and Vohra (2012), and Athey et al. (2013), considered subsidies and set-asides as constraints in optimization. Subsidies can be interpreted as a device that reflects the heterogeneity in welfare weight between agents, and set-asides can be interpreted as a device to integrate complementary criteria into a baseline criterion. The results of this study clarify when and how to use subsidies and set-asides as incentive devices.

To summarize:

1) This study investigates the implementation problem of SCRs in the dominant strategy. The central planner is unaware of which state occurs, while multiple agents exist who have information about the state as their respective types. The question of this paper is whether the central planner can construct a side-payment rule that incentivizes agents to be honest, thereby implementing the SCR.

2) This study departs from the previous literature on auction and mechanism design in that in this study, the central planner places importance on evaluating social welfare ethically rather than financially. The central planner does not integrate money into the social welfare evaluation, and puts heterogenous welfare weights on agents' willingness to pay. The welfare evaluations depend on the state; therefore, the central planner is ex ante unaware of them. This study aims to clarify the possibility that ethically desirable SCRs are implementable in the dominant strategy by demonstrating a new side-payment rule design.

3) This study intensively considers the multiunit auction problem with a single-unit demand. Assuming the single-crossing condition on the welfare evaluation as well as the willingness to pay, we show that a variant of the uniform-price auction successfully implements any SCR that respects a single ethical criterion.

4) As the main part of this study, we investigate the case in which the central planner faces multiple ethical criteria that have their respective advantages but conflict with one another. According to the method of procedure explored by Matsushima (2021), the central planner determines a reasonable SCR that emphasizes a consistent respect for individual criteria across problems.

According to a virtual scenario of multiple markets backed by multiple criteria, we demonstrate a new side-payment rule design, which successfully implements any SCR that can be induced by the method of procedure; that is, any SCR specified as a reasonable compromise between conflicting criteria.

The remainder of this paper is organized as follows. Section 2 presents the general framework of mechanism design and proposes a theorem. It shows that a slight modification of the VCG mechanism implements the SCR associated with a single ethical criterion, provided agents' welfare weights are independent of the state. From Section 3 onward, we eliminate this state independence and instead assume a single-crossing condition. Section 3 investigates single-unit auctions and shows that any SCR associated with a single criterion is implementable in the dominant strategy. Section 4 extends this result to the multiunit auction problem with a single-unit demand.

Section 5 is the highlight of this study. It investigates the case of multiple conflicting criteria in a multiunit auction. Subsection 5.1 explains how to specify an SCR as a reasonable compromise between criteria. Subsection 5.2 demonstrates a new design for the side-payment rule. Subsection 5.3 shows the main theorem, implying that the central planner can implement the specified SCR by using the new side-payment rule design. Finally, Section 6 concludes.

2. The Model

Let $N=\{1,...,n\}$ denote the finite set of all agents, where $n\geq 2$. Let Ω_i denote the set of possible types for each agent $i\in N$, where $\omega_i\in\Omega_i$ denotes a type for agent i. Let $\Omega=\Omega_1\times\cdots\times\Omega_n$ denote the set of all states, where a state is defined as a type profile $\omega=(\omega_i)_{i\in N}\in\Omega$. Let A denote the set of all allocations. Each agent $i\in N$ has a *quasi-linear* utility function with *private values*, given by

$$v_i(a,\omega_i)+s_i,$$

where $a\in A$, and $s_i\in R$ denotes the side payment to agent i.

A *social choice rule* (SCR) is defined as $C:\Omega\to A$.[4] A *side-payment rule* is defined as $t=(t_i)_{i\in N}$, where $t_i:\Omega\to R$ for each $i\in N$. This study investigates the *direct mechanism* defined as a combination of an SCR and a side-payment rule (C,t). The central planner determines the allocation $C(\tilde{\omega})\in A$ and makes the monetary payment $t_i(\tilde{\omega})\in R$ to each agent $i\in N$, provided that each agent $j\in N$ announces $\tilde{\omega}_j\in\Omega_j$ in the direct mechanism (C,t), where we denote $\tilde{\omega}=(\tilde{\omega}_j)_{j\in N}$. In this case, each agent $i's$ payoff is given by $v_i(C(\tilde{\omega}),\omega_i)+t_i(\tilde{\omega})$.

The direct mechanism (C,t) is said to be *strategy-proof* if truth-telling is a dominant strategy for each agent; that is, for every $i\in N$, $\omega_i\in\Omega_i$, and $\tilde{\omega}\in\Omega$,

$$v_i(C(\omega_i,\tilde{\omega}_{-i}),\omega_i)+t_i(\omega_i,\tilde{\omega}_{-i})\geq v_i(C(\tilde{\omega}),\omega_i)+t_i(\tilde{\omega}).$$

An SCR C is said to be *implementable in the dominant strategy* if there exists a side-payment rule t such that the direct mechanism (C,t) is strategy-proof.

This section and the subsequent two sections assume that the central planner has a *single ethical criterion*, which is expressed by a vector of *welfare weights* $(w_i(\omega))_{i\in N}\in R^n_+$. The central planner sees the welfare evaluation for each agent i as their willingness to pay multiplied by their welfare weight $w_i(\omega)v_i(a,\omega_i)$.[5]

The ethical criterion (i.e., the vector of welfare weights) is said to be *homogeneous* across agents if $w_i(\omega)=w_1(\omega)$ for all $i\in N\setminus\{1\}$ and $\omega\in\Omega$. The divergence between welfare evaluation and consumer sovereignty is well expressed by the *heterogeneity in welfare weight* between agents; that is, the inequality of $w_i(\omega)\neq w_1(\omega)$ for some $i\in N\setminus\{1\}$ and $\omega\in\Omega$.

[4] Unlike Matsushima (2021), this study does not investigate inter-problem regularities explicitly. Hence, we fix a specific allocation problem and define an SCR, or a state-contingent social choice rule (SSCR) according to the terminology of Matsushima (2021), as a function of the state rather than the problem.

[5] For convenience, this study assumes that the welfare weights are independent of the allocation. This assumption is irrelevant in the substances of this study because we consider chiefly the single-unit demand case. However, this assumption might be restrictive in other cases. See Section 6.

To determine the SCR C, the central planner maximizes the sum of the welfare evaluations with respect to $a \in A$. For every state $\omega \in \Omega$, the corresponding desirable allocation $C(\omega) \in A$ is determined by the following inequalities:

$$\sum_{i \in N} w_i(\omega) v_i(C(\omega), \omega_i) \geq \sum_{i \in N} w_i(\omega) v_i(a, \omega_i) \text{ for all } a \in A.$$

The central planner forbids all agents from redistributing their allocations (reselling the commodities). The central planner's concern is just the achievement of welfare optimal in-kind assignments. They do not consider monetary transfers in the welfare evaluation. The ethical framework of this study is related to previous works such as Diamond and Mirrlees (1971), Saez and Stantcheva (2016), and Dworczak et al. (2020) that considered issues of efficiency-equity tradeoffs where monetary transfers are restricted for some exogenous reasons, and agents have heterogenous welfare weights. However, this study differs substantially from these works in that we do not incorporate money into the social welfare in question. We target only the direct effects of in-kind allocation for the welfare evaluation.

It is important to note that the willingness to pay multiplied by their welfare weight $w_i(\omega)v_i(a, \omega_i)$ is not necessarily the expression of agent $i's$ welfare in monetary value. There may exist a positive real number $l > 0$ such that $lw_i(\omega)v_i(a, \omega_i)$ is the monetary equivalent. In this case, we can decompose the welfare evaluation for each agent $i \in N$ into two parts: willingness to pay $v_i(a, \omega_i)$ and the externality due to the activity they engage in after the allocation determination $a \in A$, that is, $\{lw_i(\omega) - 1\}v_i(a, \omega_i)$. However, as the central planner ignores monetary transfers in the welfare evaluation, they do not need to know l; that is, the monetary equivalent of each agent's welfare. The central planner does not need to know the correct externality effect in this study's problems.

The following condition implies the simplest form of the welfare weight: state independence.

Condition 1: The ethical criterion $(w_i(\omega))_{i \in N}$ is independent of $\omega \in \Omega$, that is, there exists an $n-$dimensional vector $(w_i)_{i \in N} \in R^n_+$ such that

$$w_i = w_i(\omega) \text{ for all } i \in N \text{ and } \omega \in \Omega .$$

Under homogeneity in welfare weight between agents, the central planner can implement the welfare maximization in the dominant strategy by constructing the *VCG mechanism* (Vickrey, 1961; Clarke, 1971; Groves, 1973; Krishna, 2002). The VCG mechanism can successfully internalize the externality effect of each agent's announcement through its impact on the allocation determination, provided that the private values for willingness to pay are assumed. The following theorem shows that we can apply this internalization in the case of heterogeneity in welfare weight.

Theorem 1: *Under Condition 1, the direct mechanism* (C,t) *is strategy-proof if for every* $i \in N$ *, there exists* $e_i : \Omega_{-i} \to R$ *such that*

$$t_i(\omega) = \sum_{j \neq i} \frac{w_j}{w_i} v_j(C(\omega), \omega_j) + e_i(\omega_{-i}) \text{ for all } \omega \in \Omega .$$

Proof: For every $i \in N$, $\tilde{\omega} \in \Omega$, and $\omega_i \in \Omega_i$, we have

$$v_i(C(\tilde{\omega}), \omega_i) + t_i(\tilde{\omega}) - e_i(\tilde{\omega}_{-i})$$

$$= \frac{1}{w_i}\{w_i v_i(C(\tilde{\omega}), \omega_i) + \sum_{j \neq i} w_j v_j(C(\tilde{\omega}), \tilde{\omega}_j)\}$$

$$\leq \frac{1}{w_i}\{w_i v_i(C(\omega_i, \tilde{\omega}_{-i}), \omega_i) + \sum_{j \neq i} w_j v_j(C(\omega_i, \tilde{\omega}_{-i}), \tilde{\omega}_j)\}$$

$$= v_i(C(\omega_i, \tilde{\omega}_{-i}), \omega_i) + t_i(\omega_i, \tilde{\omega}_{-i}) - e_i(\tilde{\omega}_{-i}),$$

which implies that (C,t) is strategy-proof.

Q.E.D.

3. Single-Unit Auction

Specifically, this section investigates a *single-unit auction*, where we assume that

$$A = N ,$$

for each $i \in N$,

$$\boldsymbol{\Omega_i = [0,1]},$$

$$\boldsymbol{v_i(i,\omega_i) = \omega_i} \text{ for all } \boldsymbol{\omega_i \in \Omega_i},$$

and

$$\boldsymbol{v_i(j,\omega_i) = 0} \text{ for all } \boldsymbol{\omega_i \in \Omega_i} \text{ and } \boldsymbol{j \neq i}.$$

According to the welfare maximization explained in Section 2, the central planner determines the SCR $\boldsymbol{C}$ such that for every $\boldsymbol{\omega \in \Omega}$ and $\boldsymbol{i \in N}$,

$$\boldsymbol{C(\omega) = i} \qquad \text{if } \boldsymbol{w_i(\omega)\omega_i > w_j(\omega)\omega_j} \text{ for all } \boldsymbol{j \in N \setminus \{i\}}.$$[6]

We introduce the *single-crossing condition for the welfare evaluation*, which is weaker than Condition 1 because we permit $\boldsymbol{(w_i(\omega))_{i \in N}}$ to depend on the state $\boldsymbol{\omega}$.

Condition 2: For each $\boldsymbol{i \in N}$, $\boldsymbol{\omega_{-i} \in \Omega_{-i}}$, and $\boldsymbol{j \in N \setminus \{i\}}$,

$$\boldsymbol{w_i(\omega)\omega_i - w_j(\omega)\omega_j}$$

is increasing in $\boldsymbol{\omega_i}$.

Condition 2 implies that the difference between an agent $\boldsymbol{i}$ and another agent $\boldsymbol{j \neq i}$ in the welfare evaluation is increasing in $\boldsymbol{\omega_i}$. Hence, both the differences in welfare evaluation and in willingness to pay are increasing in $\boldsymbol{\omega_i}$; that is, they both satisfy the single-crossing property. However, this study incorporates a substantial difference between welfare evaluation and willingness to pay: the welfare evaluation for each agent depends on the state; that is, it satisfies *interdependent values*, while each agent's willingness to pay satisfies private values. In the remainder of this study, we assume Condition 2 instead of Condition 1.

We define $\boldsymbol{\omega_i(\omega_{-i}) \in \Omega_i}$ by

$$\boldsymbol{w_i(\omega_i(\omega_{-i}), \omega_{-i})\omega_i(\omega_{-i}) = \max_{j \neq i}[w_j(\omega_i(\omega_{-i}), \omega_{-i})\omega_j]}.$$

Condition 2 guarantees that $\boldsymbol{\omega_i(\omega_{-i}) \in \Omega_i}$ exists uniquely. Clearly, we have

$$\boldsymbol{C(\omega) = i} \qquad \text{if } \boldsymbol{\omega_i > \omega_i(\omega_{-i})}.$$

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[6] For simplicity, we ignore the tie-breaking case. We can eliminate this ignorance at the expense of irrelevant complexity such as random assignments.

Note that whenever $\omega_i > \omega_i(\omega_{-i})$, there exists no $j \neq i$ such that $\omega_j > \omega_j(\omega_{-j})$.

To implement the SCR C, the central planner specifies the side-payment rule t in the manner that for every $i \in N$ and $\omega \in \Omega$,

$$t_i(\omega) = -\frac{\max_{j\neq i}[w_j(\omega_i(\omega_{-i}),\omega_{-i})\omega_j]}{w_i(\omega_i(\omega_{-i}),\omega_{-i})} \qquad \text{if } C(\omega) = i,$$

and

$$t_i(\omega) = 0 \qquad \text{if } C(\omega) \neq i.$$

The following theorem shows that in a single-unit auction, the central planner can implement welfare maximization (the specified SCR C) in dominant strategy. Note that $\frac{\max_{j\neq i}[w_j(\omega_i(\omega_{-i}),\omega_{-i})\omega_j]}{w_i(\omega_i(\omega_{-i}),\omega_{-i})}$ is independent of agent $i's$ announcement, which, along with the definition of $\omega_i(\omega_{-i})$, is crucial for proving the theorem.

Theorem 2: *Under Condition 2, the specified direct mechanism* (C,t) *is strategy-proof.*

Proof: Suppose that $C(\tilde{\omega}) = i$, that is, $\tilde{\omega}_i > \omega_i(\tilde{\omega}_{-i})$. We have

$$v_i(C(\tilde{\omega}),\omega_i) + t_i(\tilde{\omega})$$

$$= \frac{1}{w_i(\omega_i(\tilde{\omega}_{-i}),\tilde{\omega}_{-i})}\{w_i(\omega_i(\tilde{\omega}_{-i}),\tilde{\omega}_{-i})\omega_i - \max_{j\neq i}[w_j(\omega_i(\tilde{\omega}_{-i}),\tilde{\omega}_{-i})\tilde{\omega}_j]\},$$

which is positive if and only if $\omega_i > \omega_i(\tilde{\omega}_{-i})$, because of Condition 2. Note that $v_i(C(\tilde{\omega}),\omega_i) + t_i(\tilde{\omega})$ is independent of $\tilde{\omega}_i$, provided that $\tilde{\omega}_i > \omega_i(\tilde{\omega}_{-i})$. Conversely, suppose that $C(\tilde{\omega}) \neq i$; that is, $\tilde{\omega}_i < \omega_i(\tilde{\omega}_{-i})$. We have

$$v_i(C(\tilde{\omega}),\omega_i) + t_i(\tilde{\omega}) = 0.$$

These observations imply that truth-telling is a dominant strategy.

Q.E.D.

Under homogeneity in welfare weight, we have

$$\omega_i(\omega_{-i}) = \max_{j\neq i}[\omega_j],$$

and therefore, we have

$$t_i(\omega) = -\max_{j \neq i}[\omega_j] \quad \text{if} \quad C(\omega) = i .$$

Hence, we can regard the mechanism in this section to be a natural extension of the *second-price auction* to the case of heterogeneity in welfare weight.

This mechanism is related to the *generalized VCG mechanism* studied by Crémer and McLean (1985, 1988), which modified the second-price auction for the interdependent value case. Both studies are common in that the central planner sees welfare maximization as a matter of interdependent values. However, in this study, unlike that of Crémer and McLean (1985, 1988), the central planner sees agents' incentives as a matter of private values. Owing to this difference, this study can prove our result using dominant strategy, whereas Crémer and McLean (1985, 1988) used an ex-post equilibrium, which is a weaker equilibrium concept than dominant strategy in interdependent values, for proving their result.

4. Multiunit Auction

This section investigates a *multiunit auction*, where H units of homogeneous commodity exist, and each agent has a single-unit demand, where we assume $H < n$. An allocation is defined as a subset of agents $a \subset N$, where $|a| = H$, and only agents in a are assigned. Hence, we specify

$$A = \{a \subset N \mid |a| = H\} .$$

We assume that for each $i \in N$,

$$\Omega_i = [0,1] ,$$

$$v_i(a, \omega_i) = \omega_i \qquad \text{if} \quad i \in a ,$$

and

$$v_i(a, \omega_i) = 0 \qquad \text{if} \quad i \notin a .$$

For each $h \in \{1,..., H\}$, we denote the h^{th} greatest welfare evaluation among all agents except for agent i by

$$\max_{j \neq i}[w_j(\omega)\omega_j \mid h] ,$$

where we have

$$w_{j'}(\omega)\omega_{j'} \geq \max_{j\neq i}[w_j(\omega)\omega_j \mid h]$$

for at least h agents j' in $N\setminus\{i\}$,

and

$$w_{j'}(\omega)\omega_{j'} \leq \max_{j\neq i}[w_j(\omega)\omega_j \mid h]$$

for at least $n-h+1$ agents j' in $N\setminus\{i\}$.

According to the welfare maximization, the central planner specifies the SCR C such that

$$i \in C(\omega) \qquad \text{if } \omega_i > \frac{\max_{j\neq i}[w_j(\omega)\omega_j \mid H]}{w_i(\omega)},$$

and

$$i \notin C(\omega) \qquad \text{if } \omega_i < \frac{\max_{j\neq i}[w_j(\omega)\omega_j \mid H]}{w_i(\omega)}.$$

The central planner assigns the commodities to the top- H agents in the welfare evaluation.

In a similar manner to the definition of $\omega_i(\omega_{-i})$ in Section 3, for each $h \in \{1,...,H\}$, we define $\omega_i(h,\omega_{-i}) \in \Omega_i$ as

$$w_i(\omega_i(h,\omega_{-i}),\omega_{-i})\omega_i(h,\omega_{-i}) = \max_{j\neq i}[w_j(\omega_i(h,\omega_{-i}),\omega_{-i})\omega_j \mid h].$$

Note that

$$i \in C(\omega) \qquad \text{if } \omega_i > \omega_i(H,\omega_{-i}),$$

and

$$i \notin C(\omega) \qquad \text{if } \omega_i < \omega_i(H,\omega_{-i}).$$

Further, note that $\omega_i(h,\omega_{-i})$ is decreasing in $h \in \{1,...,H\}$.

To implement the SCR C, the central planner specifies the side-payment rule t in the manner that for each $i \in N$ and $\omega \in \Omega$,

$$t_i(\omega) = -\frac{\max_{j\neq i}[w_j(\omega_i(H,\omega_{-i}),\omega_{-i})\omega_j \mid H]}{w_i(\omega_i(H,\omega_{-i}),\omega_{-i})} \qquad \text{if } i \in C(\omega),$$

and

$$t_i(\omega) = 0 \qquad \text{if } i \notin C(\omega).$$

The following theorem shows that in a multiunit auction with a single-unit demand, the central planner can implement welfare maximization in the dominant strategy. Note that $\frac{\max_{j \neq i}[w_j(\omega_i(H, \omega_{-i}), \omega_{-i})\omega_j \mid H]}{w_i(\omega_i(H, \omega_{-i}), \omega_{-i})}$ is independent of agent $i's$ announcement, which, along with the definition of $\omega_i(H, \omega_{-i})$, is crucial for proving the theorem.

Theorem 3: *Under Condition 2, the specified direct mechanism* (C, t) *is strategy-proof.*

Proof: Suppose $i \in C(\tilde{\omega})$; that is, $\tilde{\omega}_i > \omega_i(H, \tilde{\omega}_{-i})$. We have

$$v_i(C(\tilde{\omega}), \omega_i) + t_i(\tilde{\omega})$$
$$= \frac{w_i(\omega_i(H, \tilde{\omega}_{-i}), \tilde{\omega}_{-i})\omega_i - \max_{j \neq i}[w_j(\omega_i(H, \tilde{\omega}_{-i}), \tilde{\omega}_{-i})\tilde{\omega}_j \mid H]}{w_i(\omega_i(H, \tilde{\omega}_{-i}), \tilde{\omega}_{-i})},$$

which is positive if and only if $\omega_i > \omega_i(H, \tilde{\omega}_{-i})$, because of Condition 2. Note that $v_i(C(\tilde{\omega}), \omega_i) + t_i(\tilde{\omega})$ is independent of $\tilde{\omega}_i$, provided that $\tilde{\omega}_i > \omega_i(H, \tilde{\omega}_{-i})$. Conversely, suppose that $i \notin C(\tilde{\omega})$; that is, $\tilde{\omega}_i < \omega_i(H, \tilde{\omega}_{-i})$. We have

$$v_i(C(\tilde{\omega}), \omega_i) + t_i(\tilde{\omega}) = 0.$$

These observations imply that truth-telling is a dominant strategy.

Q.E.D.

Under homogeneity in welfare weight, we have

$$t_i(\omega) = -\max_{j \neq i}[\omega_j \mid H] = -\max_{j \in N}[\omega_j \mid H+1] \quad \text{if } i \in C(\omega).$$

Hence, we can regard the mechanism in this section as a natural extension of the *uniform-price auction* to the case of heterogeneity in welfare weight. In auction theory, it is well known that under homogeneity in welfare weight, truth-telling is the dominant strategy in the uniform-price auction with a single-unit demand. Theorem 3 generalizes this result to the case of heterogeneity in welfare weight.

5. Multiple Criteria

Continuing from Section 4, we investigate a multiunit auction with a single-unit demand. This section assumes that *multiple* ethical criteria coexist that conflict with one another. The central planner must make a reasonable compromise between these criteria in the determination of an SCR. In this respect, the model in this section follows Matsushima (2021), which is the first work to characterize SCRs when multiple criteria coexist.[7]

There exist $\bar{d}$ different ethical criteria that are given by their respective vectors of welfare weights; that is, $(w_i^d(\omega))_{i\in N} \in R_+^n$ for each criterion $d \in D \equiv \{1,...,\bar{d}\}$ and each state $\omega \in \Omega$, where we assume $\bar{d} \leq H$. Associated with each criterion $d \in D$, we define the corresponding *priority order over agents* $\pi_d(\omega) : N \rightarrow \{1,...,n\}$ as follows: for each $i \in N$ and $j \in N \setminus \{i\}$,

$$\pi_d(i,\omega) < \pi_d(j,\omega) \text{ if } w_i^d(\omega)\omega_i > w_j^d(\omega)\omega_j .$$

That is, at each criterion $d \in D$, agent i is prioritized to agent j if the corresponding welfare evaluation for agent i is greater than that for agent j.

To make our ethical conflicts substantial, we assume it is impossible to aggregate these criteria into a single artifically created ethical criterion in a reasonable manner. Hence, to make a compromise without such aggregation, the central planner adapts the *method of procedure* explored by Matsushima (2021). That is, we introduce an arbitrary priority order over criteria, or a *procedure*, given by

$$\gamma : \{1,...,H\} \rightarrow D .$$

The central planner uses the criterion $\gamma(1) \in D$ to pick the first agent to be assigned, then uses the criterion $\gamma(2) \in D$ to pick the second agent to be assigned, and so on.[8]

5.1. Specification of the Social Choice Rule

[7] We describe the criteria as their respective welfare maximization, which is crucial for the results of this section.

[8] To eliminate irrelevant criteria, we assume that for every criterion $d \in D$, there exists $h \in \{1,...,H\}$ such that $\gamma(h) = d$.

Associated with the predetermined priority order over criteria γ, the central planner specifies the SCR C according to the following multiple steps. Fix an arbitrary state $\omega \in \Omega$. In step 1, the top-ranked agent at criterion $\gamma(1) \in D$ among N is selected. This agent is denoted by $\tau(1,\omega) \in N$. At each step $h \in \{2,...,n\}$, the top-ranked agent at criterion $\gamma(h) \in D$ among the set of remaining agents $N \setminus \{\tau(1,\omega),...,\tau(h-1,\omega)\}$ is selected sequentially. This agent is denoted by

$$\tau(h,\omega) \in N \setminus \{\tau(1,\omega),...,\tau(h-1,\omega)\}.$$

We then define $C(\omega) \subset N$ by

$$C(\omega) = \{\tau(1,\omega),...,\tau(H,\omega)\}.$$

That is, the first H agents selected through these steps are assigned commodities.

The above specification originates from the reserve systems in emergency (Pathak et al., 2020), which were subsequently generalized by Matsushima (2021). Matsushima termed it *the method of procedure*. To emphasize consistent respect for individual *criteria* across various problems associated with different numbers of agents and units, Matsushima (2021) axiomatized the method of procedure from the viewpoint of inter-problem regularities. Matsushima then showed that this method is dominant for specifying reasonable SCRs, whenever it is impossible to aggregate conflicting criteria into a single criterion and, therefore, impossible to emphasize consistent respect for individual *agents* across problems. See Matsushima (2021) for details.

5.2. Specification of the Side-payment Rule

To implement the SCR C, the central planner specifies the side-payment rule t_i for each agent $i \in N$ according to the following steps. Fix an arbitrary profile of the other agents' types $\omega_{-i} \in \Omega_{-i}$. In step 1, the top-ranked agent at criterion $\gamma(1) \in D$ among $N \setminus \{i\}$ is selected. This agent is denoted by $\tau(1,\omega_{-i},i) \in N \setminus \{i\}$. At each step $h \in \{2,...,n-1\}$, the top-ranked agent at criterion $\gamma(h) \in D$ among the set of remaining agents $N \setminus \{i,\tau(1,\omega_{-i},i),...,\tau(h-1,\omega_{-i},i)\}$ is selected. This agent is denoted by $\tau(h,\omega_{-i},i) \in N \setminus \{i,\tau(1,\omega_{-i},i),...,\tau(h-1,\omega_{-i},i)\}$.

We select $\tau(i,d,\omega_{-i}) \in N \setminus \{i\}$ and $h(i,d,\omega_{-i}) \in \{1,...,H\}$ in the manner that

$$\tau(i,d,\omega_{-i}) = \tau(h(i,d,\omega_{-i}),\omega_{-i},i),$$

$$\gamma(h(i,d,\omega_{-i})) = d,$$

and

$$\gamma(h) \neq d \quad \text{for all} \quad h \in \{h(i,d,\omega_{-i})+1,...,H\}.$$

In other words, the agent $\tau(i,d,\omega_{-i})$ is selected in step $h(i,d,\omega_{-i})$ based on (justified by) criterion d. After step $h(i,d,\omega_{-i})$, criterion d is never used to select assigned agents. Hence, we can regard agent $\tau(i,d,\omega_{-i})$ as the last agent to be assigned based on criterion d, provided agent i is absent. Notably, we have

$$i \in C(\omega) \qquad \text{if there exists a criterion } d \in D \text{ such that}$$

$$\pi_d(i,\omega) < \pi_d(\tau(i,d,\omega_{-i}),\omega),$$

and

$$i \notin C(\omega) \qquad \text{if} \quad \pi_d(i,\omega) > \pi_d(\tau(i,d,\omega_{-i}),\omega) \quad \text{for all} \quad d \in D.$$

We denote the priority of the agent $\tau(i,d,\omega_{-i})$ at criterion d among all agents except for agent i by

$$H(i,d,\omega_{-i}) \in \{1,...,H\}.$$

We must note that

$$H(i,d,\omega_{-i}) = \pi_d(\tau(i,d,\omega_{-i}),\omega)$$

$$\text{if} \quad \pi_d(i,\omega) > \pi_d(\tau(i,d,\omega_{-i}),\omega),$$

and

$$H(i,d,\omega_{-i}) = \pi_d(\tau(i,d,\omega_{-i}),\omega) - 1$$

$$\text{if} \quad \pi_d(i,\omega) < \pi_d(\tau(i,d,\omega_{-i}),\omega).$$

Hence, we have

$$i \in C(\omega) \qquad \text{if}$$

$$\omega_i > \min_{d \in D} \frac{\max_{j \neq i}[w_j^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})\omega_j \mid H(i,d,\omega_{-i})]}{w_i^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})},$$

and

$$i \notin C(\omega) \qquad \text{if}$$

$$\omega_i < \min_{d \in D} \frac{\max_{j \neq i}[w_j^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})\omega_j \mid H(i,d,\omega_{-i})]}{w_i^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})}.$$

Based on these notations, to implement the SCR C, we specify the side-payment rule t_i for agent i as follows:

$$t_i(\omega) = -\min_{d \in D} \frac{\max_{j \neq i}[w_j^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})\omega_j \mid H(i,d,\omega_{-i})]}{w_i^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})}$$

$$\text{if } i \in C(\omega),$$

and

$$t_i(\omega) = 0 \qquad \text{if } i \notin C(\omega).$$

The interpretation is as follows. Fix $\omega \in \Omega$ and $i \in N$ arbitrarily. Virtually, there exist $\bar{d}$ markets, where in each market $d' \in D$, there exist $H(i,d',\omega_{-i})$ units of commodities to be sold. Agent i can purchase the commodity in any market $d' \in D$ for the price given by

$$\min_{d \in D} \frac{\max_{j \neq i}[w_j^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})\omega_j \mid H(i,d,\omega_{-i})]}{w_i^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})}.$$

Here, we can regard

$$\frac{\max_{j \neq i}[w_j^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})\omega_j \mid H(i,d,\omega_{-i})]}{w_i^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})}$$

as a variant of the uniform price in market d. Agent i can purchase the commodity for the lowest price across markets, regardless of the market they purchase in; that is, which criterion justifies their assignments. This specification of the side-payment rule guarantees that each winner pays the same price irrespective of which criterion justifies their assignment, thereby playing a central role in incentivizing each agent to be honest. Note that $H(i,d,\omega_{-i})$ is not greater than H, and it may be even less than H in any market $d \in D$, whereas $\sum_{d \in D} H(i,d,\omega_{-i})$ is never less than H.

5.3. Main Theorem

The following theorem shows that the central planner can successfully implement a reasonable compromise between the conflicting criteria; that is, the specified SCR C, in the dominant strategy. The lowest price for each agent i, given by

$$\min_{d\in D}\frac{\max_{j\neq i}[w_j^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})\omega_j \mid H(i,d,\omega_{-i})]}{w_i^d(\omega_i(H(i,d,\omega_{-i}),\omega_{-i}),\omega_{-i})},$$

is independent of their announcements. Hence, they pay the same monetary amount irrespective of their announcements whenever they can win the commodity. This independence, along with the definitions of $\omega_i(H(i,d,\omega_{-i})$, is crucial for proving the theorem.

Theorem 4: *Suppose that all criteria* $d\in D$ *satisfy Condition 2. The specified direct mechanism* (C,t) *is strategy-proof.*

Proof: Consider arbitrary $i\in N$, $\omega_i\in\Omega_i$, and $\tilde{\omega}\in\Omega$. Suppose that $i\notin C(\tilde{\omega})$; then, we have

$$v_i(C(\tilde{\omega}),\omega_i)+t_i(\tilde{\omega})=0.$$

Conversely, suppose that $i\in C(\tilde{\omega})$; then, we have

$$v_i(C(\tilde{\omega}),\omega_i)+t_i(\tilde{\omega})$$

$$=\omega_i-\min_{d\in D}\frac{\max_{j\neq i}[w_j^d(\omega_i(H(i,d,\tilde{\omega}_{-i}),\tilde{\omega}_{-i}),\tilde{\omega}_{-i})\tilde{\omega}_j \mid H(i,d,\tilde{\omega}_{-i})]}{w_i^d(\omega_i(H(i,d,\tilde{\omega}_{-i}),\tilde{\omega}_{-i}),\tilde{\omega}_{-i})},$$

which is positive if and only if $i\in C(\omega_i,\tilde{\omega}_{-i})$; that is,

$$\omega_i>\min_{d\in D}\frac{\max_{j\neq i}[w_j^d(\omega_i(H(i,d,\tilde{\omega}_{-i}),\tilde{\omega}_{-i}),\tilde{\omega}_{-i})\tilde{\omega}_j \mid H(i,d,\tilde{\omega}_{-i})]}{w_i^d(\omega_i(H(i,d,\tilde{\omega}_{-i}),\tilde{\omega}_{-i}),\tilde{\omega}_{-i})}.$$

Note that $v_i(C(\tilde{\omega}),\omega_i)+t_i(\tilde{\omega})$ is independent of $\tilde{\omega}_i$, provided that $i\in C(\tilde{\omega})$; that is,

$$\tilde{\omega}_i>\min_{d\in D}\frac{\max_{j\neq i}[w_j^d(\omega_i(H(i,d,\tilde{\omega}_{-i}),\tilde{\omega}_{-i}),\tilde{\omega}_{-i})\tilde{\omega}_j \mid H(i,d,\tilde{\omega}_{-i})]}{w_i^d(\omega_i(H(i,d,\tilde{\omega}_{-i}),\tilde{\omega}_{-i}),\tilde{\omega}_{-i})}.$$

These observations imply that truth-telling is a dominant strategy.

Q.E.D.

5.4. Discussion

The side-payment rule specified in Subsection 5.3 is an extension of that specified in Section 4. By considering two or more criteria simultaneously, instead of considering one criterion exclusively, not only who are the winners but also how much the winners pay will change. This change occurs through two routes, which have the following opposite effects.

The first route is because the winner pays only the lowest price in all markets, regardless of which criterion justifies their assignment. This device is necessary because it provides each agent with an incentive to be honest. Clearly, this device lowers the payment of an existing winner, compared to the case with fewer criteria, thereby subsidizing them. Interestingly, regardless of how much an additional criterion is considered in the procedure, the existing winners' payments can be dramatically reduced compared to when the criterion is not considered at all. The following example is helpful for understanding the first route.

Example 1 (Subsidies): Assume that there exist two criteria (i.e., $D = \{1, 2\}$) and for each $i \in N$,

$$\omega_i = 1,$$

$$w_i^1 = n - i + 1,$$

$$w_i^2 = n - i + 1 + y \quad \text{if } i \in \{1, ..., H\},$$

and

$$w_i^2 = n - i + 1 \quad \text{if } i \in \{H + 1, ..., n\},$$

where we assume $y > 0$. All agents have the same willingness to pay and, regardless of the criterion, each agent $i \in \{1, ..., n-1\}$ is prioritized to agent $i + 1$, that is,

$$\pi_1(i, \omega) = \pi_2(i, \omega) = i.$$

Regardless of the specification of a procedure γ, the assignment of the commodities to the top-H agents (i.e., agents 1, 2, ..., H) is optimal. The substantial difference between the criteria is that criterion 2 evaluates the welfare of the top- H agents greater than criterion 1. This serves to discount the winners' payments. We specify γ by

$$\gamma(1)=\gamma(2)=\cdots=\gamma(H-1)=1 \text{ and } \gamma(H)=2.$$

The central planner uses criterion 2 only in step H. In this case, the side-payment rule for each winner $i \in \{1,...,H\}$ is given by

$$t_i(\omega)=-\frac{w_{H+1}^2}{w_i^2}=-\frac{n-H}{n-i+1+y}.$$

Conversely, if the central planner considers only criterion 1, any winner $i \in \{1,...,H\}$ must pay $\frac{n-H}{n-i+1}$, which is greater than $\frac{n-H}{n-i+1+y}$ because of $y>0$.

The second route is because of the possibility that $H(i,d,\tilde{\omega}_{-i})$ is less than H at some criterion $d \in D$. This case occurs when the criteria are in great conflict over who should be prioritized. Note that (without an agent i), at each criterion $d \in D$, the top-$H(i,d,\tilde{\omega}_{-i})$ agents are assigned the commodities, and the $H(i,d,\tilde{\omega}_{-i})^{th}$ agent (i.e., agent $\tau(i,d,\omega_{-i})$) is the lowest-ranked agent who is assigned based on criterion d. If the criteria are in great conflict, there may exist agents who are assigned based on other criteria than criterion d but whose ranks are lower than $H(i,d,\tilde{\omega}_{-i})$ at criterion d. In this case, $H(i,d,\tilde{\omega}_{-i})$ must be less than H. Hence, the second route increases the winners' payments as more criteria are added. The following example is helpful for understanding the second route.

Example 2 (Set-Asides): We modify Example 1 by changing criterion 2 by

$$w_i^2=i \text{ for each } i \in \{1,...,H\}.$$

Hence, criteria 1 and 2 are in greatest conflict with one another, because

$$\pi_1(1,\omega)<\pi_1(2,\omega)<\cdots<\pi_1(n,\omega),$$

while

$$\pi_2(1,\omega)>\pi_2(2,\omega)>\cdots>\pi_2(n,\omega).$$

As the central planner uses criterion 2 only in step H, the top-($H-1$) agents at criterion 1 (i.e., agents 1, 2, ..., $H-1$) are assigned based on criterion 1, while the top-ranked agent at criterion 2 (i.e., agent n), who is the lowest-ranked at criterion 1, is assigned based on criterion 2. The side-payment rule for each agent $i \in \{1,...,H-1\}$, who wins the commodity based on criterion 1, is given by

$$t_i(\omega) = -\frac{w_H^1}{w_i^1} = -\frac{n-H+1}{n-i+1},$$

where $H(i,d,\tilde{\omega}_{-i}) = H-1 < H$ holds. Conversely, if the central planner considers only criterion 1, agent H becomes the winner instead of agent n, and any winner $i \in \{1,...,H\}$ pays $\frac{n-H}{n-i+1}$, which is cheaper than $\frac{n-H+1}{n-i+1}$.

Example 2 can be interpreted as a situation in which the central planner sets aside the commodity for agents who are judged by criterion 1 to be vulnerable. The related studies in this respect are those of Ayres and Cramton (1996), Pai and Vohra (2012), and Athey, et al. (2013). A substantial difference between these studies and ours is that we do not differentiate agents in advance: whether to set aside depends on the state. As the state is unknown to the central planner, they attempt to determine this decision endogenously by properly designing the incentive mechanism as outlined in this section.

6. Conclusion

This study investigated a mechanism design with side payments, where the central planner, who is ex-ante unaware of the state, attempts to implement ethically desirable SCR in the dominant strategy. As a significant departure from the previous literature on mechanism design, we assumed that the central planner determines which allocation is ethically desirable by making a strict distinction between the direct social merit of the allocation and individual agents' willingness to pay. Focusing on the multiunit auction problem with a single-unit demand, we showed that any reasonable SCR, which can be induced by the method of procedure explored by Matsushima (2021), is implementable in the dominant strategy, where we demonstrated a new side-payment rule design as a powerful incentive device.

Future research should consider the extension from the multiunit action with a single-unit demand to more general mechanism design problems. In Theorem 1 in Section 3, we proved that in general environments, any SCR associated with a single criterion is implementable in the dominant strategy, provided that the central planner is aware of agents' welfare weights even ex ante. Hence, it is important for future research to investigate more general environments where the central planner is ex-ante unaware of their (state-contingent) welfare weights.

Although not explicitly proven in this study, with the single-criterion assumption, it is almost self-evident that the possibility result in the multiunit auction with a single-unit demand (i.e., Theorem 3) can be extended to the position auction (multi-item auction) problem that Edelman et al. (2007) and Varian (2007) explored, by simply replacing the variant of the generalized VCG mechanism with a variant of their generalized second-price auction. Meanwhile, the extension of the result in the case with multiple conflicting criteria (i.e., Theorem 4) should be considered an essential issue.

In addition, by removing the single-unit demand assumption, we expect that a new research direction will be developed. A related study is that of Prendergast (2017), who designed and implemented a food bank system. This system distributes fake money to the persons in charge, and encourages them to participate in auctions for various kinds of food under the premise of not reselling to anyone other than the poor.

Along with the above-mentioned extensions, it is important to consider the case in which the welfare weights depend not only on the state but also on the allocation; that is, $\boldsymbol{w_i(\omega)}$ is replaced by $\boldsymbol{w_i(a,\omega)}$. In this case, the central planner is concerned about not only the state-contingent importance of each agent in the welfare evaluation but also on the state-contingent importance of which items are assigned to whom in the welfare evaluation.

Additional studies such as these are expected in the future.

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