Lecture 2: Asymmetric Treatment of Students Using Preference Rank Classes

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October 20, 2023

Introduction

The two main types of affirmative action policies are

1. quota/reserve-based
2. priority-based

In Lecture 1 we examined quota-based and reserve-based mechanisms.

In this lecture we look at a novel and less explicit type of affirmative action than either quota/reserve-based or priority-based policies.

This kind of policy treats students asymmetrically in terms of their reported preferences.

Overview: PRP Mechanisms

Based on **Ayoade and Pápai (2023)** [*Games and Economic Behavior*]:

"School Choice With Preference Rank Classes"

- Introduction of Preference Rank Partitioned (PRP) mechanisms:
    1. Definition
    2. Example
    3. Special Members and Classes
- Rank-Partition Stability and Modified Priority Profiles
- Near-Boston mechanisms: the only Pareto-efficient PRP mechanisms
- Incentives and the PRP Manipulation Theorem
- Equitable PRP mechanisms and PP-Stability

Overview: Asymmetric Treatment of Students Using PRP Mechanisms

- Non-equitable PRP mechanisms and Affirmative Action
- Favored (Minority) Student PRP Mechanisms

Overview: Asymmetric Treatment of Students Using PRP Mechanisms

- Non-equitable PRP mechanisms and Affirmative Action
- Favored (Minority) Student PRP Mechanisms

Next lecture: two more applications of PRP mechanisms to affirmative action:

1. Non-explicit "affirmative action" policies using PRP mechanisms:

   Progressive Choice mechanisms
   - based on **Liang and Pápai (2023)** [work in progress].

2. A more explicit reserve-based (PRP) mechanism "between" DA-R and IA-DA-R: demonstrating trade-offs

“School Choice With Preference Rank Classes” by Nickesha Ayoade and Szilvia Pápai

Introduction of
Preference Rank Partitioned (PRP) Mechanisms

Introduction of PRP Mechanisms

We study a large family of matching mechanisms:

Preference Rank Partitioned (PRP) mechanisms

- PRP mechanisms are Deferred Acceptance mechanisms with choice functions.
- Choice function for each school: specifies the set of selected students from each applicant pool.

Introduction of PRP Mechanisms

We study a large family of matching mechanisms:

Preference Rank Partitioned (PRP) mechanisms

- PRP mechanisms are Deferred Acceptance mechanisms with choice functions.
- Choice function for each school: specifies the set of selected students from each applicant pool.
- Choice functions are based on a partition of both student preference ranks and school priority ranks.
- The choice functions select students from the applicant pool lexicographically:
  Step 1: based on the **priority classes**
  Step 2: based on the **preference classes**
  Step 3: based on the **tie-breaker: strict priority ordering**

Unified Framework

The family of PRP mechanisms provides a unified framework to study many known (classes of) matching mechanisms as well as new ones.

The family of PRP mechanisms includes:

1. Deferred Acceptance (Gale and Shapley, 1962)
2. Boston/Immediate Acceptance (Abdulkadiroğlu and Sönmez, 2003)
3. First-Preference-First (Pathak and Sönmez, 2013)
4. Application-Rejection/Parallel (Chen and Kesten, 2017)
5. Secure Boston (Dur et al., 2019)
6. French Priority (Bonkoungou, 2019)

Papers on Previously Used or Studied PRP Mechanisms

Pathak and Sönmez (2013):
Studies the First-Preference-First mechanisms which were banned in school choice in England.

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Proposes the Secure Boston mechanism to improve upon the Boston/IA mechanism.

Bonkoungou (2019):
Analyzes the French Priority mechanisms used until recently in university admissions in France.

Deferred Acceptance (DA) with School Choice Functions

- **Step 1:**
  Each student applies to her first-ranked school. Each school tentatively assigns its seats according to its *choice function.* Any remaining applicants are rejected.
- **Step t:**
  Each student who was rejected in the previous step applies to her next-ranked school. Each school considers the students who are tentatively assigned to the school, if any, together with its new applicants (the "applicant pool") and tentatively assigns its seats according to its *choice function.* Any remaining applicants are rejected.

The algorithm terminates when each student is either tentatively assigned to some school or has been rejected by each school and thus remains unassigned.

Preference Rank Partitioned (PRP) Mechanisms

1. **Priority (rank) classes:** partition each school's priority ordering $\succ_c$ by specifying the number of consecutively ranked students in each member of the partition.
   $\Rightarrow$ **priority partition profile v**
2. **Preference (rank) classes:** partition each student's preference ordering P_i by specifying the number of consecutively ranked schools in each member of the partition.
   $\Rightarrow$ **preference partition profile x**

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   $\Rightarrow$ **preference partition profile x**

These partitions are applied to the profile $(\succ, P)$ consisting of a strict priority profile $\succ$ and a strict preference profile P of a given matching problem.

Each pair of a priority partition profile and a preference partition profile (v,x) specifies a matching mechanism within the class of PRP mechanisms.

Preference Rank Partitioned Mechanisms

Based on the priority and preference partition profiles (v, x), run the DA with the PRP choice function corresponding to (v, x) for each school.

Each school $c \in C$ selects among students in its applicant pool in each round as follows:

1. First selects students up to the fixed capacity of the school in its highest **priority classes,** given v.

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1. First selects students up to the fixed capacity of the school in its highest **priority classes,** given v.
2. If the priority classes do not determine the selection, then selects students who rank school c in the highest possible **preference classes,** given x.
3. If the preference partition still does not determine the selection then the choice is resolved based on the **strict priority ordering** $\succ_c$. This is a tie-breaking step.

PRP Mechanisms

Example (PRP choice functions)

School priorities $\succ$

$\succ_a$	$\succ_b$	$\succ_c$
1	3	2
2	2	1
3	4	4
4	1	3

Student preferences P

P_1	P_2	P_3	P_4
b	d	b	b
c	b	a	a
a	a	d	d
d	c	c	c

Assume that school a has capacity 1. Let the applicant pool for school a be $\{1, 2, 4\}$.

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Student 4 is eliminated based on the priority classes of school a. This leaves students 1 and 2.

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Assume that school a has capacity 1. Let the applicant pool for school a be $\{1, 2, 4\}$.

Student 4 is eliminated based on the priority classes of school a. This leaves students 1 and 2.

Student 2 is selected based on preference classes: 2 ranks a in the highest preference class and 1 ranks a in the second highest preference class.

PRP Mechanisms

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3	4	4
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Student preferences P

P_1	P_2	P_3	P_4
b	d	b	b
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d	c	c	c

Now consider the same problem but a different PRP mechanism: agent 2's preference classes are different.

Given applicant pool $\{1, 2, 4\}$ for school a, student 4 is eliminated based on the priority classes of school a, as before. This leaves students 1 and 2.

PRP Mechanisms

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Student preferences P

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b	d	b	b
c	b	a	a
a	a	d	d
d	c	c	c

Now consider the same problem but a different PRP mechanism: agent 2's preference classes are different.

Given applicant pool $\{1, 2, 4\}$ for school a, student 4 is eliminated based on the priority classes of school a, as before. This leaves students 1 and 2.

But now a selection cannot be made between 1 and 2 based on the preference classes, since both rank a in the second preference class.

Break the tie based on the strict priority order $\succ_a$: student 1 is selected.

Exogenous Coarse Priorities (a detour)

PRP mechanisms can be defined with exogenously given **coarse priorities**.

- If coarse priorities are given exogenously, adopt these as the priority rank classes in a PRP mechanism (primitives).
- A PRP mechanism is specified by $(x, \succ)$: a preference rank partition profile and a strict priority profile for tie-breaking.
- This is the approach of Bonkoungou (2019) for French Priority mechanisms (for which preference partitions are restricted to be the finest).

(Standard) DA as a PRP Mechanism

Choice function:

Step 1 or **Step 3:** Select students from the applicant pool up to the capacity according to the given strict priorities.

Skip **Step 2**.

DA Mechanism	Priority Partition (Step 1)	Preference Partition (Step 2)
	Finest/Coarsest	Coarsest

Note: The priority partition is arbitrary, since there is no selection based on preference classes.

Boston/IA as a PRP Mechanism

Choice function:

Step 1: Skip.

Step 2: Select students from the applicant pool based on the preference rankings.

Step 3: If selection is not resolved then apply the strict priority tie-breaking.

Boston/IA Mechanism	Priority Partition (Step 1)	Preference Partition (Step 2)
	Coarsest	Finest

First-Preference-First as PRP Mechanisms

Choice function:

Step 1:
- Equal-preference schools select students from their applicant pool based on the finest priorities (resolved).
- Preference-first schools don't make any selection (not resolved).

Step 2: Preference-first schools select based on the preference rankings (just like Boston/IA).

Step 3: If selection is still not resolved for preference-first schools then apply the strict priority tie-breaking.

First-Preference-First Mechanisms	Priority Partition (Step 1)	Preference Partition (Step 2)
	Equal-preference schools: finest Preference-first schools: coarsest	Finest

Secure Boston/IA as a PRP Mechanism

Choice function:

Step 1: Each school c selects students from the applicant pool such that it first chooses the top q_c-ranked students in the applicant pool.

Step 2: If selection is not resolved, select based on the preference rankings (just like Boston/IA).

Step 3: If selection is still not resolved then apply the strict priority tie-breaking.

Secure Boston/IA Mechanism	Priority Partition (Step 1)	Preference Partition (Step 2)
	For each school c: finest for top q_c, then coarsest	Finest

French Priority Mechanisms as PRP Mechanisms

Choice function:

Step 1: Select students from the applicant pool based on the priority classes (or exogenously given coarse priorities).

Step 2: If selection is not resolved, select based on the preference rankings (just like Boston/IA).

Step 3: If selection is still not resolved then apply the strict priority tie-breaking.

French Priority Mechanisms	**Priority Partition (Step 1)**	**Preference Partition (Step 2)**
	Arbitrary	Finest

Application-Rejection (Parallel) Mechanisms as PRP Mechanisms

Choice function:

Step 1: Skip.

Step 2: Select students from the applicant pool based on the preference rank classes that are homogeneous across students.

Step 3: If selection is not resolved then go to the strict priority tie-breaker.

Application-Rejection Mechanisms	**Priority Partition (Step 1)**	**Preference Partition (Step 2)**
	Coarsest	Homogeneous (same for each student)

Summary: Special Members/Classes of PRP Mechanisms

PRP Mechanisms	**Priority Partition**	**Preference Partition**
DA	Finest/Coarsest	Coarsest
Boston/IA	Coarsest	Finest
First-Preference-First	Equal-preference schools: finest Preference-first schools: coarsest	Finest
Secure Boston/IA	For each school c: finest for top q_c, then coarsest	Finest
French Priority	Arbitrary	Finest
Application-Rejection	Coarsest	Homogeneous (same for each student)

Rank-Partition Stability and Modified Priority Profiles

Rank-Partition Stability

Definition

A matching mechanism f is **rank-partition-stable** if there exist priority and preference rank partition profiles (v, x) such that

1. the selected matching is stable at each preference profile with respect to the modified strict priority profile $\bar{\succ}$ which
   - preserves students' priorities across priority rank classes
   - within each priority rank class orders students according to their preference rank partitions at the given preference profile
   - uses the given strict priorities to break ties if necessary
2. the selection of the stable matching only depends on this modified strict priority profile at each preference profile.

Construction of a Modified Strict Priority Profile

Example

School priorities $\succ$

$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$
4	3	4	4
1	1	1	5
2	5	5	3
3	2	2	1
5	4	3	2

Student preferences P

P_1	P_2	P_3	P_4	P_5
b	b	b	b	d
c	c	a	a	a
a	a	d	c	0
d	d	c	d	

Modified priority profile at P

$\bar{\succ}_a$	$\bar{\succ}_b$	$\bar{\succ}_c$	$\bar{\succ}_d$
1	3	1	5
2	1	4	4
4	2	2	3
5	4	3	1
3	5	5	2

Stability and Optimality Properties of PRP Mechanisms

Proposition 1

Each PRP mechanism $f^{v,x}$ is rank-partition stable and $f^{v,x}(\succ, P)$ is the unique (v, x)-optimal rank-partition stable matching at each profile $(\succ, P)$.

Near-Boston Mechanisms: the Only Pareto-Efficient PRP Mechanisms

Efficiency Properties of PRP Mechanisms

Near-Boston (Near-IA) mechanisms are PRP mechanisms such that

1. each school has the coarsest priority partition;
2. there exists $i \in S$ such that each student $s \in S \setminus \{i\}$ has the finest preference partition (while student i has an arbitrary preference partition).

Efficiency Properties of PRP Mechanisms

Near-Boston (Near-IA) mechanisms are PRP mechanisms such that

1. each school has the coarsest priority partition;
2. there exists $i \in S$ such that each student $s \in S \setminus \{i\}$ has the finest preference partition (while student i has an arbitrary preference partition).

Theorem 1

A rank-partition stable mechanism is Pareto-efficient if and only if it is a Near-Boston mechanism.

Thus, only the Near-Boston mechanisms are Pareto-efficient within the class of PRP mechanisms.

Incentives and the PRP Manipulation Theorem

Incentive Properties of PRP Mechanisms

Definitions

For matching mechanism φ, given a profile $(\succ, P)$, if there is a student $s \in S$ and an alternative preference ranking $P'_s \in \mathcal{P}_s$ such that $\varphi_s(\succ, (P'_s, P_{-s}))P_s\ \varphi_s(\succ, P)$ then s can **manipulate** φ at P via P'_s, and mechanism φ is **manipulable** at P.

We will also say that s **can manipulate** at P **to obtain school** $\varphi_s(\succ, (P'_s, P_{-s}))$.

If a matching mechanism is not manipulable at any preference profile P then it is **strategyproof.**

Incentive Properties of PRP Mechanisms

Theorem 2

The only strategyproof rank-partition stable mechanism is the DA.

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Note:

- The theorem implies that the only strategyproof PRP mechanism is the DA, which generalizes a similar result by Chen and Kesten (2017) on Application-Rejection mechanisms.
- The intuition for this result is that the **DA is the only PRP mechanism whose choice function is independent of the preferences.**
- The preference partition does not play any role in the selection of students in the DA, since it is the coarsest for each student.

Manipulability of PRP Mechanisms

Theorem 3: PRP Manipulation Theorem

Let f be a PRP mechanism. Let $i \in S$, $c \in C$ and let P, P'_i be such that $c \; P_i \; f_i(P)$ and c is in the same or a lower preference class in P'_i than in P_i. Then $f_i(P'_i, P_{-i}) \neq c$.

Manipulability of PRP Mechanisms

Theorem 3: PRP Manipulation Theorem

Let f be a PRP mechanism. Let $i \in S$, $c \in C$ and let P, P_i' be such that $c\ P_i\ f_i(P)$ and c is in the same or a lower preference class in P_i' than in P_i. Then $f_i(P_i', P_{-i}) \neq c$.

- The theorem says that student i cannot manipulate a PRP mechanism to obtain a seat at school c by placing school c in the same or a lower preference rank class than where it is truthfully.
- Thus, manipulation is only possible if the school is reported to be in a higher preference rank class.

Manipulability of PRP Mechanisms: Corollaries

Corollary: Students with the coarsest preference partition

Given a PRP mechanism, if student i has the coarsest preference partition then i cannot manipulate at any profile.

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Given a PRP mechanism, if student i has the coarsest preference partition then i cannot manipulate at any profile.

Corollary: Schools in the top preference rank class

Given a PRP mechanism, if student i ranks some school c in her top preference rank class at some preference profile P then i cannot manipulate at P to obtain c.

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Corollary: Students with the coarsest preference partition

Given a PRP mechanism, if student i has the coarsest preference partition then i cannot manipulate at any profile.

Corollary: Schools in the top preference rank class

Given a PRP mechanism, if student i ranks some school c in her top preference rank class at some preference profile P then i cannot manipulate at P to obtain c.

Corollary: Reshuffle within preference rank classes

Given a PRP mechanism, if student i misrepresents her preferences by reporting a reshuffle of the schools within her preference rank classes then i cannot manipulate.

Manipulability of PRP Mechanisms

- By the PRP Manipulation Theorem, a **seat at a school may only be obtainable by manipulation when reporting the school to be in a higher preference class than it truthfully is.**

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- By the PRP Manipulation Theorem, a **seat at a school may only be obtainable by manipulation when reporting the school to be in a higher preference class than it truthfully is.**
- This implies that the (standard) **Deferred Acceptance mechanism is strategyproof,** since for the DA each student has the coarsest preference partition.
- At the other extreme, the Boston/IA mechanism is the PRP mechanism for which each student has the finest preference partition, and thus the theorem sheds light on **why the Boston/IA mechanism is very manipulable theoretically:**
    - each swap in the reported preference ordering results in placing at least one school in a higher preference class;
    - with the exception of the top-ranked school, all schools may be obtainable by manipulation when the Boston/IA mechanism is used.

Equitable PRP Mechanisms and PP-Stability

Equitable PRP Mechanisms

Definition

An **Equitable PRP mechanism** is a PRP mechanism with a homogeneous preference partition across students (i.e., the same preference partition for each student).

Note: All previously used and studied PRP mechanisms are Equitable PRP mechanisms (see the summary table).

We will characterize Equitable PRP mechanisms using a weaker axiom than the standard stability notion, which we call **PP-Stability** (Priority/Preference-Rank Stability).

Fairness and Stability: Standard Definitions

- Student i has **justified envy** in μ at $(\succ, P)$ if there exist school $c \in C$ and student $j \in S$ such that:
  - $c \; P_i \; \mu_i$
  - $i \succ_c j$
  - $\mu_j = c$

Note: In other words, j *violates* i's *priority* (at school c) in matching μ.

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Note: In other words, j *violates* i's *priority* (at school c) in matching μ.

- A matching μ is **fair** at $(\succ, P)$ if there is no student i who has justified envy in μ at $(\succ, P)$.
- A matching μ is **stable** at $(\succ, P)$ if it is individually rational, non-wasteful and fair at $(\succ, P)$.

PP-Stability: A Weaker Axiom than Stability

- Student i has **PP-justified envy** in μ at $(\succ, P)$ if there exist school $c \in C$ and student $j \in S$ such that:
  - $c \; P_i \; \mu_i$
  - $r_i(c) \leq r_j(c)$ (i ranks c at least as high as j)
  - $i \succ_c j$
  - $\mu_j = c$

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  - $i \succ_c j$
  - $\mu_j = c$
- A matching μ is **PP-fair** at P if there is no student i who has PP-justified envy in μ at $(\succ, P)$.
- A matching μ is **PP-stable** at P if it is individually rational, non-wasteful and PP-fair at $(\succ, P)$.

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- A matching μ is **PP-fair** at P if there is no student i who has PP-justified envy in μ at $(\succ, P)$.
- A matching μ is **PP-stable** at P if it is individually rational, non-wasteful and PP-fair at $(\succ, P)$.

Proposition 2

A PRP mechanism is PP-stable if and only if it is an Equitable PRP mechanism.

Stability Properties of PRP Mechanisms

Definition

If the preference rank partition profile used in the construction of the modified priority profile is homogeneous, then a rank-partition stable matching mechanism is **equitable-rank-partition stable**.

Stability Properties of PRP Mechanisms

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If the preference rank partition profile used in the construction of the modified priority profile is homogeneous, then a rank-partition stable matching mechanism is **equitable-rank-partition stable**.

Theorem 4

A rank-partition stable matching mechanism is PP-stable if and only if it is equitable-rank-partition stable.

Note: This generalizes Proposition 2 on Equitable PRP mechanisms.

Corollary to Theorems 1 and 4

An equitable-rank-partition stable mechanism is Pareto-efficient if and only if it is the Boston/IA mechanism.

Note: It is implied that the only Equitable PRP mechanism that is Pareto-efficient is the Boston/IA mechanism.

Non-Equitable PRP Mechanisms and Affirmative Action

Example of a PRP mechanism which is not PP-Stable

Example

School priorities $\succ$

$\succ_a$	$\succ_b$	$\succ_c$	$\succ_d$
4	3	4	4
1	1	1	5
2	4	5	3
3	2	2	1
5	5	3	2

Student preferences P

P_1	P_2	P_3	P_4	P_5
b	b	b	b	d
c	c	a	a	a
a	a	d	c	0
d	d	c	d	

Each school has capacity 1.

PRP steps				
Step	a	b	c	d
1		$1, 2, \underline{3}, 4$		$\underline{5}$
2	$\underline{4}$	$\underline{3}$	$\underline{1}, 2$	$\underline{5}$
3	$\underline{2}, 4$	$\underline{3}$	$\underline{1}$	$\underline{5}$
4	$\underline{2}$	$\underline{3}$	$\underline{1}, 4$	$\underline{5}$
5	$\underline{2}$	$\underline{3}$	$\underline{1}$	$\underline{5}, 4$

Example of a PRP Mechanism which is Not PP-Stable

Example

PRP steps				
Step	*a*	*b*	*c*	*d*
1		1, 2, $\underline{3}$, 4		$\underline{5}$
2	$\underline{4}$	$\underline{3}$	$\underline{1}$, 2	$\underline{5}$
3	$\underline{2}$, 4	$\underline{3}$	$\underline{1}$	$\underline{5}$
4	$\underline{2}$	$\underline{3}$	$\underline{1}$, 4	$\underline{5}$
5	$\underline{2}$	$\underline{3}$	$\underline{1}$	$\underline{5}$, 4

PP-stability is violated since

- 2 is assigned to school *a* which 4 envies
- 4 ranks school *a* higher than 2
- 4 has a higher priority for school *a* than 2

Non-Equitable PRP Mechanisms and Affirmative Action

- The mechanism in the example is a Non-Equitable PRP mechanism by Proposition 2.
- Since it **treats students asymmetrically** due to their different preference rank partitions, such a mechanism could be used for affirmative action.

Non-Equitable PRP Mechanisms and Affirmative Action

- The mechanism in the example is a Non-Equitable PRP mechanism by Proposition 2.
- Since it **treats students asymmetrically** due to their different preference rank partitions, such a mechanism could be used for affirmative action.
- Non-Equitable PRP mechanisms provide a **novel way to do affirmative action.**
- While less direct than quota/reserve-based or priority-based affirmative action policies, Non-Equitable PRP mechanisms may be more palatable in a climate where affirmative action is controversial (or banned).
- We explore a simple (and extreme) Non-Equitable PRP mechanism next, but there are many Non-Equitable PRP Mechanisms that the designer may choose from.

Favored (Minority) Students PRP Mechanisms

Favored Students PRP Mechanisms

PRP Mechanisms	Priority Partition	Preference Partition
DA	Finest/Coarsest	Coarsest
Boston/IA	Coarsest	Finest
First-Preference-First	Equal-preference schools: finest Preference-first schools: coarsest	Finest
Secure Boston/IA	For each school c: finest for top q_c, then coarsest	Finest
French Priority	Arbitrary	Finest
Application-Rejection	Coarsest	Homogeneous
Favored Students	Coarsest	Favored students: coarsest Remaining students: finest

Favored Minority Students Mechanisms

Given that in our model students are partitioned into minority and majority students, it is natural to let minority students be the favored students.

Step 1: Skip.

Step 2: Select students from the applicant pool based on preference rank classes as follows:

- **Minority students:** coarsest preference partition (each school is in the first preference class).
- **Majority students:** finest preference partition.

Step 3: If selection is not resolved then apply the strict priority tie-breaking.

Favored Minority Students Mechanisms: Properties

Efficiency:

- A Favored Minority Students mechanism is only Pareto-efficient if there is at most one favored/minority student.
- If there are no minority students then this mechanism becomes the Boston/IA mechanism, which is Pareto-efficient.
- The intuition is that the fewer minority students there are, the more efficient the mechanism becomes (although the exact relationship may not hold).

Favored Minority Students Mechanisms: Properties

Manipulation:

- Since minority students have the coarsest preference partition, it follows from the PRP Manipulation Theorem that the Favored Minority Students mechanism is stratergyproof for minority students.

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Manipulation:

- Since minority students have the coarsest preference partition, it follows from the PRP Manipulation Theorem that the Favored Minority Students mechanism is stratergyproof for minority students.
- However, it is obviously manipulable by majority students.
- This contrasts with the IA-DA-R mechanism which is obviously manipulable by minority students, and not obviously manipulable by majority students.
- The intuition is that the fewer minority students there are, the more manipulable the mechanism becomes (although the exact relationship may not hold).

Summary

- Unified framework to analyze the stability, efficiency and incentive properties of matching mechanisms for which student selection is based on preference ranks in addition to priorities: PRP mechanisms.
- Many PRP mechanisms are (or were) used in practice in school placement or university admissions.
- New insights into the manipulability of some prominent matching mechanisms (DA, Boston/IA) and other knownn PRP mechanisms.
- Priority and preference partitions in PRP mechanisms indicate trade-offs: stability/incentives versus efficiency:
  - For fewer priority violations and better incentives choose finer priority partitions and/or coarser preference partitions.
  - For more efficiency choose coarser priority partitions and/or finer preference partitions.

Implications for Affirmative Action Design

- Non-Equitable PRP mechanisms provide an alternative method to implement an affirmative action policy and, more generally, a preferential treatment policy.
- There are many ways to treat students asymmetrically through the choice of their individual preference rank partitions, so the designer has flexibility when choosing the mechanism.
- These mechanisms may be less controversial than quota/reserve-based or direct priority-based mechanisms as they are less explicit.
- An extreme affirmative action mechanism which lets minority students have the coarsest preference partition, the Favored Minority Student mechanism, guarantees strategyproofness for minority students.

References I

Abdulkadiroğlu, A. and Sönmez, T. (2003). School choice: A mechanism design approach. *American Economic Review*, 93(3):729–747.

Ayoade, N. and Pápai, S. (2023). School choice with preference rank classes. *Games and Economic Behavior*, 137:317–341.

Bonkoungou, S. (2019). Deferred acceptance under the French tie-breaking rule.

Chen, Y. and Kesten, O. (2017). Chinese college admissions and school choice reforms: A theoretical analysis. *Journal of Political Economy*, 125(1):99–139.

Dur, U., Hammond, R. G., and Morrill, T. (2019). The secure Boston mechanism: Theory and experiments. *Experimental Economics*, 22(4):918–953.

Gale, D. and Shapley, L. S. (1962). College admissions and the stability of marriage. *The American Mathematical Monthly*, 69(1):9–15.

References II

Liang, Y. and Pápai, S. (2023). Progressive choices in school choice problems.

Pathak, P. A. and Sönmez, T. (2013). School admissions reform in Chicago and England: Comparing mechanisms by their vulnerability to manipulation. *American Economic Review*, 103(1):80–106.