Lecture 1: Axiomatic Affirmative Action Design in School Choice

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October 16, 2023

Overview: Part 1 - Introduction

- School choice model (many-to-one matching)
- Introduction of basic matching mechanisms:
  1. Deferred Acceptance (DA)
  2. Efficiency-Adjusted DA (EADA)
  3. Immediate Acceptance (IA)
- Different types of affirmative action policies
- Minimal responsiveness of affirmative action policies
- Mechanisms with quotas or reserves

School Choice Model

A standard many-to-one matching model first studied by Abdulkadiroğlu and Sönmez (2003).

- Set of students S
- Set of schools C; capacities $q = (q_c)$
- Each student $s \in S$ has a **strict preference ordering** P_s over $C \cup \{0\}$, where 0 means staying unmatched
- $P = (P_s)_{s \in N}$ is a preference profile of students
- $\mathcal{P}$ is the set of all preference profiles
- Each school $c \in C$ has a **strict priority ordering** $\succ_c$ over S
- $\succ = (\succ_c)_{c \in C}$ is a priority profile of schools
- Π is the set of all priority profiles
- A **matching** μ is an assignment of students to schools such that no more than q_c students are assigned to each school $c \in C$
- $\mathcal{M}$ is the set of matchings

Basic Matching Mechanisms

Deferred Acceptance Mechanism (DA)

Gale and Shapley (1962):

- *Step* 1:
  Each student applies to her first-ranked school. Each school **tentatively** assigns its seats according to its priorities up to its capacity. Any remaining applicants are rejected.
- *Step* t: $(t \geq 2)$
  Each student who was rejected in the previous step applies to her next-ranked acceptable school. Each school considers the students who are tentatively assigned to the school, if any, together with its new applicants (the "applicant pool") and **tentatively** assigns its seats according to its priorities up to its capacity. Any remaining applicants are rejected.

- The algorithm terminates when each student is either tentatively assigned to some school or has been rejected by each acceptable school and thus remains unmatched.

- The tentative assignments in the last step become final assignments.

Efficiency-Adjusted DA (EADA)

Kesten (2010):

- EADA runs the DA algorithm multiple times iteratively, after adjustments are made to the preferences of specific *interrupters* in each round, until the final matching becomes Pareto-efficient.
- **Interrupter:** An agent whose application causes another agent to be rejected by a school, but eventually this agent is also rejected by this school.
- Interrupters may cause efficiency loss.
- Find all the interrupters in the last step of the DA algorithm where there are interrupters, and drop the school at which they are interrupters to the bottom of the preference list (or make the school unacceptable).
- Re-run the DA algorithm with this (efficiency) adjusted preference profile. Repeat as many times as necessary.
- This procedures guarantees a Pareto-efficient matching, which (weakly) Pareto-dominates the DA matching.

Immediate Acceptance Mechanism (IA) (also known as Boston)

- *Step* 1:
  Each student applies to her first-ranked school. Each school **permanently** assigns its seats according to its priorities up to its capacity. Any remaining applicants are rejected.
- *Step* t: $(t \geq 2)$
  Each student who was rejected in the previous step applies to her next-ranked acceptable school. Each school **permanently** assigns its remaining seats according to its priorities up to its capacity. Any remaining applicants are rejected.

- The algorithm terminates when each student is either assigned to some school or has been rejected by each acceptable school and thus remains unmatched.

Different Types of Affirmative Action Policies

School Choice Model with Majority/Minority Students

A simple model for affirmative action first studied by Kojima (2012).

New:

- The set of students S is divided into majority students S^M and minority students S^m.

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Two Main Types of Affirmative Action Policies

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In today's lecture we will focus on quota-based and reserve-based affirmative action policies.

Minimal Responsiveness of Affirmative Action Policies

Minimal Responsiveness: Kojima (2012)

The minimal responsiveness axiom for mechanisms with an affirmative action policy was first proposed by Kojima (2012).

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- **Intuition**: a stronger affirmative action policy should not harm the minority students.
- **Weak requirement:** it only requires that at least one minority student is better off if the matching is affected at all for minority students.
- What is a **"stronger" affirmative action policy?** It depends on the type of affirmative action policy used.
- For quota-based and reserve-based affirmative action it means an unambiguous increase in the quotas or reserves.
- For priority-based affirmative action it means an increase in the priorities of minority students, while the priorities among majority students, as well as among minority students, remain the same.

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- What is a **"stronger" affirmative action policy?** It depends on the type of affirmative action policy used.
- For quota-based and reserve-based affirmative action it means an unambiguous increase in the quotas or reserves.
- For priority-based affirmative action it means an increase in the priorities of minority students, while the priorities among majority students, as well as among minority students, remain the same.
- A perplexing paper, Kojima (2012), shows that no stable mechanism with a quota-based or priority-based affirmative action policy is minimally responsive.

Affirmative Action Policies with Quotas and Reserves

Affirmative Action Policies

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- **Majority quotas** q_c^M: specify the maximum number of seats that can be given to majority students at each school c.
- **Minority reserves** r_c: specify the number of seats for which minority students are prioritized over majority students at each school c.

Mechanisms with Quotas or Reserves

DA-based mechanisms: DA-Q, DA-R

acceptances are temporary

Efficiency improvements over DA-R: MDA, EIDA

à la Kesten (2010), similar to EADA

IA-based mechanisms: IA-Q, IA-R

acceptances are permanent

Affirmative Action Mechanisms Based on Deferred Acceptance: DA-Q and DA-R

DA with Majority Quotas (DA-Q):

- The DA-Q mechanism is based on the DA. The only difference is that it does not let the number of majority students exceed the majority quota at any school.
- The DA-Q is due to Abdulkadiroğlu and Sönmez (2003) in a more general model and was first studied in this form by Kojima (2012).

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DA with Minority Reserves (DA-R):

- The DA-R mechanism is a DA mechanism with the following choice function: each school in each round first accepts minority students for the minority reserves, and then accepts students from the rest of the applicant pool for the remaining seats.
- DA-R was proposed by **Hafalir, Yenmez and Yildirim (2013)** to reduce the inefficiency of the DA-Q; it eliminates the wastefulness of DA-Q.

Affirmative Action Mechanisms Based on Efficiency Improvements: MDA and EIDA

Modified DA with Minority Reserves (MDA):

- The MDA mechanism modifies the DA-R mechanism based on the concept of **interferers**.
- An interferer is a minority student who causes the school to reject a majority student in some step of the DA-R algorithm due to minority reserves, but in a later step this minority student is also rejected by this school (compare to Kesten's interrupters).
- By treating specific interferers as majority students in iterative steps (in the spirit of EADA), the DA-R matching obtained in the last iteration is the MDA matching.
- This mechanism was proposed by **Doğan (2016);** it is the first minimally responsive mechanism in the literature.

Affirmative Action Mechanisms Based on Efficiency Improvements: MDA and EIDA

Efficiency Improved DA with Minority Reserves (EIDA):

- EIDA implements efficiency improvements over the DA-R, exactly as the EADA mechanism of Kesten (2010) improves DA.
- Proposed by Ju et al. (2018).
- It was shown by Ding et al. (2019) that it is not minimally responsive.

Affirmative Action Mechanisms Based on Immediate Acceptance: IA-Q and IA-R

IA with Majority Quotas (IA-Q):

- The IA-Q mechanism is based on the IA mechanism. The only difference is that once a school accepts enough majority students to fill its majority quota, it does not accept more majority students.
- Ergin and Sönmez (2006) introduced the IA mechanism with a fixed quota for each type in a more general model. The IA-Q mechanism only has a cap for majority students.

Affirmative Action Mechanisms Based on Immediate Acceptance: IA-Q and IA-R

IA with Minority Reserves (IA-R):

- The IA-R mechanism is an IA mechanism with the following choice function: each school in each round first accepts minority students for the minority reserves, and then accepts students from the rest of the applicant pool for the remaining seats. The difference from the DA-R is that acceptances are **permanent** in each round.
- It is the same as the IA Mechanism with Affirmative-Action-Target introduced by Doğan and Klaus (2018).

Both IA-Q and IA-R are special cases of the class of combined quota-based and reserve-based IA mechanisms studied by Afacan and Salman (2016).

“Affirmative Action Policies in School Choice: Immediate versus Deferred Acceptance”

by Muntasir Chaudhury and Szilvia Pápai

Overview: Part 2

Based on **Chaudhury and Pápai (2023)** [working paper]:

"Affirmative Action Policies in School Choice:
Immediate versus Deferred Acceptance"

- We study **three welfare axioms** for mechanisms with an affirmative action policy that sets aside seats for minority students:
  1. non-wastefulness
  2. respecting the affirmative action policy
  3. minimal responsiveness

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- We show that all previously studied mechanisms with an affirmative action policy fail to satisfy at least one of the three welfare axioms.

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- We introduce the **Immediate and Deferred Acceptance Mechanism with Minority Reserves (IA-DA-R)** which satisfies all three axioms.

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- We show that all previously studied mechanisms with an affirmative action policy fail to satisfy at least one of the three welfare axioms.
- We introduce the **Immediate and Deferred Acceptance Mechanism with Minority Reserves (IA-DA-R)** which satisfies all three axioms.
- Fairness and incentive axioms are also analyzed, and we establish some possibility and impossibility results.

Affirmative Action Policies: Majority Quotas and Minority Reserves

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Minority allotments: a common framework

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Minority allotments: a common framework

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Minority allotments: a common framework

majority quotas - the number of seats that remain in excess of the majority quotas: $v_c = q_c - q_c^M$

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Feasible minority allotment policies: $v = (v_c)_{c \in C}$ such that for each school $c \in C$, $0 \leq v_c \leq q_c$

Let $\mathcal{V}$ denote the set of feasible minority allotment policies.

Mechanisms with Minority Allotments

A **mechanism with minority allotments** is $\varphi : \mathcal{V} \times \mathcal{P} \times \Pi \to \mathcal{M}$.

It assigns a matching μ to each minority allotment policy $v \in \mathcal{V}$ and each profile $(P, \succ) \in \mathcal{P} \times \Pi$.

Three Welfare Axioms

Welfare Axiom 1: Non-Wastefulness

Non-Wastefulness

A mechanism φ is non-wasteful if for all $v \in \mathcal{V}$, $(P, \succ) \in \mathcal{P} \times \Pi$, $s \in S$ and $c \in C$, if $c \; P_s \; \varphi_s(v, P, \succ)$ then $|\mu_c| = q_c$, where $\varphi(v, P, \succ) = \mu$.

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- This is a basic efficiency requirement which is typically easy to satisfy without an affirmative action policy.
- A non-wasteful mechanism may become wasteful when it incorporates an affirmative action policy.
- Quota-based mechanisms are wasteful.
- /Note: We define non-wastefulness to imply *individual rationality* (i.e., no student is assigned to an unacceptable school)./

Welfare Axiom 2: Respecting the Affirmative Action Policy

Respecting the Affirmative Action Policy

Mechanism φ respects the affirmative action policy if for all $v \in \mathcal{V}$, $(P, \succ) \in \mathcal{P} \times \Pi$, $i \in S^m$ and $c \in C$, if $c \; P_i \; \mu_i$ then $|\mu_c^m| \geq v_c$, where $\varphi(v, P, \succ) = \mu$ and μ_c^m is the set of minority students assigned to school c in matching μ.

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- The axiom requires that minority students are prioritized for the number of school seats specified by the minority allotment at each school.
- Basic requirement; otherwise the affirmative action policy is ineffective.

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- The axiom requires that minority students are prioritized for the number of school seats specified by the minority allotment at each school.
- Basic requirement; otherwise the affirmative action policy is ineffective.
- Some version of this axiom is typically included in fairness/stability conditions for mechanisms with minority allotments.
- Surprisingly, not all mechanisms with a minority allotment policy proposed in the literature satisfy this basic intuitive property.

Welfare Axiom 3: Minimal Responsiveness

v' represents a *weakly stronger affirmative action policy* than v if for all $c \in C$, $v'_c \geq v_c$, i.e., $v' \geq v$.

Minimal Responsiveness

A mechanism φ is minimally responsive if for all $v, v' \in \mathcal{V}$ such that $v' \geq v$ and all $(P, \succ) \in \mathcal{P} \times \Pi$ such that $\varphi_{S^m}(v, P, \succ) \neq \varphi_{S^m}(v', P, \succ)$, there exists $i \in S^m$ such that $\varphi_i(v', P, \succ)\ P_i\ \varphi_i(v, P, \succ)$.

Welfare Axiom 3: Minimal Responsiveness

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- The axiom requires that a weakly stronger affirmative action policy does not result in a Pareto-inferior outcome for the minority students at any profile (weak requirement).
- It was first proposed for quota-based affirmative action policies by Kojima (2012), and was later extended to reserved-based affirmative action policies by Doğan (2016).

Results on Previous Affirmative Action Mechanisms

Results on DA-Q and DA-R

Proposition 1

The DA-Q mechanism respects the affirmative action policy, but it is wasteful and not minimally responsive.

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Proposition 2

The DA-R mechanism is non-wasteful and respects the affirmative action policy, but it is not minimally responsive.

Results on DA-Q and DA-R

Proposition 1

The DA-Q mechanism respects the affirmative action policy, but it is wasteful and not minimally responsive.

Proposition 2

The DA-R mechanism is non-wasteful and respects the affirmative action policy, but it is not minimally responsive.

- The statements in these two propositions follow from the findings of Kojima (2012), Hafalir et al. (2013), and Doğan (2016).

Results on MDA and EIDA

Proposition 3

The MDA mechanism is non-wasteful, minimally responsive, but it does not respect the affirmative action policy.

Results on MDA and EIDA

Proposition 3

The MDA mechanism is non-wasteful, minimally responsive, but it does not respect the affirmative action policy.

Proposition 4

The EIDA mechanism is non-wasteful, but it does not respect the affirmative action policy, and it is not minimally responsive.

Results on MDA and EIDA

Proposition 3

The MDA mechanism is non-wasteful, minimally responsive, but it does not respect the affirmative action policy.

Proposition 4

The EIDA mechanism is non-wasteful, but it does not respect the affirmative action policy, and it is not minimally responsive.

- The above results on non-wastefulness and minimal responsiveness follow from Doğan (2016), Ju et al. (2018) and Ding et al. (2019).
- We show with the next example that MDA and EIDA do not respect the affirmative action policy.

MDA Does Not Respect the Affirmative Action Policy

Example (MDA)

Let $S^M = \{a\}$ and $S^m = \{i_1, i_2\}$ be the sets of majority and minority students.
Let $C = \{c_1, c_2, c_3\}$ with capacity $q = (1, 1, 1)$ and minority reserves $r = (1, 0, 0)$.

P_a	P_{i_1}	P_{i_2}	$\succ_{c_1}$	$\succ_{c_2}$	$\succ_{c_3}$
$\boxed{c_1}$	$\boxed{c_3}$	c_1	a	a	a
$\underline{c_3}$	$\underline{c_1}$	$\underline{\boxed{c_2}}$	i_1	i_2	i_1
c_2	0	c_3	i_2	i_1	i_2

The DA-R matching is underlined.
Minority student i_2 is an interferer for school c_1. Considering i_2 a majority student at school c_1, MDA yields the matching indicated by the squares.
Since $c_1 \; P_{i_2} \; c_2$, $r_{c_1} = 1$, and the majority student a is matched to school c_1, MDA does not respect the affirmative action policy.

EIDA Does Not Respect the Affirmative Action Policy

Example (EIDA)

Same setup: $S^M = \{a\}$ and $S^m = \{i_1, i_2\}$ are the sets of majority and minority students.
Let $C = \{c_1, c_2, c_3\}$ with capacity $q = (1, 1, 1)$ and minority reserves $r = (1, 0, 0)$.

P_a	P_{i_1}	P_{i_2}		$\succ_{c_1}$	$\succ_{c_2}$	$\succ_{c_3}$
$\boxed{c_1}$	$\boxed{c_3}$	c_1		a	a	a
$\underline{c_3}$	$\underline{c_1}$	$\underline{\boxed{c_2}}$		i_1	i_2	i_1
c_2	0	c_3		i_2	i_1	i_2

EIDA produces the same matching as MDA at $(P, \succ)$ with $r = (1, 0, 0)$.
Minority student i_2 is an interrupter for school c_1. Dropping c_1 to the bottom of i_2's preference list, EIDA yields the matching indicated by the squares.
Therefore, EIDA does not respect the affirmative action policy.

Results on IA-Q and IA-R

Proposition 5

The IA-Q mechanism respects the affirmative action policy, but it is wasteful and not minimally responsive.

Results on IA-Q and IA-R

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Proposition 6

The IA-R mechanism is non-wasteful and minimally responsive but it does not respect the affirmative action policy.

Results on IA-Q and IA-R

Proposition 5

The IA-Q mechanism respects the affirmative action policy, but it is wasteful and not minimally responsive.

Proposition 6

The IA-R mechanism is non-wasteful and minimally responsive but it does not respect the affirmative action policy.

- The IA-Q mechanism is wasteful, similarly to DA-Q, due to the rigidity of majority quotas. However, it respects the affirmative action policy since it imposes the majority quota cap, while a minority applicant is never rejected by a school with any empty seats.
- The IA-R mechanism is non-wasteful, similarly to DA-R, as it lets majority students have the seats that are not filled with minority students. Minimal responsiveness was proved by Afacan and Salman (2016).

IA-Q is Wasteful and Not Minimally Responsive

Example

Let $S^M = \{a_1, a_2\}$ and $S^m = \{i\}$ be the sets of majority and minority students. Let $C = \{c_1, c_2, c_3\}$ with capacities $q = (1, 1, 1)$.

P_{a_1}	P_{a_2}	P_i
c_2	$\boxed{c_1}$	c_1
c_3	0	$\boxed{c_3}$
$\boxed{0}$		0

$\succ_{c_1}$	$\succ_{c_2}$	$\succ_{c_3}$
a_2	a_1	i
i	i	a_1
a_1	a_2	a_2

P_{a_1}	P_{a_2}	P_i
$\underline{c_2}$	$\underline{\boxed{c_1}}$	c_1
$\boxed{c_3}$	0	$\underline{c_3}$
0		$\boxed{0}$

$\succ_{c_1}$	$\succ_{c_2}$	$\succ'_{c_3}$
a_2	a_1	a_1
i	i	i
a_1	a_2	a_2

IA-Q is wasteful

Given $q^M = (1, 0, 1)$, a_1 is unassigned and the only seat at c_2 is left empty. Since $c_2 P_{a_1} 0$, the IA-Q mechanism is wasteful.

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P_{a_1}	P_{a_2}	P_i
c_2	$\boxed{c_1}$	c_1
c_3	0	$\boxed{c_3}$
$\boxed{0}$		0

$\succ_{c_1}$	$\succ_{c_2}$	$\succ_{c_3}$
a_2	a_1	i
i	i	a_1
a_1	a_2	a_2

P_{a_1}	P_{a_2}	P_i
$\underline{c_2}$	$\underline{\boxed{c_1}}$	c_1
$\boxed{c_3}$	0	$\underline{c_3}$
0		$\boxed{0}$

$\succ_{c_1}$	$\succ_{c_2}$	$\succ'_{c_3}$
a_2	a_1	a_1
i	i	i
a_1	a_2	a_2

IA-Q is wasteful

Given $q^M = (1, 0, 1)$, a_1 is unassigned and the only seat at c_2 is left empty. Since $c_2 P_{a_1} 0$, the IA-Q mechanism is wasteful.

IA-Q is Not Minimally Responsive

With $\bar{q}^M = (1, 1, 1)$, $\bar{\mu}_i = c_3$ and with $q^M = (1, 0, 1)$, i is unassigned. Since $c_3 \ P_i \ 0$, the IA-Q mechanism is not minimally responsive.

IA-R Does Not Respect the Affirmative Action Policy

Example

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P_{a_1}	P_{a_2}	P_i	$\succ_{c_1}$	$\succ_{c_2}$	$\succ_{c_3}$
$\boxed{c_2}$	$\boxed{c_1}$	c_1	a_2	a_1	a_1
c_3	0	c_2	i	i	i
0		$\boxed{c_3}$	a_1	a_2	a_2

Given $r = (0, 1, 0)$, the IA-R matching at this profile is $\mu_{a_1} = c_2$, $\mu_{a_2} = c_1$ and $\mu_i = c_3$.

Since $c_2 \ P_i \ c_3$ and the minority reserve $r_{c_2} = 1$ is not used by the minority student (student i), the IA-R mechanism does not respect the affirmative action policy.

Summary Table

None of the previous mechanisms satisfies all three of the welfare axioms:

Mechanism	**Welfare Axioms**		
	Non-Wasteful	*Respects AA*	*Minimally Responsive*
DA-Q	×	✓	×
DA-R	✓	✓	×
MDA	✓	×	✓
EIDA	✓	×	×
IA-Q	×	✓	×
IA-R	✓	×	✓

On Truhtful Reporting and Comparisons

Do the mechanisms actually satisfy the welfare axioms?

Theory:

- DA-Q and DA-R are strategyproof - but not obviously strategyproof (Li, 2017; Ashlagi and Gonczarowski, 2018)
- MDA and EIDA are not strategyproof - but not obviously manipulable
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Experimental evidence:

- Growing experimental literature: even the strategyproof DA mechanism is often manipulated (e.g., Dreyfuss et al. (2022)).
- Non-truth-telling equilibria may not predict behavior well in certain school choice settings, thus an equilibrium analysis may be misleading. In some realistic settings both DA and IA are manipulated at similar rates (Featherstone and Niederle, 2016).

Discussion:
Motivation for a New Mechanism

Immediate vs Deferred Acceptance, Reserves vs Quotas: Conflicts

- Minority reserves guarantee non-wastefulness, while majority quotas ensure that the affirmative action policy is respected.

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- Immediate (permanent) acceptances allow for the minority reserves to be minimally responsive, but they do not respect the affirmative action policy, since majority students may be permanently accepted ahead of minority students who apply later.
- Note that the only two previous mechanisms that are minimally responsive, MDA and IA-R, do not respect the affirmative action policy.

The IA-DA-R Mechanism

Resolving the conflicts: IA-DA-R Mechanism

Question: **Can we satisfy all three welfare axioms simultaneously?**

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Immediate and Deferred Acceptance Mechanism with Minority Reserves
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Immediate and Deferred Acceptance Mechanism with Minority Reserves
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- The main idea of the IA-DA-R mechanism is to assign minority students to minority reserve seats permanently, as in the IA mechanism, while all other seats are assigned tentatively, as in the DA mechanism, including the assignment of majority students to empty minority reserve seats.
- The IA-DA-R mechanism is a DA mechanism which incorporates immediate acceptance for minority students when they are assigned to minority reserve seats.

IA-DA-R Mechanism - Definition

IA-DA-R Mechanism

Fix a minority reserve policy r and a profile $(P, \succ)$.

Step 1:

- Every student applies to her most preferred school according to P.

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- Any remaining applicants are rejected.

The mechanism terminates when there is no more rejection by any school. All tentative matches in the final step become final matches.

IA-DA-R Mechanism: An Illustrative Example

Example

Let $S^M = \{a_1, \ldots, a_5\}$ and $S^m = \{i_1, \ldots, i_4\}$ be the sets of majority and minority students. Let $C = \{c_1, \ldots, c_4\}$ with capacities $q = (3, 2, 3, 1)$.

Consider the following profile:

P_{a_1}	P_{a_2}	P_{a_3}	P_{a_4}	P_{a_5}	P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}
$\underline{\boxed{c_1}}$	$\underline{c_1}$	c_1	$\underline{\boxed{c_4}}$	c_4	c_4	c_4	c_4	$\boxed{c_1}$
c_2	$\boxed{c_3}$	$\underline{\boxed{c_2}}$	c_3	$\underline{c_1}$	$\underline{c_2}$	c_2	$\underline{\boxed{c_3}}$	$\underline{c_3}$
c_3	c_2	c_3	c_2	$\boxed{c_2}$	c_1	$\boxed{c_1}$	c_2	c_2
c_4	c_4	c_4	c_1	c_3	$\boxed{c_3}$	$\underline{c_3}$	c_1	c_4

$\succ_{c_1}$	$\succ_{c_2}$	$\succ_{c_3}$	$\succ_{c_4}$
a_1	a_5	a_5	a_4
a_5	a_3	a_2	a_1
a_2	a_1	i_3	a_2
a_3	i_3	a_1	a_3
i_1	i_1	a_3	i_1
i_2	i_2	a_4	i_2
a_4	a_4	i_1	i_3
i_3	a_2	i_2	a_5
i_4	i_4	i_4	i_4

IA-DA-R Mechanism: An Illustrative Example

Example (Steps of the IA-DA-R mechanism with minority reserves $r = (2, 0, 0, 0)$)

	c_1	c_2	c_3	c_4
Step 1	a_1, a_2, ~~a_3~~, $\enclose{circle}{i_4}$			a_4, ~~a_5~~, ~~i_1~~, ~~i_2~~, ~~i_3~~
Step 2	a_1, ~~a_2~~, a_5, $\enclose{circle}{i_4}$	a_3, i_1, ~~i_2~~	i_3	a_4
Step 3	a_1, ~~a_5~~, $\enclose{circle}{i_2}$, $\enclose{circle}{i_4}$	a_3, i_1	a_2, i_3	a_4
Step 4	a_1, $\enclose{circle}{i_2}$, $\enclose{circle}{i_4}$	a_3, a_5, ~~i_1~~	a_2, i_3	a_4
Step 5	a_1, ~~i_1~~, $\enclose{circle}{i_2}$, $\enclose{circle}{i_4}$	a_3, a_5	a_2, i_3	a_4
Step 6	a_1, $\enclose{circle}{i_2}$, $\enclose{circle}{i_4}$	a_3, a_5	a_2, i_1, i_3	a_4

The IA-DA-R matching is given by $\mu_{c_1} = \{a_1, i_2, i_4\}$, $\mu_{c_2} = \{a_3, a_5\}$, $\mu_{c_3} = \{a_2, i_1, i_3\}$, and $\mu_{c_4} = \{a_4\}$.

(Permanently accepted minority students for minority reserve seats are circled.)

IA-DA-R Mechanism - Properties

The IA-DA-R mechanism ensures that

1. minority reserve seats are not wasted, since majority students can be assigned to minority reserve seats temporarily, implying **non-wastefulness**;

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Theorem 1

The **IA-DA-R** mechanism is non-wasteful, respects the affirmative action policy and is minimally responsive.

Summary Table

Mechanism	**Welfare Axioms**		
	Non-Wasteful	*Respects AA*	*Minimally Responsive*
DA-Q	×	✓	×
DA-R	✓	✓	×
MDA	✓	×	✓
EIDA	✓	×	×
IA-Q	×	✓	×
IA-R	✓	×	✓
IA-DA-R	✓	✓	✓

Fairness - Acceptable Priority Violations

Justified Envy and Priority Violation

- Student i has **justified envy** in μ at $(\succ, P)$ if there exist school $c \in C$ and student $j \in S$ such that
  - $c \; P_i \; \mu_i$
  - $i \succ_c j$
  - $\mu_j = c$
- We will say that ***j* violates *i*'s priority** (at school c) in matching μ when i has justified envy in μ due to j as defined above.

Priority Violations and Minority Fairness

We define the following simple fairness axiom.

Minority Fairness

Given a minority allotment policy $v \in \mathcal{V}$ and a profile $(P, \succ) \in \mathcal{P} \times \Pi$, we call a **matching** μ **minority fair** with respect to v and $(P, \succ)$ if, at profile $(P, \succ)$, it satisfies the following conditions:

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A **mechanism** φ is **minority fair** if for all minority allotment policies $v \in \mathcal{V}$ and all profiles $(P, \succ) \in \mathcal{P} \times \Pi$, $\varphi(v, P, \succ)$ is minority fair with respect to v and $(P, \succ)$.

Existence Theorem

Theorem 2: Existence

There exists a mechanism with minority allotments which is non-wasteful, respects the affirmative action policy, is minimally responsive, and minority fair.

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- The theorem holds since the IA-DA-R mechanism satisfies all the properties.
- This is an existence (possibility) result, which contrasts with a related impossibility result of Doğan (2016).
- The main difference between the two results is due to the fact that the fairness condition of Doğan (2016) requires that **minority students do not violate other minority students' priorities.**
- This is not required by Minority Fairness (and it is not satisfied by IA-DA-R).
- Note that this may not be a desirable feature when affirmative action benefits only the most privileged members of the minority group.

Incentives

On Strategyproofness

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- Minority students can manipulate due to the immediate acceptances, similarly to IA manipulations.
- It is less obvious that majority students can also manipulate it, since majority students can only be accepted tentatively for any seat, whether reserved or not, as in the DA mechanism.
- A majority student may be able to manipulate because her reported preferences may affect when a minority student applies to a school with available minority reserve seats which may, in turn, affect the majority student's assignment in a profitable manner.

IA-DA-R Can Be Manipulated by Majority Students

Example

Let $S^M = \{a_1, a_2\}$ and $S^m = \{i_1, i_2\}$ be the sets of majority and minority students.
Let $C = \{c_1, c_2, c_3\}$ with capacities $q = (1, 1, 1)$.

P_{a_1}	P_{a_2}	P_{i_1}	P_{i_2}	P'_{a_2}	$\succ_{c_1}$	$\succ_{c_2}$	$\succ_{c_3}$
c_2	$\boxed{c_3}$	c_1	c_1	c_1	a_1	a_1	a_1
$\underline{\boxed{c_1}}$	c_1	$\boxed{c_2}$	$\underline{c_2}$	c_3	a_2	i_1	i_1
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Given $r = (0, 1, 0)$, the IA-DA-R matching at P is underlined.

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Given $r = (0, 1, 0)$, the IA-DA-R matching at P is underlined.

If majority student a_2 reports P'_{a_2} then the IA-DA-R matching at the resulting profile is as indicated by the squares.

By reporting false preferences P'_{a_2}, majority student a_2 gets his first choice instead of being unmatched.

Strategyproofness: Impossibility Results

However, it is not possible to find a mechanism that is strategyproof for either type of student, in addition to the other properties.

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Theorem 3: Impossibility

1. There is no mechanism with minority allotments which is non-wasteful, respects the affirmative action policy, is minimally responsive, minority fair, and **strategyproof for minority students**.
2. There is no mechanism with minority allotments which is non-wasteful, respects the affirmative action policy, is minimally responsive, minority fair, and **strategyproof for majority students**.

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- This theorem strengthens a result by Doğan (2016); his fairness axiom requires that *minority students do not violate other minority students' priorities,* while ours does not require this.
- It is also stronger than the impossibility result of Doğan (2016) because strategyproofness is only required for one student type at a time.

Incentive Properties of the IA-DA-R Mechanism

Non-obvious manipulation (Troyan and Morrill, 2020)

A mechanism is not obviously manipulable by agent $s \in S$, if all (true) preferences $\hat{P}_s$ and all successful manipulation strategies P'_s satisfy:

1. $B(\hat{P}_s; \hat{P}_s)\ \hat{R}_s\ B(P'_s; \hat{P}_s)$;
2. $W(\hat{P}_s; \hat{P}_s)\ \hat{R}_s\ W(P'_s; \hat{P}_s)$.

Here, for all preference orderings P_s:

$B(P_s; \hat{P}_s)$ denotes the **best assignment** for s according to $\hat{P}_s$

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when reporting P_s, with respect to all reports by other agents (P_{-s}).

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Proposition 7

The IA-DA-R mechanism is not obviously manipulable by majority students.

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However, it is obviously manipulable by minority students.

Summary

- None of the previous mechanisms with a quota/reserve-based affirmative action policy satisfy three basic welfare axioms: non-wastefulness, respecting the affirmative action policy, and minimal responsiveness.
- We propose a new mechanism, **IA-DA-R**, and show that it **satisfies all three axioms.**
- The IA-DA-R mechanism is **minority fair.**
- The IA-DA-R mechanism is not strategyproof for either minority or majority students, but it is **not obviously manipulable by majority students**.
- It is **impossible to reconcile the three welfare axioms with minority fairness and strategyproofness for either minority or majority students.**

Policy Recommendation?

- If minimal responsiveness is not considered important but strategyproofness is, use the DA-R mechanism.
- If minimal responsiveness is considered important, use the IA-DA-R mechanism.

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