# The Effect of Framing in Sealed-Bid Auctions: <br> <br> Theory and Experiments 

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# The Effect of Framing in Sealed-Bid Auctions: Theory and Experiments ${ }^{1}$ 

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#### Abstract

We investigate strategic games with imperfect information, such as sealed-bid auctions, wherein players are not good at hypothetical thinking and are therefore unable to select even dominant strategies unless devices to guide them are put in place. We propose a measure to encourage such bounded-rational players to engage in hypothetical thinking. We consider a frame as a multi-stage game format with imperfect information within a range consistent with an inherent strategic game. We showed that a well-specified frame has a significant effect on the promotion of hypothetical thinking. By comparing the second-price auction as a non-frame format and the ascending proxy auctions as framed formats, we theoretically demonstrate that framing can eliminate overbidding and encourage bidders to act more sincerely. These theoretical findings were confirmed by both online and laboratory experimental results.


Keywords: Sealed-Bid Auctions, Hypothetical Thinking, Cautious Undominance, Frames, Online and Laboratory Experiments.
JEL Classification Numbers: C90, D44, D78, D82, D91

[^0]
## 1. Introduction

We investigate strategic games wherein we assume imperfect information, in that each player cannot observe the other players' decisions during their play. If each player wants to make rational decisions, they are forced to make various hypotheses about strategies that the other players will select, and depending on each hypothesis, consider which strategy they prefer. However, actual players are not good at such hypothetical thinking. It is thought that the failure of hypothetical thinking is one of the reasons why anomalies are observed wherein individuals make irrational decisions in various strategic situations. For example, a bidder tends to overbid in a second-price auction, but suffers a loss by participating in it. One reason for their overbidding is that they incorrectly infer that overbidding relaxes the other bidders' competitive spirit and therefore lowers their bids. However, such inferences are irrational because the bidders cannot observe each other's bids (imperfect information).

In this study, we propose a method for designing measures (i.e., frames) to guide players who are not good at hypothetical thinking, so that they can make better inferences that are as close to the correct hypothetical thinking as possible. Specifically, we present strategic situations as multi-stage games with imperfect information that details all players' decision-making procedures, such as the (elaborately set) ascending proxy auction format, rather than more abstract strategic game formats, such as the (oversimplified) second-price auction format. We show both theoretically and experimentally the possibility that a well-designed multi-stage game with imperfect information effectively acts as a frame for prompting hypothetical thinking in boundedrational players.

A frame divides each player's strategy into multi-stage action selections. We consider each player's strategy as a combination of the multiple actions that they sequentially decide on through a multi-stage procedure with imperfect information. The frame synchronizes players' sequential decisions with each other in their epistemology. We continue to hold the imperfect information assumption in any framing device. Importantly, within a frame, at each stage, each player recognizes (in their epistemology) that the other players have already decided on their previous stage actions, whereas they
have not yet decided on current and future actions. We argue that this sequential and synchronized nature of a frame plays a significant role in helping players practice hypothetical thinking. We categorized hypothetical thinking into two types. First, the type that concerns actions that other players have decided upon before the current stage. Second, the type that concerns the actions that players decide upon in current and future stages. We assume that players can correctly practice the first type of hypothetical thinking but fail to practice the second type of hypothetical thinking.

Each player correctly recognizes that the actions selected in past stages can no longer be changed. However, players do not perceive the actions they will decide on at the current and future stages as immutable. This misperception prevents them from correctly perceiving that the action they take at the current stage is not related to these actions. Thus, players can practice the first type of hypothetical thinking but fail to practice the second.

We present cautious (un)dominance (C-undominance) as a new concept to relate bounded rationality to frame guidance. A player's strategy is said to be C-undominated if they give up selecting any other action at any stage by changing from optimistic to pessimistic predictions about current and future actions. C-undominance is a weaker concept than the standard notion of undominance. In fact, in the second-price auction without a frame, sincere bidding is the only undominated strategy, whereas all bidding strategies, whether sincere bidding, overbidding, or underbidding, are C-undominated strategies.

We characterize C-undominated strategies in a wide range of multi-stage games with imperfect information (i.e., frames), which are abstracted by a second-price auction game and support various specifications of proxy auction formats. We demonstrate that such framing generally eliminates overbids. Overbidding can be C-undominated only if a bidder (player) anticipates that, by overbidding, they will be able to change others' bids to lower prices. However, by setting low-bidding decisions as early-stage actions, we can eliminate such anticipations in favor of overbidding.

It should also be noted that while encouraging (first type) hypothetical thinking, the framing expressed by multi-stage games can have a negative effect of weakening the player's self-control ability because it induces a physical passage of time in decisionmaking under imperfect information. This lack of self-control allows underbidding to be C-undominated. Bidders pessimistically think that if they stop underbidding, they may
become enthusiastic and overbid in the next stages. Fearing the eventuality of their own "stay up to the end," they cannot refuse to underbid. To allow players to restore selfcontrol when deviating from their default strategies, we define weakly cautious (un)dominance (WC-undominance) as a solution concept that retains more rationality than C-undominance in terms of self-control. We show that an alternating-play frame succeeds in making the second mover sincerely bid, suggesting that framing makes players more sincerely.

Based on the above theoretical findings and considerations, we conduct laboratory experiments to compare the second-price auction format (without a frame) and an ascending proxy auction format (framed). Our experimental results are as follows:
i) Without frame, experimental subjects tend to overbid.
ii) With a frame, unlike the case without a frame, only a few experimental subjects overbid.
iii) With a frame, the experimental subjects tend to bid more sincerely compared to the case without a frame.
iv) With a frame, unlike the case without a frame, many experimental subjects tend to underbid.

We further conducted online experiments wherein each subject was permitted to participate at their convenient time and competed with a hypothetical virtual player, being explicitly instructed to image as if they were playing with an actual player. Our finding is that there is no substantial difference in the results between the online and laboratory experiments, although subjects are more likely to make mistakes in the online experiments with frames.

These experimental results were consistent with our theoretical findings and considerations. The novelties of this study are summarized as follows:
v) As a cognitive method for promoting hypothetical thinking in strategic situations, we propose the use of polite descriptions of multi-stage games with imperfect information as a frame.
vi) By presenting the characterization theorems of C -undominated and WCundominated strategies for sealed-bid auctions, we theoretically show that our framing method can eliminate irrational behaviors such as overbidding, whereas it supports sincere bidding as well as underbidding.
vii) Our theoretical findings and consideration are consistent with both online and laboratory experimental results.

## 2. Literature Review

The failure of hypothetical thinking, such as violation of the sure-thing principle due to the disjunction effect (Tversky and Shafir, 1992) contains clues for discovering the origin of various anomalies in real life and laboratory experiments, such as the winner's curse (Charness and Levin, 2009), non-pivotal voting (Esponda and Vespa, 2014), market failure caused by informational asymmetry (Ngangoue and Weizsacker, 2015), ambiguity, and loss aversion (Esponda and Vespa, 2016). In game theory, equilibrium analyses consider bounded rationality in hypothetical thinking, as demonstrated in Jehiel (2005), Eyster and Rabin (2005), Esponda (2008), and Li (2017).

Li (2017) demonstrates obvious (un)dominance (in our terminology), wherein a player misperceives the other players' behavior optimistically when they select the default strategy and pessimistically when selecting any alternative strategy. The essential difference between Li and the present study is that we are aware that framing can mitigate this misperception and show a positive result in terms of rationality, even under the assumption of imperfect information. Hence, we should no longer make hasty decisions such that designing institutions with perfect information (i.e., open-bid auctions) is better than designing institutions with imperfect information (i.e., sealed-bid auctions). This study revealed the positive significance of the social implementation of sealed-bid auctions.

The present study is related to that of Glazer and Rubinstein (1996), who showed that extensive game representation (perfect information) serves as a guide for solving a normal game (imperfect information), wherein both games are practically the same from the viewpoint of rational players. To help bounded-rational players correctly reason for the iterative elimination of dominated strategies, Glazer and Rubinstein propose an extensive game that mimics this iterative elimination procedure. Glazer and Rubinstein assumed that players could perform hypothetical thinking correctly, whereas this study assumed that players were not good at either hypothetical thinking or high-order
reasoning. We focus on games with imperfect information and consider only a one-round elimination of dominant strategies, not iterative eliminations.

Our experimental results concerning the sealed-bid second-price auction format, which support the tendency for overbidding, are consistent with many previous studies in laboratory experiments, such as Kagel, Harstad, and Levin (1987) and Li (2017). However, the field experiments by Lucking-Reiley (1999) do not provide results in support of overbidding; they compare the sealed-bid second-price auction and the openbid ascending auction and obtain experimental results that support the revenue equivalence theorem. This finding agrees with our experimental results if the field experiment is considered a type of framed experiment.

The remainder of this paper is organized as follows: Section 3 defines the frames, C-undominance, and WC-undominance. Section 4 considers sealed-bid auctions, introduces the second-price auction format as a non-frame format, and introduces the ascending proxy auction format as a framed format. We then characterize the C undominated and WC-undominated strategies based on the given formats. Section 5 presents experimental results. Finally, section 6 concludes the study.

## 3. Multi-Stage Games with Imperfect Information

We define a multi-stage game with imperfect information as $\Gamma=\left(N, T,\left(\left(A_{i, t}\right)_{t=1}^{T}, u_{i}\right)_{i \in N}\right) . N=\{1, \ldots, n\}$ denotes the finite set of players, where $n \geq 2$. The positive integer $T$ denotes the number of stages at which each player sequentially selects parts of their strategy as their actions. Let $A_{i, t}$ denote the finite set of actions that player $i$ selects at stage $t$. Let $A_{i} \equiv \underset{t=1}{T} A_{i, t}$ denote the finite set of strategies of player $i$, $a_{i} \equiv\left(a_{i, t}\right)_{t=1}^{T} \in A_{i}$ and $A \equiv \underset{i \in N}{\times} A_{i}$. Let $u \equiv\left(u_{i}\right)_{i \in N}$, where $u_{i}: A \rightarrow R$ denotes the payoff function of player $i$. If players select an action profile $\left(a_{i, t}\right)_{i \in N} \in \underset{i \in N}{\times} A_{i, t}$ at each stage $t \in\{1, \ldots, T\}$, that is, if they select a strategy profile $a=\left(a_{i}\right)_{i \in N} \in A$, then each player $i \in N$ receives the payoff $u_{i}(a) \in R$. Let $A_{-i} \equiv \underset{j \neq i}{\times} A_{j}$ and $a_{-i} \equiv\left(a_{j}\right)_{j \neq i} \in A_{-i}$. We assume
imperfect information in that each player cannot observe the other players' actions during their play. We consider a multi-stage game with imperfect information $\Gamma$ as a frame of the abstract strategic game given by $(N, A, u)$. We define the standard notion of undominance, according to which a rational player selects their strategy, as follows.

Definition 1: A strategy $a_{i} \in A_{i}$ of player $i$ is said to be undominated in $\Gamma$ if there exists no $\hat{a}_{i} \in A_{i} \backslash\left\{a_{i}\right\}$ such that

$$
u_{i}\left(\hat{a}_{i}, a_{-i}\right) \geq u_{i}\left(a_{i}, a_{-i}\right) \text { for all } a_{-i} \in A_{-i}
$$

and

$$
u_{i}\left(\hat{a}_{i}, \tilde{a}_{-i}\right)>u_{i}\left(a_{i}, \tilde{a}_{-i}\right) \text { for some } a_{-i} \in A_{-i} .
$$

It needs to be noted that the definition of undominance does not depend on the specification of the frame $\Gamma$, that is, it depends only on its abstract strategic game ( $N, A, u$ ).

We consider bounded rationality, in which a player is incapable of hypothetical thinking, as follows: we define the notion of obvious undominance (O-undominance) after Li (2017), according to which a bounded-rational player selects their strategy, wherein each player anticipates the other players' strategy selections optimistically when they select the default strategy, but pessimistically when they select any other strategy.

Definition 2: A strategy $a_{i} \in A_{i}$ of player $i$ is said to be O-undominated in $\Gamma$ if there exists no $\hat{a}_{i} \in A_{i} \backslash\left\{a_{i}\right\}$ such that

$$
\min _{a_{-i} \in A_{-i}} u_{i}\left(\hat{a}_{i}, a_{-i}\right) \geq \max _{a_{-i} \in A_{i}} u_{i}\left(a_{i}, a_{-i}\right),
$$

and

$$
u_{i}\left(\hat{a}_{i}, \tilde{a}_{-i}\right)>u_{i}\left(a_{i}, \tilde{a}_{-i}\right) \text { for some } a_{-i} \in A_{-i} .
$$

The definition of O-undominance does not depend on the specifications of the frame $\Gamma$, that is, it depends only on ( $N, A, u$ ), similar to that of undominance.

Following is a milder assumption about bounded rationality in hypothetical thinking.

We assume that at each stage, every player can correctly consider that the actions that the other players have already selected at the past stages can no longer be changed. However, every player misunderstands that at each stage, their action selection influences the current and future action selections of the other players, despite the imperfect information environment. Further, we consider a possibility that a player loses self-control over their own future action selections. Wary of this possibility, they will make a cautious action selection at each stage. Formally, we define a solution concept termed cautious undominance (C-undominance) as follows. Let $a_{i}^{t} \equiv\left(a_{i, 1}, \ldots a_{i, t}\right) \in A_{i}^{t} \equiv \underset{\tau=1}{\star} A_{i, \tau}$ denote a player $i^{\prime} s$ action history up to stage $t \in\{1, \ldots, T\}$. Let $a^{t} \equiv\left(a_{i}^{t}\right)_{i \in N} \in A^{t} \equiv \underset{i \in N}{\times} A_{i}^{t}$ denote an all-players' action history up to stage $t \in\{1, \ldots, T\}$. Let $A_{i}\left(a_{i}^{t}\right) \equiv\left\{\tilde{a}_{i} \in A_{i} \mid \tilde{a}_{i}^{t}=a_{i}^{t}\right\}$ denote the set of strategies of player $i$ that are consistent with their action history $a_{i}^{t}$ up to stage $t$ Let $a_{-i}^{t} \equiv\left(a_{j}^{t}\right)_{j \neq i} \in A_{-i}^{t} \equiv \underset{j \neq i}{\times} A_{j}^{t}$, and $A_{-i}\left(a_{-i}^{t}\right) \equiv \underset{j \neq i}{\times} A_{j}\left(a_{j}^{t}\right)$.

Definition 3: A strategy $a_{i} \in A_{i}$ of player $i$ is said to be C-undominated if there exists no $t \in\{1, \ldots, T\}, a_{-i}^{t-1} \in A_{-i}^{t-1}$, and $\hat{a}_{i, t} \in A_{i, t} \backslash\left\{a_{i, t}\right\}$, such that

$$
\min _{\tilde{a}_{-i} \in A_{i}\left(a_{i}^{-i}\right), \hat{a}_{i} \in A_{i}\left(a_{i}^{t-1}, \hat{a}_{i,}\right)} u_{i}\left(\hat{a}_{i}, \tilde{a}_{-i}\right) \geq \max _{\tilde{a}_{-i} \in A_{-i}\left(a_{i-i}^{-l}\right)} u_{i}\left(a_{i}, \tilde{a}_{-i}\right),
$$

and

$$
\min _{\hat{a}_{i} \in A_{i}\left(a_{i}^{-1}, \hat{a}_{i}\right)} u_{i}\left(\hat{a}_{i}, \tilde{a}_{-i}\right)>u_{i}\left(a_{i}, \tilde{a}_{-i}\right) \text { for some } \hat{a}_{-i} \in A_{-i}\left(a_{-i}^{t-1}\right) .
$$

In C-undominance, we assume a failure of the second type of hypothetical thinking in that at each stage $t$, each player $i$ anticipates the other players' current and future action selections optimistically when they select according to the default strategy $a_{i}$, but pessimistically when they select any other action $\hat{a}_{i, t}$. It should be noted that we also consider a failure of self-control in that a player has sufficient self-control when playing the default strategy $a_{i}$, but loses self-control when changing to other strategies. That is, at each stage $t$, when a player $i$ selects any other action $\hat{a}_{i, t}$ instead of the default action
$a_{i, t}$, they lose self-control and therefore, anticipates their own future action selections $\left(\hat{a}_{i, t+1}, \ldots, \hat{a}_{i, T}\right)$ pessimistically. In contrast, we assume that each player can practice the first type of hypothetical thinking.

Proposition 1: Suppose that $n=2, T \geq 2, A_{1, t}$ is a singleton for all $t \neq 1$, and $A_{2, t}$ is a singleton for all $t \neq T$. Then, a strategy $a_{1} \in A_{1}$ of player 1 is C -undominated if and only if it is O-undominated. A strategy $a_{2} \in A_{2}$ of player 2 is C-undominated if and only if it is undominated.

Proof: Because player 1 does not face the first type of hypothetical thinking, we have an equivalence between C-undominance and O-undominance for player 1. Since player 2 does not face the second type of hypothetical thinking, we have an equivalence between C-undominance and undominance for player 2:
Q.E.D.

Proposition 1 implies that in a frame consisting of first and second movers, the second mover, who does not need the second type of hypothetical thinking, behaves rationally, whereas the first mover, who does need the second type of hypothetical thinking in its entirety, remains irrational.

We weaken C-undominance by restoring self-control as follows.

Definition 4: A strategy $a_{i} \in A_{i}$ of player $i$ is said to be weakly cautious-undominated (WC-undominated) if there exists no $t \in\{1, \ldots, T\}, a_{-i}^{t-1} \in A_{-i}^{t-1}$, and $\hat{a}_{i} \in A_{i}\left(a_{i}^{t-1}\right)$ such that $a_{i, t} \neq \hat{a}_{i, t}$,

$$
\min _{\tilde{a}_{-i} \in \mathcal{A}_{i}\left(a_{-i}^{-1}\right)} u_{i}\left(\hat{a}_{i}, \tilde{a}_{-i}\right) \geq \max _{\tilde{a}_{-i} \in A_{-i}\left(a_{-i}^{-1}\right)} u_{i}\left(a_{i}, \tilde{a}_{-i}\right),
$$

and

$$
u_{i}\left(\hat{a}_{i}, \tilde{a}_{-i}\right)>u_{i}\left(a_{i}, \tilde{a}_{-i}\right) \text { for some } \tilde{a}_{-i} \in A_{-i}\left(a_{-i}^{t-1}\right) .
$$

In WC-undominance, we assume a failure of the second type of hypothetical
thinking, such as C -undominance, in that at each stage $t$, each player $i$ anticipates the other players' current and future action selections optimistically when they select according to the default strategy $a_{i}$ but pessimistically when they select any alternative strategy $\hat{a}_{i} \neq a_{i}$, which is consistent with their past action history $\hat{a}_{i}^{t-1}=a_{i}^{t-1}$. Unlike Cundominance, players retain self-control even if they deviate from their default strategies. In WC-undominance, they can intentionally change not only their current actions $\hat{a}_{i, t} \neq a_{i, t}$, but also their future actions $\left(\hat{a}_{i, t+1}, \ldots, \hat{a}_{i, T}\right)$ without fear of loss of self-control. Clearly, undominance means WC-undominance, but not vice versa. WC-undominance means C-undominance, but not vice versa.

## 4. Sealed-Bid Auction

## 4. 1. Ascending Proxy Auction

We model a single-unit sealed-bid auction as the frame $\Gamma$, which is defined as: fixing an arbitrary positive integer $W$. We assume that for every $i \in N$ and $t \in\{1, \ldots, T\}$

$$
\begin{aligned}
& A_{i, t} \subset\{0,1, \ldots, W\}, \\
& 0 \in A_{i, t},
\end{aligned}
$$

and

$$
A_{i, t} \cap A_{i, t^{\prime}}=\{0\} \text { for all } t^{\prime} \neq t
$$

We assume that for every $w \in\{1, \ldots, W\}$, there exists $t \in\{1, \ldots, T\}$ such that

$$
w \in A_{i, t},
$$

and that for each $t \in\{1, \ldots, T\}$, there exists $i \in N$ such that $A_{i, t} \neq\{0\}$.
At each stage, $t \in\{1, \ldots, T\}$, a fictitious auctioneer requests that each player (bidder) $i \in N$ select the maximal price among $A_{i, t} \backslash\{0\}$ that they can pay for the commodity to be auctioned. If no such price exists (i.e., player $i$ does not want to purchase the commodity at any price in $A_{i, t} \backslash\{0\}$ ), player $i$ selects 0 . If $A_{i, t}=\{0\}$, that is, if the auctioneer requests nothing from player $i$ at stage $t$, player $i$ automatically selects a 0 .

The auctioneer increases the price step-by-step; that is, we assume that for each $i \in N$ and $t \in\{1, \ldots, T\}$, if $A_{i, t} \neq\{0\}$, then

$$
\arg \min A_{i, t} \backslash\{0\}=\arg \max \bigcup_{t^{\prime}=1}^{t-1} A_{i, t^{\prime}}+1
$$

We allow the auctioneer to increase the prices at different speeds for different players. However, the difference in speed is limited, and the auctioneer asks players as equally as possible. That is, we assume that for each $i \in N$ and $t \in\{1, \ldots, T\}$, if $A_{i, t} \neq\{0\}$,

$$
\begin{equation*}
\arg \max A_{i, t} \geq \arg \max \bigcup_{t^{\prime}=1}^{t-1} A_{j, t^{\prime}} \geq \arg \max \bigcup_{t^{\prime}=1}^{t-1} A_{i, t^{\prime}} \text { for all } j \in N . \tag{1}
\end{equation*}
$$

Inequality (1) implies the limitation of the speed difference such that the auctioneer will increase the price to the bidder for whom they asked the lowest price compared to the other bidders in priority, and the auctioneer therefore makes this bidder's price level catch up with the other bidders' price levels. This limitation minimizes the possibility of selfcontrol failure in C-undominance. (Assumption (1) is not necessary for the discussion of WC-undominance because it restores self-control.) For example, consider a three-bidder case and suppose $A_{1,1}=A_{2,1}=\{0\}$ and $A_{3,1}=\{0,1\}$. In this case, $A_{3,2}=\{0\}$ must hold. If $A_{1,2}=\{0,1\}$, there exists $w \in\{0, \ldots, W\}$ such that $A_{2,2}=\{0, \ldots, w\}$, that is, $A_{2,2}$ can be any set of non-negative integers less than or equal to connected from zero.

We define the stage at which player $i$ leaves the auction, $t_{i}\left(a_{i}\right) \in\{1, \ldots, T+1\}$, as

$$
t_{i}\left(a_{i}\right)=T+1 \quad \text { if } a_{i, t}=\max A_{i, t} \text { for all } t \in\{1, \ldots, T\}
$$

and

$$
t_{i}\left(a_{i}\right)=\min \left\{t \in\{1, \ldots, T\} \mid a_{i, t}<\max A_{i, t}\right\} \text { otherwise. }
$$

We then define the (proxy) bid of player $i, b_{i}\left(a_{i}\right) \in\{0, \ldots, W\}$, as

$$
\begin{array}{ll}
b_{i}\left(a_{i}\right)=W & \text { if } t_{i}\left(a_{i}\right)=T+1, \\
b_{i}\left(a_{i}\right)=\min A_{i, t_{i}\left(a_{i}\right)} \backslash\{0\}-1 & \text { if } a_{i, t_{i}\left(a_{i}\right)}=0,
\end{array}
$$

and

$$
b_{i}\left(a_{i}\right)=a_{i, t_{i}\left(a_{i}\right)} \quad \text { otherwise }
$$

Hence, by playing $a_{i} \in A_{i}$, player $i$ expresses their willingness to pay as $b_{i}\left(a_{i}\right) \in\{0, \ldots, W\}$.

We consider the winner to be the player whose bid is the highest (i.e., $\left.b_{i}\left(a_{i}\right)=\max _{j \in N} b_{j}\left(a_{j}\right)\right)$. If there are multiple players whose bids are the highest, the auctioneer randomly determines the winner among these players.

Let $v_{i} \in[1, W]$ denote the valuation of player $i$ for the commodity. For simplicity, we assume that $v_{i}$ is not an integer. The winner pays the highest bid among all the remaining players (i.e., the second price). Therefore, we specify the payoff function $u_{i}$ for player $i$ as follows:

$$
\begin{array}{ll}
u_{i}(a)=0 & \text { if } b_{i}\left(a_{i}\right)<\max _{j \neq i} b_{j}\left(a_{j}\right), \\
u_{i}(a)=v_{i}-\max _{j \neq i} b_{j}\left(a_{j}\right) & \text { if } b_{i}\left(a_{i}\right)>\max _{j \neq i} b_{j}\left(a_{j}\right),
\end{array}
$$

and

$$
\begin{array}{ll}
u_{i}(a)=\frac{v_{i}-b_{i}\left(a_{i}\right)}{m} & \text { if } b_{i}\left(a_{i}\right)=\max _{j \neq i} b_{j}\left(a_{j}\right) \text { and } \\
& m=\left|\left\{j \in N \mid b_{j}\left(a_{j}\right)=\max _{h \in N} b_{h}\left(a_{h}\right)\right\}\right| .
\end{array}
$$

Second Price Auction Format (SP): A special case $\Gamma=\Gamma^{S P}$, specified by

$$
T=1,
$$

corresponds to the second price auction format (SP), where the auctioneer requests every player to select the maximal price that they are willing to pay at once. We have $A_{i}=A_{i, 1}=\{0,1, \ldots, W\}$ and $b_{i}\left(a_{i}\right)=a_{i, 1}=a_{i}$. We can consider $\Gamma^{S P}$ as the auction format without frame.

Ascending Proxy Auction Format (AP): Another special case $\Gamma=\Gamma^{A P}$ corresponds to the ascending proxy auction format (AP), which is specified by

$$
T=W,
$$

and

$$
A_{i, t}=\{0, t\} \text { for all } i \in N \text { and } t \in\{1, \ldots, W\} .
$$

At each stage $t$, the auctioneer simultaneously asks each player whether to accept price $t$. Note that the first stage in which player $i$ rejects the price corresponds to $t_{i}\left(a_{i}\right)$, which
is defined as

$$
t_{i}\left(a_{i}\right)=W+1 \quad \text { if } a_{i}=(1, \ldots, W)
$$

and

$$
t_{i}\left(a_{i}\right)=\min \left\{t \in\{1, \ldots, W\} \mid a_{i, t}=0\right\} \quad \text { otherwise. }
$$

The proxy bid of player $i$ is expressed by

$$
b_{i}\left(a_{i}\right)=W \quad \text { if } t_{i}\left(a_{i}\right)=W+1,
$$

and

$$
b_{i}\left(a_{i}\right)=t_{i}\left(a_{i}\right)-1 \quad \text { otherwise } .
$$

Irrespective of the frame specification, it is clear that the strategy $a_{i} \in A_{i}$ of player $i$ is undominated if and only if it is sincere bidding (i.e., $b_{i}\left(a_{i}\right)=\left[v_{i}\right]$ ).

Proposition 2: Irrespective of frame specifications, every strategy $a_{i} \in A_{i}$ of player $i$ is O-undominated.

Proof: If player $i$ selects the default strategy $a_{i}$, they optimistically anticipate that any other player $j \neq i$ selects $b_{j}\left(a_{j}\right)=0$, and therefore, they win the auction for the price 0 , earning the positive payoff $v_{i}>0$. If player $i$ selects any other strategy, they pessimistically anticipate that any other player $j \neq i$ selects $b_{j}\left(a_{j}\right)=W$, and therefore, they lose the auction, earning nothing. Hence, any strategy $a_{i}$ is O-undominated.
Q.E.D.

## 4. 2. C-Undominance

We characterize the set of all C-undominated strategies as follows: For each $x \in(1, W)$, we define $\tau_{i}(x) \in\{1, \ldots, T\}$ as the stage $t$ at which the auctioneer asks player $i$ for the acceptance of the price defined as the integer part of $x,[x]$ (i.e., $[x] \in A_{i, t}$ and $0 \leq x-[x]<1)$.

Proposition 3: A strategy $a_{i} \in A_{i}$ of player $i$ is C-undominated if and only if any one of the following six conditions holds:

C1: $\quad$ The strategy $a_{i}$ implies the sincere bidding (i.e., $b_{i}\left(a_{i}\right)=\left[v_{i}\right]$ ).
C2: $\quad$ The strategy $a_{i}$ implies the underbidding (i.e., $b_{i}\left(a_{i}\right)<\left[v_{i}\right]$ ),

$$
b_{i}\left(a_{i}\right)=\arg \max A_{i, \tau_{i}\left(b_{i}\left(a_{i}\right)\right)},
$$

and

$$
\min _{j \neq i}[\arg \max \underbrace{\tau_{i}\left(b_{i}\left(a_{j}\right)+1\right)-1}_{t=1} A_{j, t}] \leq b_{i}\left(a_{i}\right) .
$$

C3: The strategy $a_{i}$ implies the underbidding,

$$
b_{i}\left(a_{i}\right)=\arg \max A_{i, \tau_{i}\left(b_{i}\left(a_{i}\right)\right)},
$$

and

$$
\min _{j \neq i}\left[\arg \max \bigcup_{t=1}^{\tau_{i}\left(b_{i}\left(a_{i}\right)+1\right)-1} A_{j, t}\right]=b_{i}\left(a_{i}\right)+1 .
$$

C4: The strategy $a_{i}$ implies the underbidding,

$$
b_{i}\left(a_{i}\right) \leq \arg \max A_{i, \tau_{i}\left(b_{i}\left(a_{i}\right)\right)}-1,
$$

and

$$
\min _{j \neq i}[\arg \max \underbrace{\tau_{i}\left(b_{i}\left(a_{j}\right)\right)-1}_{t=1} A_{j, t}] \leq b_{i}\left(a_{i}\right) .
$$

C5: The strategy $a_{i}$ implies the underbidding,

$$
b_{i}\left(a_{i}\right)=\arg \max A_{i, \tau_{i}\left(b_{i}\left(a_{i}\right)\right)}-1,
$$

and

$$
\min _{j \neq i}[\arg \max \underbrace{\tau_{i}\left(b_{i}\left(a_{j}\right)\right)-1}_{i=1} A_{j, t}]=b_{i}\left(a_{i}\right)+1 .
$$

C6: $\quad$ The strategy $a_{i}$ implies the overbidding (i.e., $b_{i}\left(a_{i}\right)>\left[v_{i}\right]$ ), and

$$
\min _{j \neq i}\left[\arg \max \bigcup_{t=1}^{\tau_{i}\left(b_{i}\left(a_{i j}\right)\right)-1} A_{j, t}\right] \leq\left[v_{i}\right] .
$$

C1 implies that sincere bidding is a C-undominated strategy, because it is undominated. $\mathrm{C} 2, \mathrm{C} 3, \mathrm{C} 4$, and C 5 correspond to the necessary and sufficient conditions
under which the underbidding is C -undominated. Under either C 3 or C 5 , the failure of self-control causes players to play the underbids. Under either C2 or C4, even the selfcontrolled players underbid because of the failure of the second type of hypothetical thinking. C6 corresponds to the necessary and sufficient condition under which overbidding is C -undominated, which implies that we can eliminate numerous overbidding strategies because players can practice the first type of hypothetical thinking and recognize that there is no room for their opponents to take actions that would be detrimental to them. Appendix A provides the proof of Proposition 3.

We focus on the class of frames in which the auctioneer asks each bidder at most a single price at every stage, such as AP, and show that C-undominance eliminates most of the overbidding, whereas it tolerates underbidding as well as sincere bidding.

Proposition 4: Suppose that for every $i \in N$ and $t \in\{1, \ldots, T\}$, either $A_{i, t}=\{0\}$, or $A_{i, t} \backslash\{0\}$ is a singleton.

Then, a strategy $a_{i} \in A_{i}$ of player $i$ is C-undominated if and only if

$$
b_{i}\left(a_{i}\right) \leq\left[v_{i}\right]+1 .
$$

Proof: Suppose $b_{i}\left(a_{i}\right)<\left[v_{i}\right]$ (underbidding). Note

$$
b_{i}\left(a_{i}\right)=\arg \max A_{i, \tau_{i}\left(b_{i}\left(a_{i}\right)\right)} .
$$

From (1), we have

$$
b_{i}\left(a_{i}\right)+1 \geq \arg \max \underbrace{\tau_{i}\left(b_{i}\left(a_{i}\right)+1\right)-1}_{t^{\prime}=1} A_{j, t^{\prime}} \text { for all } j \neq i,
$$

which implies that any one of C2 and C3 holds.
Suppose $b_{i}\left(a_{i}\right)>\left[v_{i}\right]$ (overbidding). Note

$$
b_{i}\left(a_{i}\right)=\arg \max A_{i, \tau_{i}\left(b_{i}\left(a_{i}\right)\right)} .
$$

From (1), we have

$$
\min _{j \neq i}\left[\arg \max \bigcup_{i=1}^{\tau_{i}\left(b_{i}\left(a_{i j}\right)\right)-1} A_{j, t}\right]=b_{i}\left(a_{i}\right)-1 \geq\left[v_{i}\right] .
$$

This is consistent with C. 6 if and only if $b_{i}\left(a_{i}\right)=\left[v_{i}\right]+1$. From these observations and Proposition 3, we prove the proposition.
Q.E.D.

Theorem 1: In $\Gamma^{S P}$, every strategy $a_{i} \in A_{i}$ of player $i$ is C-undominated. In $\Gamma^{A P}$, a strategy $a_{i} \in A_{i}$ of player $i$ is C-undominated if and only if:

$$
b_{i}\left(a_{i}\right) \leq\left[v_{i}\right]+1 .
$$

Proof: Consider $\Gamma^{S P}$. Since only the second type of hypothetical thinking matters, we have equivalence between C -undominance and O -undominance.

Consider $\Gamma^{A P}$. Because $\Gamma^{A P}$ satisfies the condition in Proposition 4, the inequality $b_{i}\left(a_{i}\right) \leq\left[v_{i}\right]+1$ is necessary and sufficient for C-undominance.
Q.E.D.

By replacing the second-price auction format (no-frame format, SP) with the ascending proxy auction format (framed format, AP), we can eliminate most overbidding strategies from the set of C-undominated strategies. This framing restores the first type of hypothetical thinking, eliminating the misperception that overbidding induces other bidders to lower their prices and discourages players from overbidding.

## 4. 3. WC-Undominance

We can characterize the set of all WC-undominated strategies as follows.

Proposition 5: The strategy $a_{i} \in A_{i}$ of player $i$ is WC-undominated if and only if any one of $\mathrm{C} 1, \mathrm{C} 2, \mathrm{C} 4$, and C 6 is satisfied.

Failure of self-control causes a player to play underbid under either C3 or C5. Because players restore self-control in WC-undominance, Proposition 5 is established immediately from Proposition 3. We show the full proof of Proposition 5 in Appendix B.

Theorem 2: In $\Gamma^{S P}$, every strategy $a_{i} \in A_{i}$ of player $i$ is WC-undominated. In $\Gamma^{A P}$, a
strategy $a_{i} \in A_{i}$ of player $i$ is WC-undominated if and only if:

$$
b_{i}\left(a_{i}\right) \leq\left[v_{i}\right]+1 .
$$

Proof: Consider $\Gamma^{S P}$. Since only the second type of hypothetical thinking matters, we have equivalence between WC-undominance and O-undominance.

Consider $\Gamma^{A P}$. Note

$$
b_{i}\left(a_{i}\right)=\arg \max A_{i, \tau_{i}\left(b_{i}\left(a_{i}\right)\right)} .
$$

Suppose $b_{i}\left(a_{i}\right)<\left[v_{i}\right]$ (underbidding). Note

$$
b_{i}\left(a_{i}\right)=\arg \max \bigcup_{t^{\prime}=1}^{\tau_{i}\left(b_{i}\left(a_{i}\right)+1\right)-1} A_{j, t^{\prime}} \text { for all } j \neq i
$$

which implies C2. Suppose $b_{i}\left(a_{i}\right)>\left[v_{i}\right]$ (overbidding). Note

$$
\min _{j \neq i}\left[\arg \max {\left.\underset{i=1}{\tau_{i}\left(b_{i}\left(a_{i j}\right)\right)-1} A_{j, t}\right]=b_{i}\left(a_{i}\right)-1 \geq\left[v_{i}\right] . . . . . . . .}\right.
$$

This is consistent with C. 6 if and only if $b_{i}\left(a_{i}\right)=\left[v_{i}\right]+1$. From these observations and Proposition 5, we prove Theorem 2.
Q.E.D.

It should be noted that Proposition 4 in Subsection 4.2, which implies that the tendency to underbid cannot be ruled out in C-undominance, does not necessarily hold if we replace C -undominance with WC -undominance. The following proposition states that, with two players (i.e., $n=2$ ) in an alternating-play frame, sincere bidding is the only WC-undominated strategy for the second mover:

Alternating Ascending Proxy Auction Format (AAP): A special case $\Gamma=\Gamma^{A A P}$ corresponds to the alternating ascending proxy auction format (AAP) wherein $n=2$, $T=2 W$, and for each $w \in\{1, \ldots, W\}$,

$$
A_{1,2(w-1)+1}=A_{2,2 w}=\{0, w\} \text { and } A_{2,2(w-1)+1}=A_{1,2 w}=\{0\} .
$$

In AAP, the auctioneer asks player 1 at each odd stage, whereas they ask player 2 at each even stage.

Proposition 6: Consider $\Gamma=\Gamma^{A A P}$. A strategy $a_{1} \in A_{1}$ of player 1 is WC-undominated if and only if

$$
b_{1}\left(a_{1}\right) \leq\left[v_{1}\right]+1 .
$$

A strategy $a_{2} \in A_{2}$ of player 2 is WC-undominated if and only if it implies the sincere bidding, that is,

$$
b_{2}\left(a_{2}\right)=\left[v_{2}\right] .
$$

Proof: In the same manner as Theorem 2, we can show that a strategy $a_{1} \in A_{1}$ of player 1 is WC-undominated if and only if $b_{1}\left(a_{1}\right) \leq\left[v_{1}\right]+1$.

Consider player 2. Note

$$
b_{2}\left(a_{2}\right)=\arg \max A_{2, \tau_{2}\left(b_{2}\left(a_{2}\right)\right)} .
$$

Suppose $b_{2}\left(a_{2}\right)<\left[v_{2}\right]$ (underbidding). Note

$$
b_{2}\left(a_{2}\right)+1=\arg \max \underbrace{\tau_{2}\left(b_{2}\left(a_{2}\right)+1\right)-1}_{t^{\prime}=1} A_{1, t^{\prime}} .
$$

which contradicts C2. Suppose $b_{2}\left(a_{2}\right)>\left[v_{2}\right]$ (overbidding). Note

$$
\arg \max \bigcup_{t=1}^{\tau_{2}\left(b_{2}\left(a_{2}\right)\right)-1} A_{1, t}=b_{2}\left(a_{2}\right)>\left[v_{2}\right],
$$

which contradicts C6.
Q.E.D.

Without a frame, a bounded-rational player cannot rule out any strategy selection. A frame allows a bounded-rational player to eliminate overbidding strategies by restoring the first type of hypothetical thinking. Furthermore, by framing, bounded-rational but self-controlled players tend to exclude underbidding strategies as well, and allow only sincere bidding strategies. The difference between players 1 and 2 is whether the opponent has already answered the same single price at each stage. If this difference is considered trivial, underbidding can be primarily attributed to the failure of self-control. Based on this theoretical consideration, we test the behavioral hypotheses that, compared with SP (no-frame), subjects tend to bid more sincerely in AP (framed).

## 5. Laboratory and Online Experiments

We report the results of our online and laboratory experiments comparing SP (noframe) and AP (framed). We propose the behavioral hypotheses that the experimental subjects tend to overbid in SP more often than in AP, underbid in AP more often than in SP, and bid more sincerely in AP than SP. Our experimental results indicate that the hypotheses hold true regardless of whether the experiments were conducted online or in laboratories.

## 5. 1. Experimental Design

Eight sessions of computer-based experiments were conducted for this study. ${ }^{5}$ Four sessions were conducted in the laboratories in December 2019, and the others were conducted online in December 2020. ${ }^{6}$ We recruited 112 subjects for the laboratory experiments and 212 subjects for the online experiments. Both subjects consisted of randomly selected undergraduate and graduate students from various departments at Ritsumeikan University in Japan, and they were informed of the experiments through university-wide e-mail announcements. Participants were paid according to their performance in the experiments (at a fixed rate of 100 JPY per point). In addition to the performance-based payment, our subjects were paid a participation fee of 1,500 JPY for the laboratory experiments, and $1,000 \mathrm{JPY}$ for the online experiments. On average, the subjects earned $4,979 \mathrm{JPY}$ for the laboratory experiment and 4,621 JPY for the online experiments.

Each session consisted of five games that proceeded in the following order: prisoner's dilemma games 1 and 2, guessing games 1 and 2, and auction games. Each game was two-player in which a pair of subjects was randomly and anonymously matched to play the game. In laboratory experiments, each subject was randomly matched with a partner player before the start of each game. In the online experiments, partner players

[^1]were not determined during the sessions but were determined later, and each subject was allowed to access the experimental web page to join the experiments at any time between 9:00 and 21:00 on the assigned day using a PC individually, and was asked to leave their responses online. Then, after the session closed, their response for each game was matched with that of another player who was randomly chosen as their partner player among the subjects who joined the experiment in the same session. Although the online subjects did not play games concurrently with the partner player, each subject was explicitly instructed to imagine that they were playing with an anonymous partner player concurrently during the experiments.

The experimental design is between-subjects. Approximately half of the subjects (56 subjects in the laboratory experiment and 103 subjects in the online experiment) were assigned to the treatment group and played framed games: prisoner's dilemma games 1 and 2 in a sequential-decision format, guessing games 1 and 2 in a sequential-decision format, and the auction game that mimics the ascending proxy auction format (AP) discussed in Section 4. The remaining subjects were assigned to the control group and played prisoner's dilemma games 1 and 2 in a simultaneous-decision format, guessing games 1 and 2 in a simultaneous-decision format, and a no-frame second price auction (SP). Full experimental instructions are provided in the online appendix. Because our aim in this study was to examine C-undominance and WC-undominance, we focused only on auction games.

In either SP or AP auction games, the subjects were given five points in advance. Each subject is then paired with a partner player and competes against the partner in the assigned auction format. Both players of a pair bid for a monetary prize, or points, denoted by "winner point." The winner point depends on each player, which mimics their respective valuations of the virtual commodity to be traded in a single-unit seal-bid auction. Each player's winner point is determined experimentally in advance, and each player is informed of their own winner point before the auction starts but not of the winner point of their partner. The winner points vary between paired players; in each pair of players, 9.5 point are randomly assigned to one player, while 6.5 point are assigned to the other player.

The winner of an auction earns the winner point, but abandons certain points as the winner's payment. The winner of the auction and their payment are determined according
to the auction format to which the pair of subjects is assigned. Subjects assigned to the control group play the conventional second-price auction format (SP), in which each subject is asked to choose an integer between 0 and 15 as a bid. Subsequently, the subject is determined to be the winner if the integer chosen by them exceeds the integer chosen by their partner. If the integers were identical among the paired players, the computer randomly determined the winner from the two players. The winner's payment is identical to the integer (bid) chosen by the loser.

On the other hand, subjects assigned to the treatment group play the following ascending proxy auction format (i.e., AP): A fictitious auctioneer raises a hypothetical payment from 1 to 15 point by one increment, and for each payment point the subjects are asked to express whether they are willing to obtain the prize of the auction at the expense of the payment point being posted, or simply "buy" the prize. More specifically, starting with 1 payment point, both of the paired subjects are asked to express whether they are willing to buy the prize at the expense of 1 point, responding in the form of a binary answer to either "accept" or "deny." Once both subjects determined their responses, the fictitious auctioneer raised the hypothetical payment point by one and asked the subjects to express their willingness to accept the expense of two points. This process is repeated until the hypothetical payment point reaches a maximum value of 15. Finally, the players' willingness to accept was recorded in the form of binary responses corresponding to each hypothetical payment point.

The winner/loser of the auction is determined once both players finish expressing their willingness to accept fully until the hypothetical payment point reaches the maximum number. The loser is determined by the player who first expresses the denial of willingness during the fictitious auctioneer raises the hypothetical payment points one by one. If both players expressed their first denial at an identical payment point, the computer randomly determined the winner of the game. The winner's payment is determined by the payment point at which the loser first expresses its denial minus 1. If the loser maintains its willingness to accept until the payment point reaches its maximum number (i.e., 15), the auction payment is determined to be the maximum number.

The final earning of the winner in either auction is determined as the sum of the winner's points and the five points endowed at the start of the auction, minus the winner's payment, although the winner earns nothing if the value is negative. The loser's earning
is the five points endowed at the start of the auction.
Experimental instructions are provided in the online appendix. In the laboratory experiments, printed instructions were provided to participants and explained using a recorded voice before each game started. In the online experiments, the participants were asked to read and understand the instructions projected onto the screen before each game started.

### 5.2. Data Analysis

### 5.2.1. Pooled Data across Laboratory and Online experiments

We observed 323 bids of subjects and associated valuations (winner points) for the auction in the pooled data across the laboratory and online experiments. ${ }^{7}$ Figure 1 in Appendix D displays the histograms of the difference between the bid submitted by a subject and the winner point (i.e., the subject's valuation of the virtual commodity in the auction) assigned to the subject for both AP (treatment) and SP (control). A positive value implies an overbid and a negative value corresponds to an underbid in the histograms. If the subjects play the undominated strategy, the histogram is clustered around zero (i.e., a sincere bid) in both auctions. However, if the subjects behave consistently with Cundominance or WC-undominance, we observe a large mass of overbids only in SP and we observe significant underbids, sincere bids, or both in AP.

Although both histograms in Figure 1 have a certain mass at zero, both distributions are systematically biased and the directions of the biases are reversed, as we observe a large mass of overbids only in SP. For SP, $52.1 \%$ of the bids are larger than the assigned winner points at least by two, while only $17.6 \%$ are smaller at least by two, and $30.3 \%$ are around zero (between -2 and 2). Contrarily, the distribution for AP displays downward bias, implying that underbids emerge in response to the treatment. The fraction of overbids (larger than zero at least by 2 ) is only $17.7 \%$, while $45.6 \%$ are underbids (smaller than zero at least by 2 ) and $36.7 \%$ are close to sincere bids (between -2 and 2 ). The two

[^2]distributions are statistically significantly different (Kolmogorov-Smirnov test, p -value $<0.01$ ), as well as their mean values, which are 2.20 (s.e. 0.37) for SP and -1.58 (s.e. 0.35 ) for AP, as shown in Table 1 in Appendix D ( $\mathrm{p}<0.01$ ).
[Figure 1]
[Table 1]

### 5.2.2. Comparison between Laboratory Data and Online Data

In the pooled data from the laboratory and online experiments, we observed that overbids disappeared substantially and underbids emerged in response to the treatment. However, one might believe that experimental treatment could affect the heterogeneity across laboratory and online experiments. More specifically, by recognizing the substantial presence of their partner player in the laboratory experiments rather than in the online experiments, subjects could perform hypothetical thinking, restore self-control more appropriately, and make more sincere bids in the laboratory experiment. Hence, we examined whether there was any difference in the participants' responses to the treatment between the laboratory and online experiments.

We obtained 112 and 211 observations from the laboratory and online experiments, respectively. Figure 2 in Appendix D displays the histograms of the bids minus the winner points for AP and SP in the laboratory experiments, whereas Figure 3 in Appendix D displays the histograms in the online experiments. Table 1 displays the means of the bids minus the winner points and their deviations (measured in absolute terms) for the laboratory and online experiments.
[Figure 2]
[Figure 3]

In the laboratory and online experiments, the subjects reduced their overbids in
response to the treatment. The fraction of the overbids (larger than zero at least by two) was reduced from $51.8 \%$ in SP to $21.4 \%$ in AP in the laboratory experiment, as displayed in Figure 2, and was reduced from $55.3 \%$ in SP to $15.7 \%$ in AP in the online experiment in Figure 3. Consistently, as displayed in Table 1, the mean values of the histograms were significantly smaller in AP than in SP for both experiments ( p -values $<0.01$ both).

Although a reduction in overbids is common to both experiments, the treatment effect differs in the magnitudes of the deviations, as indicated in Table 1. The mean deviation in AP was significantly smaller than that in SP in the laboratory experiments (p $<0.01$ ), whereas it did not differ significantly in the online experiments ( $\mathrm{p}=0.485$ ). In response to the treatment, the bids became closer to the values in the laboratory experiments but not in the online experiments.

Given the reduction in overbids, an identical deviation in online experiments implies the emergence of underbids. As observed in Figure 3, the fraction of underbids (less than zero, at least two) is increased from $11.9 \%$ at SP to $49.0 \%$ at AP in the online experiments. However, the fraction of sincere bids (between -2 and 2) remained almost identical ( $35.8 \%$ in SP and $35.3 \%$ in AP). Indeed, the mean distribution was significantly lower than zero ( $\mathrm{p}<0.01$ ). In response to the treatment, underbids emerged, mainly behind the reduction in overbids in online experiments.

However, the reduced deviation in the laboratory experiments implies that a certain mass emerges as sincere bids or bids close to sincere bids. Indeed, as observed in Figure 2 , the fraction of the bids around zero (between -2 and 2 ) is increased from $19.6 \%$ in SP to $39.3 \%$ in AP, as well as that of the underbids is increased from $28.6 \%$ in SP to $39.3 \%$ in AP in the laboratory experiments. The mean of the distribution did not differ significantly from zero ( $\mathrm{p}=0.140$ ). In response to the treatment, sincere bids or bids close to them emerged, in addition to underbids, behind the reduction of overbids in the laboratory experiments.

Thus, the treatment effect was heterogeneous to a certain extent between the laboratory and online experiments. Overbids commonly disappear in response to treatment; however, underbids emerge mainly in online experiments, whereas sincere bids or bids close to them also emerge in addition to underbids in laboratory experiments. These experimental observations suggest that participants were more likely to lose selfcontrol online than in the laboratory.

However, the above-mentioned heterogeneous treatment effect could be spurious if we consider mistakes of subjects in AP, in which the auction results are determined based on the payment points at which the players express their denial of the willingness to buy for the first time, and which was informed to the subjects explicitly in the experimental instruction. Nonetheless, some subjects expressed their willingness to accept again, even after they have already expressed their denial at a lower payment point, although such expressions of willingness presented at higher payment points had no effect on the determination of the auction results. Perhaps they mistakenly expressed their denial at a lower payment point or mistakenly expressed their willingness at a higher payment point again; however, we cannot identify the reason why they respond irregularly.

Such irregular responses were observed unevenly across the laboratory and online experiments. In the online experiments, 29 out of 102 subjects responded irregularly, whereas 8 out of 56 subjects responded irregularly in the laboratory experiments. Because such irregular responses may reflect the intentions of the subjects incorrectly, a larger fraction of irregular responses could distort the results in online experiments more severely than in laboratory experiments.

To evaluate the behavioral difference between the online and laboratory experiments conservatively, we also present the results, omitting subjects with irregular responses for their proxy bids. Figure 4 in Appendix D displays the histograms of the bids minus the winner points for AP and SP in the laboratory experiments, whereas Figure 5 in Appendix D displays the histograms in the online experiments. Table 2 in Appendix D displays the means of the bids minus the winner points and their means of the absolute differences of them for laboratory and online data. Even in the online experiments, the absolute difference was statistically significantly smaller in AP than in SP ( $p=0.028$ ), which indicates that the bids became closer to sincere bids in response to the treatment as well as in the laboratory experiments ( $\mathrm{p}<0.01$ ). In addition, the means of the bids minus the winner points were not significantly different from zero in either experiment ( $p=0.570$ for the laboratory experiments and $\mathrm{p}=0.202$ for the online experiments).

Once the subjects with irregular responses were omitted from the data, the treatment effects were qualitatively similar across the laboratory and online experiments. Thus, the seemingly heterogeneous treatment effects across laboratory and online experiments could be due to subject mistakes. Omitting irregular responses, we observed that bids
became closer to sincere bids in response to the treatment behind the reduction of overbids commonly observed in laboratory and online experiments.
[Figure 4]
[Figure 5]
[Table 2]

## 6. Conclusion

The experimental results of this study indicate that by providing a concrete description of the decision-making procedure as a frame for a sealed-bid second-price auction, we can eliminate anomalies caused by bounded rationality and facilitate its inherent advantages, such as the achievement of efficiency and promotion of participation in social implementation. Although sincere bidding is a unique undominated strategy in second-price action, it is well known in the experimental economics literature that subjects tend to overbid. However, our results from both the laboratory and online experiments show that subjects behave more rationally when accompanied by an abstract strategic game with a suitably contrived frame for proxy-bid procedures. The effects of such framing are supported by the characterization theorems of C-undominance and CWundominance (i.e., Theorems 1 and 2) and the supplemental theoretical considerations in Section 4.

Another important experimental finding is that the framed sealed-bid auction format yields the same positive results in both the laboratory and online experiments. In the online experiment, unlike in the laboratory experiment, subjects were not required to participate and bid at the same time. This finding underscores the importance of systematic research aimed at pioneering sealed-bid auctions rather than open-bid auctions by devising a framing design. It is also desirable for future research to promote institutional design in various imperfect information games, not only from the issue of the failure of hypothetical thinking but also from various perspectives of bounded
rationality and motives other than pure self-interest, such as overcoming computational complexities and the effective use of individuals' inherent social preferences.

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## Appendix A: Proof of Proposition 3

We prove the "if" part as follows. Consider C1. Note that undominance implies Cundominance. Since the sincere bidding is an undominated strategy, it is also a Cundominated strategy.

Consider C2. Owing to underbidding, player $i$ has no incentive to lower its bids. Let player $j \neq i$ denote player, such that

$$
\arg \max \bigcup_{t=1}^{\tau_{i}\left(b_{i}\left(a_{i}\right)+1\right)-1} A_{j, t}=\underset{j^{\prime} \neq i}{\arg \min }\left[\arg \max \bigcup_{t=1}^{\tau_{i}\left(b_{i}\left(a_{i}\right)+1\right)-1} A_{j^{\prime}, t}\right] \leq b_{i}\left(a_{i}\right) .
$$

By selecting any action other than 0 at stage $\tau_{i}\left(b_{i}\left(a_{i}\right)+1\right)$, player $i$ 's anticipation changes from an optimistic view to an pessimistic view concerning player $j^{\prime} s$ strategy,
 "stay up to the end" (i.e., $t_{j}\left(a_{j}\right)=T+1$ ), provided that player $j$ does not leave and the other players leave before stage $\tau_{i}\left(b_{i}\left(a_{i}\right)+1\right)$. In this case, player $i$ loses the gain by deviating, because without such deviation they can win for the price $b_{i}\left(a_{i}\right)$ or lower.

Consider C3. Let player $j \neq i$ denote a player such that

$$
\arg \max \underbrace{\tau_{i}\left(b_{i}\left(a_{i}\right)+1\right)-1}_{t=1} A_{j, t}=\min _{j^{\prime} \neq i}[\arg \max \underbrace{\tau_{i}\left(b_{i}\left(a_{i}\right)+1\right)-1}_{t=1} A_{j^{\prime}, t}]=b_{i}\left(a_{i}\right)+1 .
$$

By selecting any action other than 0 at stage $\tau_{i}\left(b_{i}\left(a_{i}\right)+1\right)$, player $i$ 's anticipation changes from an optimistic view to an pessimistic view concerning player $j$ ' $s$ strategy,
 to "stay up to the end", provided that player $j$ does not leave and the other players leave before stage $\tau_{i}\left(b_{i}\left(a_{i}\right)+1\right)$. At the same time, player $i$ anticipates that they lose their selfcontrol and stay up to the end. Clearly, player $i$ decreases their payoff by deviating.

Consider C4. Let player $j \neq i$ denote a player such that

$$
\arg \max \underbrace{\tau_{i}\left(b_{i}\left(a_{j}\right)\right)-1}_{t=1} A_{j, t}=\min _{j^{\prime} \neq i}[\arg \max \underbrace{\tau_{i}\left(b_{i}\left(a_{i j}\right)\right)-1}_{t=1} A_{j^{\prime}, t}] \leq b_{i}\left(a_{i}\right) .
$$

By increasing their bid, player $i$ 's anticipation changes from an optimistic view to an
pessimistic view concerning player $j$ 's strategy, such that player $j$ changes their strategy from $b_{j}\left(a_{j}\right)=\arg \max \bigcup_{t=1}^{\tau_{i}\left(b_{i}\left(a_{j}\right)\right)-1} A_{j, t} \leq b_{i}\left(a_{i}\right)$ to "stay up to the end", provided that player $j$ does not leave and the other players leave before stage $\tau_{i}\left(b_{i}\left(a_{i}\right)\right)$. In this case, player $i$ decreases their payoff by deviating, because without such deviation they can win for the price $b_{i}\left(a_{i}\right)$ or lower.

Consider C5. Let player $j \neq i$ denote a player such that

$$
\arg \max \underbrace{\tau_{i}\left(b_{i}\left(a_{j}\right)\right)-1}_{t=1} A_{j, t}=\min _{j^{\prime} \neq i}[\arg \max \underbrace{\tau_{i}\left(b_{i}\left(a_{i j}\right)\right)-1}_{t=1} A_{j^{\prime}, t}]=b_{i}\left(a_{i}\right)+1 .
$$

By changing their action at stage $\tau_{i}\left(b_{i}\left(a_{i}\right)\right)$ to $\arg \max A_{i, \tau_{i}\left(b_{i}\left(a_{i}\right)\right)}$, player $i^{\prime} s$ anticipation changes from an optimistic view to an pessimistic view concerning player $j^{\prime} s$ strategy, such that player $j$ changes their strategy from $b_{j}\left(a_{j}\right)=\arg \max \bigcup_{t=1}^{\tau_{i}\left(b_{i}\left(a_{j}\right)\right)-1} A_{j, t}=b_{i}\left(a_{i}\right)+1$ to "stay up to the end", provided that player $j$ does not leave and the other players leave before stage $\tau_{i}\left(b_{i}\left(a_{i}\right)\right)$. At the same time, they pessimistically anticipate that they lose their self-control and stay up to the end. Clearly, player $i$ loses the gain by deviating.

Consider C6. Owing to overbidding, player $i$ has no incentive to increase its bids. Let player $j \neq i$ denote player, such that

$$
\arg \max \bigcup_{t=1}^{\tau_{i}\left(b_{i}\left(a_{i}\right)\right)-1} A_{j, t}=\min _{j^{\prime} \neq i}\left[\arg \max \bigcup_{t=1}^{\tau_{i}\left(b_{i}\left(a_{i}\right)\right)-1} A_{j^{\prime}, t}\right] \leq\left[v_{i}\right]
$$

By decreasing their bid less than the default bid $b_{i}\left(a_{i}\right)$, player $i$ ' $s$ anticipation changes from an optimistic view to an pessimistic view concerning player $j^{\prime} s$ strategy, such that player $j$ changes their strategy from $b_{j}\left(a_{j}\right)=\arg \max \bigcup_{i=1}^{\tau_{i}\left(b_{i}\left(a_{j}\right)\right)-1} A_{j, t} \leq\left[v_{i}\right]$ to "stay up the end," provided that player $j$ does not leave and the other players leave before stage $\tau_{i}\left(b_{i}\left(a_{i}\right)\right)$. In this case, player $i$ loses the gain by deviating because, without such a deviation, they can win for the price $\left[v_{i}\right]$ or less. We can apply the same argument even if player $i$ attempts to leave before stage $\tau_{i}\left(b_{i}\left(a_{i}\right)\right)$.

From these observations, we have proved the "if" part.
We prove the "only if" part as follows. Suppose that none of these six conditions
holds. Then, any one of the following four conditions holds:
C7: The strategy $a_{i}$ implies the underbidding,

$$
b_{i}\left(a_{i}\right)=\arg \max A_{i, \tau_{i}\left(b_{i}\left(a_{i}\right)\right)}
$$

and

$$
\min _{j \neq i}\left[\arg \max \underset{t=1}{\tau_{i}\left(b_{i}\left(a_{i}\right)+1\right)-1} A_{j, t}\right]>b_{i}\left(a_{i}\right)+1
$$

C8: The strategy $a_{i}$ implies the underbidding,

$$
b_{i}\left(a_{i}\right)<\arg \max A_{i, \tau_{i}\left(b_{i}\left(a_{i}\right)\right)}-1,
$$

and

$$
\min _{j \neq i}[\arg \max \underbrace{\tau_{i}\left(b_{i}\left(a_{j}\right)\right)-1}_{i=1} A_{j, t}]>b_{i}\left(a_{i}\right) .
$$

C9: The strategy $a_{i}$ implies the underbidding,

$$
b_{i}\left(a_{i}\right)=\arg \max A_{i, \tau_{i}\left(b_{i}\left(a_{i}\right)\right)}-1,
$$

and

$$
\min _{j \neq i}[\arg \max \underbrace{\tau_{i}\left(b_{i}\left(a_{i j}\right)\right)-1}_{i=1} A_{j, t}]>b_{i}\left(a_{i}\right)+1
$$

C10: $\quad$ The strategy $a_{i}$ implies the overbidding, and

$$
\min _{j \neq i}[\arg \max \underbrace{\tau_{i}\left(b_{i}\left(a_{j}\right)\right)-1}_{t=1} A_{j, t}]>\left[v_{i}\right] .
$$

Consider C7. Since the other players decided whether to bid less than $b_{i}\left(a_{i}\right)+2$ before stage $\tau_{i}\left(b_{i}\left(a_{i}\right)+1\right)$, even a pessimistic player $i$ is willing to change their bid from $b_{i}\left(a_{i}\right)$ to $b_{i}\left(a_{i}\right)+1$ with safety.

Consider C8. Since any other player decided whether to bid less than $b_{i}\left(a_{i}\right)+1$ before stage $\tau_{i}\left(b_{i}\left(a_{i}\right)\right)$, even a pessimistic player $i$ is willing to change their bid from $b_{i}\left(a_{i}\right)$ to $b_{i}\left(a_{i}\right)+1$ with safety.

Clearly, C9 contradicts (1).
Consider C 10 . Because any other player decided whether to bid less than $\left[v_{i}\right]+1$ before stage $\tau_{i}\left(b_{i}\left(a_{i}\right)\right)$, player $i$ has better select 0 than $b_{i}\left(a_{i}\right)$ at this stage if
$\tau_{i}\left(b_{i}\left(a_{i}\right)\right)>\tau_{i}\left(v_{i}\right)$, and they better select $\left[v_{i}\right]$ rather than $b_{i}\left(a_{i}\right)$ at this stage if $\tau_{i}\left(b_{i}\left(a_{i}\right)\right)=\tau_{i}\left(v_{i}\right)$.

From these observations, we have proved the "only if" part.

## Appendix B: Proof of Proposition 5

The proof of Proposition 5 does not depend on the failure of self-control provided that C1, C2, C4, or C6 hold. Hence, we can prove the "if" part of this theorem in the same manner as in Proposition 3.

We prove the "only if" part as follows. Because WC-undominance implies Cundominance, any WC-undominated strategy must satisfy $\mathrm{C} 1, \mathrm{C} 2, \mathrm{C} 3, \mathrm{C} 4, \mathrm{C} 5$, or C 6 . Suppose that either C3 or C5 hold. Because any other player decided whether to accept price $b_{i}\left(a_{i}\right)+1$, player $i$, who maintains self-control, can safely change their bid from $b_{i}\left(a_{i}\right)$ to $\left[v_{i}\right]$.

## Appendix C: Experimental Order

1. Laboratory Experiment

| Day | Number of Subjects | Existence of Frame (Treatment) |
| :--- | :--- | :--- |
| December 5, 2019 | 28 | Yes |
| December 6, 2019 | 28 | Yes |
| December 12, 2019 | 28 | No |
| December 13, 2019 | 28 | No |

2. Online Experiment

| Day | Number of Subjects | Existence of Frame (Treatment) |
| :--- | :--- | :--- |
| December 15, 2020 | 50 | Yes |
| December 16, 2020 | 55 | No |
| December 17,2020 | 53 | Yes |
| December 18,2020 | 54 | No |

## Appendix D: Figures and Tables

Figure 1:
Difference between the Bid and Winner Point


Figure 2:
Difference between the Bid and Winner Point: Laboratory Experiments


Figure 3:
Difference between the Bid and Winner Point: Online Experiments


Table 1: Means of Bids Minus the Winner Points and Means of Their Absolute Values

|  | AP | SP | p-value |
| :---: | :---: | :---: | :---: |
| Bid - winner point |  |  |  |
| Pooled | $-1.582(0.348)$ | $2.203(0.367)$ | 0.000 |
| Laboratory | $-0.768(0.520)$ | $1.518(0.660)$ | 0.007 |
| Online | $-2.029(0.454)$ | $2.555(0.441)$ | 0.000 |
|  |  |  |  |
| ABS (bid - winner point) | $3.658(0.226)$ | $4.342(0.222)$ | 0.031 |
| Pooled | $3.071(0.326)$ | $4.482(0.327)$ | 0.003 |
| Laboratory | $3.980(0.298)$ | $4.271(0.292)$ | 0.485 |
| Online |  |  |  |

Standard errors are indicated in parentheses. P-values for comparisons between AP and SP groups were computed using a two-sample $t$-test, which allowed for unequal variances.

Figure 4:
Difference between the Bid and Winner Point:
Laboratory Experiments (omitting irregular responses)


Figure 5:
Difference between the Bid and Winner Point: Online Experiments (omitting irregular responses)


Table 2: Means of Bids Minus the Winner Points and Means of Their Absolute Values (omitting irregular responses)

|  | AP | SP | p-value |
| :---: | :---: | :---: | :---: |
| Bid - winner point |  |  |  |
| Pooled | $-0.508(0.369)$ | $2.203(0.367)$ | 0.000 |
| Laboratory | $-0.313(0.550)$ | $1.518(0.660)$ | 0.034 |
| Online | $-0.637(0.499)$ | $2.555(0.441)$ | 0.000 |
|  |  |  |  |
| ABS (bid - winner point) |  |  |  |
| Pooled | $3.169(0.232)$ | $4.342(0.222)$ | 0.000 |
| Laboratory | $2.938(0.341)$ | $4.482(0.327)$ | 0.001 |
| Online | $3.322(0.315)$ | $4.271(0.292)$ | 0.028 |

Standard errors are indicated in parentheses. P-values for comparisons between AP and SP groups were computed using a two-sample $t$-test, which allowed for unequal variances.

# The Effect of Framing in Sealed-Bid Auctions: Theory and Experiments 

Yutaka Kayaba Jun Maekawa Hitoshi Matsushima First Version: December 10, 2017 This Version: August 23, 2023

Online Appendix:<br>English Translations of the Experimental Instructions

## 1. Laboratory Experiment with Frames: <br> Description of Experiments

## Experiment 1

We will now conduct Experiment 1.

You will be asked to choose either C or D . The person with whom you are paired will also choose between C and D . Please look at the table.

| You | C | The other person |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | C |  | D |  |
|  |  | 6 | 6 | 1 | 9 |
|  | D | 9 | 1 | 4 | 4 |

The table shows the possible combinations of selections by the two of you and the points received in each case. Your selections of C and D are presented in the relevant rows. The other person's selections of C and D are shown in the relevant columns. The values on the left in red represent your points when you have both made your selection, and the values on the right in blue represent the other person's.

When you and the other person both choose C:
You receive 6 points each
When you choose C and the other person chooses D :
You receive 1 point and the other person receives 9 points
When you choose D and the other person chooses C :
You receive 9 points and the other person receives 1 point
When you and the other person both choose D:
You receive 4 points each.

You and the other person take turns to choose either C or D. You and the other person are designated as either Player 1 or Player 2. When you are Player 1, the other person is Player 2 and vice versa.

If you are Player 1, you make your selection before the other person. The other person chooses either C or D when they know that you have already made your selection. However, the other person cannot see whether you have chosen C or D . Therefore, although the other person knows that you have already made your selection, they choose C or D without knowing the option you chose.

If you are Player 2, the other person selects either C or D before you. You choose either C or D when
you know that the other person has already made their selection. However, you cannot see whether the other person has chosen C or D . Therefore, although you know that the other person has already made their selection, you make your selection without knowing what the other person chose.

If you have a question, please raise your hand silently.
You will be paired with a new person, and the experiment will be repeated. Those who were Player 1 in the first round will now be Player 2, and those who were Player 2 will be Player 1.

When Experiment 1 is completed, we will change pairings and proceed with Experiment 2.

## Experiment 2

We will now conduct Experiment 2.

You will be asked to choose either C or D. The person with whom you are paired will also select between C and D. Please look at the table.

## The other person



The table shows the possible combinations of selections by the two of you and the points received in each case. Your selections of C and D are presented in the relevant rows. The other person's selections of C and D are presented in the relevant columns. The values on the left in red represent your points when you have both made your selection, while the values on the right in blue represent the other person's.

When you and the other person both choose C:
You receive 8 points each
When you choose C and the other person chooses D:
You receive 3 points and the other person receives 9 points
When you choose D and the other person chooses C :
You receive 9 points and the other person receives 3 points
When you and the other person both choose D:
You receive 4 points each.

You and the other person take turns to choose either C or D . You and the other person are designated as either Player 1 or Player 2. When you are Player 1, the other person is Player 2 and vice versa.

If you are Player 1 , you make your selection before the other person. The other person chooses either C or D when they know that you have already made your selection. However, the other person cannot see which option you chose. Therefore, although the other person knows that you have already made your selection, they choose C or D without knowing which option you chose.

If you are Player 2, the other person selects either C or D before you. You choose either C or D when you know that the other person has already made their selection. However, you cannot see which option the other person chose. Therefore, although you know that the other person has already made their selection, you choose C or D without knowing which of the two options the other person chose.

If you have a question, please raise your hand silently.
You will be paired with a new person, and the experiment will be repeated. Those who were Player 1 in the first round will be Player 2 in this round, and those who were Player 2 will be Player 1.

When Experiment 2 is completed, we will change pairings and proceed with Experiment 3.

## Experiment 3

We will now conduct Experiment 3.

You will be asked to choose a number between 0 and 100 . The person with whom you are paired will also choose a number between 0 and 100 . The winner is the person who had chosen the number closest to $3 / 4 x$ the average of the numbers chosen by both participants.

The winner receives 10 points, while the loser receives 0 points. In the case of a draw, both participants receive 5 points.

You and the other person take turns to choose a number. Each of you is designated as either Player 1 or Player 2. If you are Player 1 , the other person is Player 2 and vice versa.

If you are Player 1 , you choose your number before the other person. The other person chooses their number when they know that you have made your selection. However, the other person cannot see the number you have chosen. Therefore, although the other person knows that you have made your selection, they choose their number without knowing which number you chose.

If you are Player 2, the other person chooses their number first. You choose your number when you know that the other person has made their selection. However, you cannot see the number the other person has chosen. Therefore, although you know that the other person has made their selection, you choose your number without knowing which number the other person chose.

You will be paired with a new person, and the experiment will be repeated. Those who were Player 1 in the first round will be Player 2 in this round, and those who were Player 2 will be Player 1 .

If you have a question, please raise your hand silently.

When Experiment 3 is completed, we will change pairings and proceed with Experiment 4.

## Experiment 4

We will now conduct Experiment 4.

You will be asked to choose a number between 0 and 100 . The person with whom you are paired will also choose a number between 0 and 100 . The winner is the person who chose the number closest to $1 / 4 \mathrm{x}$ the average of the numbers chosen by both participants.
The winner receives 10 points, while loser receives none. In the case of a draw, both participants receive 5 points.

You and the other person take turns to choose a number. Each of you is designated as either Player 1 or Player 2. If you are Player 1, the other person is Player 2 and vice versa.
If you are Player 1 , you choose your number before the other person. The other person chooses their number when they know that you have made your selection. However, the other person cannot see which number you chose. Therefore, although the other person knows that you have made your selection, they choose their number without knowing which number you chose.

If you are Player 2, the other person chooses their number first. You choose your number when you know that the other person has made their selection. However, you cannot see which number the other person has chosen. Therefore, although you know that the other person has made their selection, you choose your number without knowing which number the other person chose.

You will be paired with a new person, and the experiment will be repeated. Those who were Player 1 in the first round will be Player 2 in this round, and those who were Player 2 will be Player 1 .

If you have a question, please raise your hand silently.

When Experiment 4 is completed, we will change pairings and proceed with Experiment 5 .

## Experiment 5

We will now conduct Experiment 5.

Everyone starts with 5 points. You and the person with whom you are paired are asked to participate in the bidding process described below. You and the other person are designated as either Bidder 1 or Bidder 2.

Bidders 1 and 2 compete for the commodity shown. The winning bidder and Points to be Paid are decided according to bidding rules set in advance.

The winner receives the commodity. However, instead of the winner receiving the actual commodity, they receive Winner's Points, as decided in advance. Winner's Points differ for Bidders 1 and 2. Neither bidder knows the amount of the other's Winner's Points.

The winner, as well as receiving Winner's Points, pays the number of Points to be Paid determined by the bidding rules. However, when the Points to be Paid are 5 or more points greater than the Winner's Points, the winner receives no points. The loser receives the 5 points given before the bidding.

These are the bidding rules.
A fictitious third party, the auctioneer, gradually raises the price from 1 point to 15 points. First, the auctioneer asks:

Do you want to buy the commodity for 1 point?
The two bidders select either yes or no. However, neither can see the other bidder's selection.
Next, the auctioneer asks:
Do you want to buy the commodity for 2 points?
The two bidders select either yes or no. However, neither can see the other bidder's selection.
The auctioneer raises the price again, asking:
Do you want to buy the commodity for 3 points?
The two bidders select either yes or no. However, neither can see the other bidder's selection.
The auctioneer continues in the same manner, raising the price one point at a time. Every person must select yes or no for all prices suggested by the auctioneer, ranging from 1 point to 15 points. The auction is completed when the price reaches 15 points.

When the auction is complete, if the price at which you first selected no is higher than the price at which the other person first selected no, you are the winner. If you and the other person first select no at the same price, the computer randomly selects one of them as the winner.

The Points to be Paid by the winner are:

## One point lesser than the price at which the loser first selected no.

This is not one point lesser than the price at which the winner first selects the No.

If you have a question, please raise your hand silently.

## 2. Laboratory Experiment without Frame: Description of Experiments

## Experiment 1

We will now conduct Experiment 1.

You will be asked to choose either C or D . The person with whom you are paired will also choose either C or D. Please look at the table.

## The other person



The table presents the possible combinations of selections by the two of you and the points received in each case. Your selections of C and D are presented in the relevant rows. The other person's selections of C and D are presented in the relevant columns. The values on the left in red represent your points when you have both made your selection, and the values on the right in blue, the other person's.

When you and the other person both choose C:
You receive 6 points each
When you choose C and the other person chooses D :
You receive 1 point and the other person receives 9 points
When you choose D and the other person chooses C :
You receive 9 points and the other person receives 1 point
When you and the other person both choose D:
You receive 4 points each.

You and the other person simultaneously choose either C or D. Neither of you can see the other person's choice.

If you have a question, please raise your hand silently.

When Experiment 1 is completed, we will change pairings and proceed with Experiment 2.

## Experiment 2

We will now conduct Experiment 2.

You will be asked to choose either C or D . The person with whom you are paired will also choose either C or D. Please look at the table.

## The other person

C D
You
C
D

| 8 | 8 | 3 | 9 |
| :--- | :--- | :--- | :--- |
| 9 | 3 | 4 | 4 |

The table presents the possible combinations of selections by the two of you and the points received in each case. Your selections of C and D are presented in the relevant rows. The other person's selections of C and D are presented in the relevant columns. The values on the left in red represent your points when you have both made your selection, and the values on the right in blue represent the other person's.

When you and the other person both choose C:
You receive 8 points each
When you choose C and the other person chooses D :
You receive 3 points and the other person receives 9 points
When you choose D and the other person chooses C :
You receive 9 points and the other person receives 3 points
When you and the other person both choose D:
You receive 4 points each.

You and the other person simultaneously choose either C or D. Neither of you can see the other person's choice.

If you have a question, please raise your hand silently.

When Experiment 2 is completed, we will change pairings and proceed with Experiment 3.

## Experiment 3

We will now conduct Experiment 3.

You will be asked to choose a number between 0 and 100 . The person with whom you have been paired will also choose a number between 0 and 100 . The winner is the person whose chosen number is closest to $3 / 4 \mathrm{X}$ the average of the numbers chosen by both participants.

The winner receives 10 points, while the loser receives none. In the case of a draw, both participants receive 5 points.

You and the other person choose your numbers simultaneously. Neither can see the other person's chosen number.

If you have a question, please raise your hand silently.

When Experiment 3 is completed, we will change pairings and proceed with Experiment 4.

## Experiment 4

We will now conduct Experiment 4.

You will be asked to choose a number between 0 and 100. The person with whom you are paired will also choose a number between 0 and 100 . The winner is the person whose chosen number is closest to $1 / 4$ $x$ the average of the numbers chosen by both participants.

The winner receives 10 points, while the loser receives none. In the case of a draw, both participants receive 5 points.

You and the other person choose your numbers simultaneously. Neither can see the other person's chosen number.

If you have a question, please raise your hand silently.

When Experiment 4 is completed, we will change pairings and proceed with Experiment 5.

## Experiment 5

We will now conduct Experiment 5 .

Everyone starts with 5 points. You and the person with whom you are paired are asked to participate in the bidding process described below.

You and the other person are designated as either Bidder 1 or Bidder 2 .
Bidders 1 and 2 compete for the commodity shown. The winning bidder and Points to be Paid are decided according to the bidding rules set in advance.

The winning bidder receives the commodity. However, instead of the winning bidder receiving the actual commodity, they receive Winner's Points, as decided in advance. Winner's Points differ for Bidder 1 and Bidder 2. Neither bidder knows the number of the other's Winner's Points.

The winning bidder, as well as receiving Winner's Points, pays the number of Points to be Paid determined by the bidding rules. However, when the Points to be Paid are 5 or more points higher than the Winner's Points, the winning bidder receives no points. The loser receives the 5 points given before the bidding.

These are the bidding rules.
Bidders 1 and 2 simultaneously choose any whole number between 0 and 15 .
If the number that you choose is higher than the number chosen by the other person, you are the winning bidder. If you and the other person choose the same number, the computer will randomly select the winning bidder.

The number of Points to be Paid by the winning bidder is the number chosen by the loser, not the number chosen by the winning bidder.

If you have a question, please raise your hand silently.

## 3. Online Experiment with Frames: <br> Description of Experiments

## Experiment 1

We will now conduct Experiment 1.

You will be asked to choose either C or D . The person with whom you are paired will also choose between C and D . Please look at the table.

## The other person

| You | C | The other person |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | C |  | D |  |
|  |  | 6 | 6 | 1 | 9 |
|  | D | 9 | 1 | 4 | 4 |

The table shows the possible combinations of selections by the two of you and the points received in each case. Your selections of C and D are presented in the relevant rows. The other person's selections of C and D are shown in the relevant columns. The values on the left in red represent your points when you have both made your selection, and the values on the right in blue represent the other person's.

When you and the other person both choose C:
You receive 6 points each
When you choose C and the other person chooses D :
You receive 1 point and the other person receives 9 points
When you choose D and the other person chooses C :
You receive 9 points and the other person receives 1 point
When you and the other person both choose D:
You receive 4 points each.

You and the other person take turns to choose either C or D. You and the other person are designated as either Player 1 or Player 2. When you are Player 1, the other person is Player 2 and vice versa.

If you are Player 1, you make your selection before the other person. The other person chooses either C or D when they know that you have already made your selection. However, the other person cannot see whether you have chosen C or D . Therefore, although the other person knows that you have already made your selection, they choose C or D without knowing the option you chose.

If you are Player 2, the other person selects either C or D before you. You choose either C or D when
you know that the other person has already made their selection. However, you cannot see whether the other person has chosen C or D . Therefore, although you know that the other person has already made their selection, you make your selection without knowing what the other person chose.

Please make your selection based on the assumption that you and the other person are participating in the experiment simultaneously. After the selection is completed, you will be paired with a new person, and the experiment will be repeated. Those who were Player 1 in the first round will be Player 2 in this round, and those who were Player 2 will be Player 1.

When Experiment 1 is completed, we will change pairings and proceed with Experiment 2.

## Experiment 2

We will now conduct Experiment 2.

You will be asked to choose either C or D. The person with whom you are paired will also select between C and D. Please look at the table.

## The other person



The table shows the possible combinations of selections by the two of you and the points received in each case. Your selections of C and D are presented in the relevant rows. The other person's selections of C and D are presented in the relevant columns. The values on the left in red represent your points when you have both made your selection, while the values on the right in blue represent the other person's.

When you and the other person both choose C:
You receive 8 points each
When you choose C and the other person chooses D:
You receive 3 points and the other person receives 9 points
When you choose D and the other person chooses C :
You receive 9 points and the other person receives 3 points
When you and the other person both choose D:
You receive 4 points each.

You and the other person take turns to choose either C or D . You and the other person are designated as either Player 1 or Player 2. When you are Player 1, the other person is Player 2 and vice versa.

If you are Player 1 , you make your selection before the other person. The other person chooses either C or D when they know that you have already made your selection. However, the other person cannot see which option you chose. Therefore, although the other person knows that you have already made your selection, they choose C or D without knowing which option you chose.

If you are Player 2, the other person selects either C or D before you. You choose either C or D when you know that the other person has already made their selection. However, you cannot see which option the other person chose. Therefore, although you know that the other person has already made their selection, you choose C or D without knowing which of the two options the other person chose.

Please make your selection based on the assumption that you and the other person are participating in the experiment simultaneously. After the selection is completed, you will be paired with a new person, and the experiment will be repeated. Those who were Player 1 in the first round will be Player 2 in this round, and those who were Player 2 will be Player 1.

When Experiment 2 is completed, we will change pairings and proceed with Experiment 3.

## Experiment 3

We will now conduct Experiment 3.

You will be asked to choose a number between 0 and 100 . The person with whom you are paired will also choose a number between 0 and 100 . The winner is the person who had chosen the number closest to $3 / 4 x$ the average of the numbers chosen by both participants.

The winner receives 10 points, while the loser receives 0 points. In the case of a draw, both participants receive 5 points.

You and the other person take turns to choose a number. Each of you is designated as either Player 1 or Player 2. If you are Player 1, the other person is Player 2 and vice versa.

If you are Player 1 , you choose your number before the other person. The other person chooses their number when they know that you have made your selection. However, the other person cannot see the number you have chosen. Therefore, although the other person knows that you have made your selection, they choose their number without knowing which number you chose.

If you are Player 2, the other person chooses their number first. You choose your number when you know that the other person has made their selection. However, you cannot see the number the other person has chosen. Therefore, although you know that the other person has made their selection, you choose your number without knowing which number the other person chose.

Please make your selection based on the assumption that you and the other person are participating in the experiment simultaneously. You will be paired with a new person, and the experiment will be repeated. Those who were Player 1 in the previous round will be Player 2 in this round, and those who were Player 2 will be Player 1 .

Those who were Player 1 in the first round will be Player 2 in this round, and those who were Player 2 will be Player 1 . Those who were Player 1 in the first round will be Player 2 in this round, and those who were Player 2 will be Player 1 .

When Experiment 3 is completed, we will change pairings and proceed with Experiment 4.

## Experiment 4

We will now conduct Experiment 4.

You will be asked to choose a number between 0 and 100. The person with whom you are paired will also choose a number between 0 and 100 . The winner is the person who chose the number closest to $1 / 4 \mathrm{x}$ the average of the numbers chosen by both participants.

The winner receives 10 points, while loser receives none. In the case of a draw, both participants receive 5 points.

You and the other person take turns to choose a number. Each of you is designated as either Player 1 or Player 2. If you are Player 1, the other person is Player 2 and vice versa.

If you are Player 1 , you choose your number before the other person. The other person chooses their number when they know that you have made your selection. However, the other person cannot see which number you chose. Therefore, although the other person knows that you have made your selection, they choose their number without knowing which number you chose.

If you are Player 2, the other person chooses their number first. You choose your number when you know that the other person has made their selection. However, you cannot see which number the other person has chosen. Therefore, although you know that the other person has made their selection, you choose your number without knowing which number the other person chose.

Please make your selection based on the assumption that you and the other person are participating in the experiment simultaneously. You will be paired with a new person, and the experiment will be repeated. Those who were Player 1 in the first round will be Player 2 in the next round, and those who were Player 2 will be Player 1.

When Experiment 4 is completed, we will change pairings and proceed with Experiment 5.

## Experiment 5

We will now conduct Experiment 5.

Everyone starts with 5 points. You and the person with whom you are paired are asked to participate in the bidding process described below. You and the other person are designated as either Bidder 1 or Bidder 2.

Bidders 1 and 2 compete for the commodity shown. The winning bidder and Points to be Paid are decided according to bidding rules set in advance.

The winner receives the commodity. However, instead of the winner receiving the actual commodity, they receive Winner's Points, as decided in advance. Winner's Points differ for Bidders 1 and 2. Neither bidder knows the amount of the other's Winner's Points.

The winner, as well as receiving Winner's Points, pays the number of Points to be Paid determined by the bidding rules. However, when the Points to be Paid are 5 or more points greater than the Winner's Points, the winner receives no points. The loser receives the 5 points given before the bidding.

These are the bidding rules.
A fictitious third party, the auctioneer, gradually raises the price from 1 point to 15 points. First, the auctioneer asks:

Do you want to buy the commodity for 1 point?
The two bidders select either yes or no. However, neither can see the other bidder's selection. Next, the auctioneer asks:

Do you want to buy the commodity for 2 points?
The two bidders select either yes or no. However, neither can see the other bidder's selection.
The auctioneer raises the price again, asking:
Do you want to buy the commodity for 3 points?
The two bidders select either yes or no. However, neither can see the other bidder's selection.
The auctioneer continues in the same manner, raising the price one point at a time. Every person must select yes or no for all prices suggested by the auctioneer, ranging from 1 point to 15 points. The auction is completed when the price reaches 15 points.

When the auction is complete, if the price at which you first selected no is higher than the price at which the other person first selected no, you are the winner. If you and the other person first select no at the same price, the computer randomly selects one of them as the winner.

The Points to be Paid by the winner are:
One point lesser than the price at which the loser first selected no.
This is not one point lesser than the price at which the winner first selects the No.

Please make your selection based on the assumption that the other person is making their selection simultaneously.

# 4. Online Experiment without Frame: <br> <br> Description of Experiments 

 <br> <br> Description of Experiments}

## Experiment 1

We will now conduct Experiment 1.

You will be asked to choose either C or D . The person with whom you are paired will also choose either C or D. Please look at the table.

## The other person



The table presents the possible combinations of selections by the two of you and the points received in each case. Your selections of C and D are presented in the relevant rows. The other person's selections of C and D are presented in the relevant columns. The values on the left in red represent your points when you have both made your selection, and the values on the right in blue, the other person's.

When you and the other person both choose C:
You receive 6 points each
When you choose C and the other person chooses D:
You receive 1 point and the other person receives 9 points
When you choose D and the other person chooses C :
You receive 9 points and the other person receives 1 point
When you and the other person both choose D:
You receive 4 points each.

Neither of you can see the other person's choice. Please make your selection based on the assumption that the other person is selecting C or D simultaneously.

When Experiment 1 is completed, we will change pairings and proceed with Experiment 2.

## Experiment 2

We will now conduct Experiment 2.

You will be asked to choose either C or D. The person with whom you are paired will also choose either C or D. Please look at the table.

## The other person

C D
You
C
D

| 8 | 8 | 3 | 9 |
| :--- | :--- | :--- | :--- |
| 9 | 3 | 4 | 4 |

The table presents the possible combinations of selections by the two of you and the points received in each case. Your selections of C and D are presented in the relevant rows. The other person's selections of C and D are presented in the relevant columns. The values on the left in red represent your points when you have both made your selection, and the values on the right in blue represent the other person's.

When you and the other person both choose C:
You receive 8 points each
When you choose C and the other person chooses D :
You receive 3 points and the other person receives 9 points
When you choose D and the other person chooses C :
You receive 9 points and the other person receives 3 points
When you and the other person both choose D:
You receive 4 points each.

Neither of you can see the other person's choice. Please make your selection based on the assumption that the other person is selecting C or D simultaneously.

When Experiment 2 is completed, we will change pairings and proceed with Experiment 3.

## Experiment 3

We will now conduct Experiment 3.

You will be asked to choose a number between 0 and 100 . The person with whom you have been paired will also choose a number between 0 and 100 . The winner is the person whose chosen number is closest to $3 / 4 \mathrm{X}$ the average of the numbers chosen by both participants

The winner receives 10 points, while the loser receives none. In the case of a draw, both participants receive 5 points.

Neither can see the other person's chosen number. Please make your selection based on the assumption that you and the other person are selecting the numbers simultaneously.

When Experiment 3 is completed, we will change pairings and proceed with Experiment 4.

## Experiment 4

We will now conduct Experiment 4.

You will be asked to choose a number between 0 and 100 . The person with whom you are paired will also choose a number between 0 and 100 . The winner is the person whose chosen number is closest to $1 / 4$ $x$ the average of the numbers chosen by both participants.

The winner receives 10 points, while the loser receives none. In the case of a draw, both participants receive 5 points.

Neither can see the other person's chosen number. Please make your selection based on the assumption that you and the other person are selecting the numbers simultaneously.

When Experiment 4 is completed, we will change pairings and proceed with Experiment 5 .

## Experiment 5

We will now conduct Experiment 5 .

Everyone starts with 5 points. You and the person with whom you are paired are asked to participate in the bidding process described below.

You and the other person are designated as either Bidder 1 or Bidder 2.
Bidders 1 and 2 compete for the commodity shown. The winning bidder and Points to be Paid are decided according to the bidding rules set in advance.

The winning bidder receives the commodity. However, instead of the winning bidder receiving the actual commodity, they receive Winner's Points, as decided in advance. Winner's Points differ for Bidder 1 and Bidder 2. Neither bidder knows the number of the other's Winner's Points.

The winning bidder, as well as receiving Winner's Points, pays the number of Points to be Paid determined by the bidding rules. However, when the Points to be Paid are 5 or more points higher than the Winner's Points, the winning bidder receives no points. The loser receives the 5 points given before the bidding.

These are the bidding rules.
Bidders 1 and 2 simultaneously choose any whole number between 0 and 15 .
If the number that you choose is higher than the number chosen by the other person, you are the winning bidder. If you and the other person choose the same number, the computer will randomly select the winning bidder.

The number of Points to be Paid by the winning bidder is the number chosen by the loser, not the number chosen by the winning bidder.

Please make your selection based on the assumption that you and the other person are participating in the experiment simultaneously.


[^0]:    ${ }^{1}$ The title of the earlier version of the study is "Framing Game Theory", which corresponds to the paper that first introduced the concept of cautious undominance. We drastically modified the original paper's contents by adding all the theorems and propositions, adding the experimental part (Section 5), and changing this version from a single-authored paper by Hitoshi Matsushima to a co-authored paper. The experiment of this study was approved by the Ritsumeikan University Ethics Review Committee for Research Involving Human Participants. Research for this study was financially supported by a grant-in-aid for scientific research (KAKENHI 25285059, 20H00070, 18K12742) from the Japan Society for the Promotion of Science (JSPS) and the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of the Japanese government, as well as by the Center of Advanced Research in Finance at the University of Tokyo. We are grateful to Professor Masa Yagasaki for his support and suggestions. All errors remain ours.
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[^1]:    ${ }^{5}$ The experiment was programmed and conducted with o-Tree (Chen et al., 2016).
    ${ }^{6}$ The data, number of subjects, and treatment for each session are demonstrated in Appendix C.

[^2]:    ${ }^{7}$ One subject who participated in the experiment on $17^{\text {th }}$, December 2020 , left in the middle of the session, thus is excluded from the data analysis.

