Mini-Course on Minimalist Market Design: Lecture 2 Evolution via Living-Donor Organ Exchange & Proof-of-Concept via US Army's Branching Process

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Kidney Exchange

- One of the most unexpected applications of market design which contributed to visibility and success of the field is kidney exchange (KE) (Roth, Sönmez & Ünver, 2004, 2005, 2007).
 - Within a few years after its introduction as a market design application, our formal approach transformed living donor kidney donation in many countries.
 - Within a decade, it started saving more than a thousand lives annually.
- Why unexpected?
 - Way outside the traditional domain of economics.
 - As in the case of the school choice reform at BPS, help from economists was volunteered as outsiders and it was not solicited.

Outreach Efforts in Kidney Exchange

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 - by using the tools they are familiar with (or at least they are comfortable to use), and
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 - by using the tools they are familiar with (or at least they are comfortable to use), and
 - without creating any issues.
- To have a realistic chance to influence policy, an aspiring market designer needs to have an in depth understanding of the mission of the institution along with a practical and transparent plan to improve it.
 - Often the history of the institution can be instructive.
 - Policy aspirations usually have strong implications on viable designs, and therefore also on the research program.

- Approved by the UNOS Board of Trustees in Fall 2000, the first kidney exchange program in the US was established in New England (UNOS Region 1) in February 2001 (Delmonico et al. 2004).
- In order to overcome barriers to living donation due to biological incompatibilities, the program made two types of arrangements:
 - 1. Paired Kidney Exchange (PKE): A direct exchange of donors between two patients with incompatible donors.
 - 2. List Exchange (LE): An indirect exchange between an incompatible pair and the deceased-donor (DD) list. (Elevated priority in the list in exchange for a kidney of the co-registered donor).

Paired Kidney Exchange (PKE)

- Originally proposed by a transplant surgeon in Rapaport (1986).
- First carried out in South Korea in 1991 (Park et al., 1999).
- Transplantation community issued a consensus statement in 2000 declaring it as ethically acceptable (Abecassis et al., 2000).
 - Considered as a high praise in medical community.
 - The consensus statement urged all four operations to be carried out simultaneously.
- The first PKE in the US was carried out in Rhode Island in 2000.

List Exchange (LE)

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 - Ethical concern: Detrimental to blood type O patients on the DD list.

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 - Ethical concern: Detrimental to blood type O patients on the DD list.
- There are four blood types A, B, AB and O.



- Type AB patients can receive a kidney of any type
- Type A patients can receive a kidney of types A or O
- Type B patients can receive a kidney of types B or O
- Type O patients can only receive a kidney of type O
- Type O patients are disadvantaged because of this "natural injustice."
- The consensus statement (Abecassis et al., 2000) highlighted the ethical concerns that involve type O patients.

• Despite the ethical concerns, New England included LE in its program. This decision was defended by its leadership as follows:

"This exchange program has a clear utilitarian goal: to have more recipients undergo successful transplantation by expanding the pool of compatible live donors."

Delmonico et al. (2004)

• Reflecting the concerns, however, much of the discussion in Delmonico et al. (2004) involves the precautions taken to mitigate the adverse impact of LE on type O patients on the DD waiting list.

- Despite being the less preferred type of KE, most transplants arranged by the New England's program in its early phases were from LE.
 - # of LE Transplants (Feb. 2001– Dec. 2003): 17
 - # of PKE Transplants (Feb. 2001– Dec. 2003): 8 in 4 PKEs
- No Database: Prior to our involvement in Fall 2004, the program did not have a unified database where participating centers could access information on patient-donor pairs co-registered in other centers.
 - Explains the small number of transplants from PKE in early years of the program.

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 - Explains the small number of transplants from PKE in early years of the program.
- While arranging a LE does not require a patient-donor database, organizing them also involved operational challenges in New England.

• A prerequisite for eligibility for LE was to assure that no PKE is feasible between the patient and any other patient registered in all 14 transplant centers in the system.

"[...] the general practice has been to ask such pairs to wait a minimum of one month, in order to avoid flooding the system with 'unnecessary' list exchanges. If no such pair is identified, the center can proceed with the live donor list exchange process." Delmonico et al. (2004)

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• Timing is Everything! Under these circumstances, we shared the first draft of RSÜ (2004) with Dr. Francis Delmonico in Fall 2003, and conveyed our interest to support them to improve their KE program.

Early Market Design Research in Kidney Exchange

- While we were both faculty members at Koç University-İstanbul, my colleague Utku Ünver visited Alvin Roth at Harvard University for the academic year 2002-2003.
- During his visit, Roth alerted him that Abdulkadiroğlu & Sönmez (1999) has an unusual application in kidney transplantation.
 - Patients with living donors are analogous to existing tenants
 - Paired-donor kidneys are analogous to occupied houses
 - Patients on DD list are analogous to newcomers
 - DD kidneys are analogous to vacant houses
- As such, YRMH-IGYT mechanism also had a potential application.

Early Market Design Research in Kidney Exchange

- Regulating the claims for "unattached" houses (either vacant or vacated during the procedure) with an exogenous priority list, YRMH-IGYT mechanism organizes two types of transactions:
 - 1. Cycle: Existing tenants trade their occupied houses
 - PKE corresponds to a cycle with two individuals
 - 2. Chain: One individual trade her priority for an "unattached" house and the remaining individuals trade their occupied houses
 - LE corresponds to a chain with two individuals
- While regulating chains through an exogenous priority list (as in YRMH-IGYT) is also a viable policy for KE, we observed that other chain selection rules may mitigate (and even eliminate) the adverse impact of LE on type O patients on the DD list.
- RSÜ (2004): Addressed both goals of the transplantation community with this generalization of the YRMH-IGYT mechanism.

The Birth of a Partnership Between Economists & Doctors

- Our informed and cautious approach resonated with Dr. Delmonico, the Chief Medical Officer at New England Organ Bank.
 Subsequently, he made the following requests:
 - 1. Given the scale of simulated welfare gains from our system, we should drop the more controversial LE altogether.
 - 2. Due to logistical constraints, we should only allow for two-way KE.
 - 3. To avoid a situation where patients and hospitals may compete for donors with certain characteristics, we must assume that patients are indifferent between all compatible donors.
- We accommodated all requests in RSÜ (2005), which formed the basis of the New England Program for Kidney Exchange (NEPKE).
 - Approved by the Renal Transplant Oversight Committee of New England in September 2004, NEPKE became the first KE system that adapted analytical techniques from market design and optimization.

Subsequent Policy Influence

- Our team coded and ran NEPKE's software for several years. Our partnership resulted in a number of additional breakthroughs.
- Larger Exchanges: With New England data, early on it became clear that inclusion of 3-way KE is especially important from a utilitarian perspective (RSÜ, 2007).
 - We convinced our medical partners to include 3-way KE to NEPKE software, and together advocated for it to the broader transplantation community in Saidman et al. (2006).
- NDD-chains: Together with our NEPKE partners, we introduced and advocated for non-simultaneous implementation of chains, when they initiate with a non-directed living donor kidney (Roth et al., 2007).
 - While NEPKE did not adopt NDD-chains, a second KE program we supported in its early years, Alliance for Paired Donation (APD), did.
 - Today, a sizable part of the welfare gains from KE are due to NDD-chains (Agarwal et al., 2019).

Limited Progress: Compatible Pairs

- Since preferences are assumed to be strict, compatible pairs participate KE in RSÜ (2004).
- RSÜ (2005) made our collaboration possible, but it restricted participation to compatible pairs.
 - Major Welfare Loss: non-O patients with O donors rarely join KE.
 - Implication: A large majority remain unmatched among O patients with non-O donors.

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 - Major Welfare Loss: non-O patients with O donors rarely join KE.
 - Implication: A large majority remain unmatched among O patients with non-O donors.
- What can be done to include these harder-to-match pairs in KE?

Global Kidney Exchange and Its Challenges

- One possibility is Global Kidney Exchange (GKE) (Rees et al., 2017): Match harder-to-match pairs with patient-donor pairs from countries where there is no possibility for living donor transplantation.
- While GKE has been heavily promoted by Michael Rees from APD and Alvin Roth, it lead to relatively modest number of transplants.
 - # of GKE transplants (01/2015 02/2022): 52 (17 Intl. & 35 US) (Rees et al., 2022)
 Reference: # of KE transplants in US (01/2015 - 02/2022): 6000+
- A big challenge for GKE is the mixed reaction in the transplantation community.
 - Many argue GKE undermines various ethical norms in transplantation.
 - Persuasion stage of reform advocacy is either bypassed under GKE or it has shown limited effectivity.

Importance of Ethical Norms

• Dr. Delmonico-key for initial collaboration between economists and medical doctors-is a leading figure in the opposition against GKE.



- According to the Opposition on GKE:
 - The program exploits poor countries and individuals
 - Helping poor patients in exchange for "donated" organs constitutes organ trafficking
 - GKE increases the risk that organs will come from paid sources

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 - The program exploits poor countries and individuals
 - Helping poor patients in exchange for "donated" organs constitutes organ trafficking
 - GKE increases the risk that organs will come from paid sources
- The contrast between the early success on KE and the opposition on GKE highlights the gravity of persuasion stage of reform advocacy.

Improving Welfare w/o Challenging Ethical Norms: Kidney

- Incentivized Kidney Exchange (Sönmez & Ünver, 2015, Sönmez, Ünver & Yenmez, 2020)
- For certain compatible patient-donor pairs, their participation in KE increases the total number of transplants.
 - Especially, non-O patients with O donors
- Main Idea: Incentivize such pairs to join KE by giving the patient some form of a priority increase in the DD list in the event of another renal failure in the future.
 - A living donor kidney functions, on average, 12 to 20 years.

Improving Welfare w/o Challenging Ethical Norms: Kidney

- In the last several years, about 1100 patients in the US received transplants via KE annually.
 - For each 10% of incentivized pairs, the number of transplants can be increased by about 180 (Sönmez, Ünver & Yenmez, 2020).
 - KE transplants can be doubled if 60% of compatible pairs can be incentivized.
- Ethics of this policy favorable discussed by several members of Canadian transplantation community in Gill et al. (2017).
- Can be considered as part of the on-going reform of the UNOS-DD allocation system for kidney.
- Challenge: Broader consensus needed to influence national policy.

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- For other organs, it may be possible to incentivize blood-type compatible pairs to join donor exchange through more local policies.

Improving Welfare w/o Challenging Ethical Norms: Liver

- Key Idea for Liver Exchange: Utilize size-compatibility requirements in living donor liver transplantation and the difference between donor risk from left vs. right-lobe donation (Ergin, Sönmez & Ünver, 2020).
- Living donors for liver typically donate (i) the larger right lobe (60-70% liver mass), (ii) the smaller left lobe (30-40% liver mass), or (iii) part of the left lobe (Segment 2/3) for small children.
 - Morbidity/Mortality risk to donor is several times higher under right lobe transplantation.
 - To survive the operation, the patient needs a graft of at least 40% of the volume of his dysfunctional liver.
- These aspects of liver transplantation result in a natural instrument to incentivize blood-type compatible pairs to join liver exchange:
 - Reducing donor risk: Instead of donating the right lobe to her intended patient, a donor can instead donate her left lobe to a smaller patient through liver exchange.

Liver Exchange at Inönü University (Malatya-Turkey)

- These ideas resulted in a partnership between our team of design economists and the liver transplant group at İnönü University (Malatya-Turkey) under the leadership of Prof. Dr. Sezai Yilmaz.
 - The second largest liver transplant group worldwide (250-300 living donor liver transplants annually)
 Reference: In 2022, the US total was an all-time high of 603.
- Agreement for Banu Bedestenci Sönmez Liver Paired Exchange (BBS-LPE) system was approved in September 2019, but the system was launched in June 2022 due to Covid-19.

Liver Exchange at Inönü University (Malatya-Turkey)

- Despite the magnitude 7.8 earthquake that hit the region in February 2023, 15 patients received transplants through BBS-LPE program in one 4-way, one 3-way and four 2-way liver exchanges in its first year.
 - The 4-way liver exchange conducted in July 2022 is a world first and the largest liver exchange to date (Yilmaz et al., 2023).

American Journal of TRANSPLANTATION

The First 4-Way Liver Paired Exchange from an Interdisciplinary Collaboration between Healthcare Professionals and Design Economists

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Proof-of-Concept for Minimalist Market Design

- Drawing on a decade-long research and policy efforts on the US Army's branching process of cadets to military specialties, I next present the first direct application and subsequent proof-of-concept of minimalist market design.
- Minimalist market design paradigm evolved though our earlier research and policy efforts in school choice.
 - Policy Impact. 2005 reform of school choice at Boston Public Schools
 - External Validity. 2007 reform of school admissions code in England
 - External Validity. 2009 reform of school choice at Chicago Public Schools
- Other successful applications after the Army's branching reform:
 - Policy Impact. Rationing of scarce medical resources during Covid-19
 - External Validity. Affirmative action in India for allocation of public positions

US Army's Branching Process

- Each year, the US Army assigns thousands of graduating cadets from the United States Military Academy (USMA) at West Point and the Reserve Officer Training Corps (ROTC) to their first job in a military occupation, or branch, through centralized systems.
 - Branch assignment is highly consequential for career progression.
- Prior to the Class of 2006, cadets were assigned positions at Army branches using a simple serial dictatorship that is induced by a cadet performance ranking known as the order of merit list (OML).
 - Under this mechanism, cadets submit their preferences over the set of branches, and the highest-OML cadet is assigned her most-preferred branch, the second highest-OML cadet is assigned her most-preferred branch among branches with remaining positions, etc.
 - Reflects the importance of hierarchy in the Army.

BRADSO Program and the 2006 Branching Reform

- In response to declining junior officer retention rates during the late 1990s and early 2000s, starting with 2006 the Army offered a menu of retention incentives to cadets at USMA and ROTC.
- The most popular incentive, which involved a reform of the branching mechanism, was the branch of choice (BRADSO) program.
 - Under this program, cadets are given higher priority for a fraction of positions at any given branch if they indicate willingness to extend their Active Duty Service Obligation (ADSO) by three years at that branch.
 - Terminology: We refer to ADSO as the price.
 - The message space of the new mechanism was also expanded by requesting cadets to report the set of branches for which they are willing to pay the increased price in exchange for receiving increased-priority at a fraction of its positions.

USMA-2006 Mechanism

- Under the USMA-2006 mechanism, the branch assignments are made through a process that resembles the previous OML-induced serial dictatorship, with one important exception:
 - Once the regular (i.e., base-price) positions are filled at any branch, cadets who indicated willingness to pay the increased price are given priority for the remaining flexible-price positions.
- The prices are subsequently determined as follows:
 - Cadets who receive a regular position are charged the base price.
 - Cadets who receive a flexible-price position are charged
 - the base price if they have not indicated willingness to pay the increased price for their assigned branch, and
 - the increased price if they have indicated willingness to pay the increased price for their assigned branch.

Shortcomings of the USMA-2006 Mechanism

- Two aspects of the USMA-2006 mechanism are not ideal:
 - **1.** Cadets are asked to whether they are willing to pay the increased price at a branch or not independent of what the alternative is.

For example, a cadet is not able to indicate

- he is willing to pay the increased price to receive a position at his first choice branch if the alternative is receiving a position at his third or lower choice branches,
- but not if the alternative is receiving a position at his second choice branch.
- 2. Cadets who indicate willingness to pay the increased price for a branch are charged the increased price upon receiving one of its flexible-price positions even if they would have received the same position at a base price in the absence of their willingness.

Shortcomings of the USMA-2006 Mechanism

- These aspects, in turn, result in a number of shortcomings of the USMA-2006 mechanism, including the following two:
 - Detectable Priority Reversal: A cadet may receive a position at the increased price, while a lower-OML cadet receives a position at the same branch at base price.
 - Failure of Incentive Compatibility: A cadet may benefit from hiding her willingness to pay the increased price (failure of BRADSO-IC) or from misrepresenting her branch preferences.
- Root Causes of Failures under the USMA-2006 Mechanism:
 - **1.** The message space is not sufficiently rich to capture cadet preferences over branch-price pairs.
 - **2.** The two elements of an assignment-the branch assignment and the price assignment-are determined sequentially rather than jointly.
Initial Proposal of the Cumulative Offer Mechanism

- Fortunately, both root causes of the failures can be addressed by foundational research on matching with contracts (Hatfield & Milgrom, 2005).
- Hence, as a remedy, Sönmez & Switzer (2013) proposed an alternative mechanism for the USMA based on Hatfield & Milgrom's celebrated cumulative offer mechanism.
 - Proposal built on a very important extension of the matching with contracts theory by Hatfield & Kojima (2010).
- The proposed mechanism is a direct mechanism where cadets submit their preferences over branch-price pairs.
 - The Army initially viewed this message space to be too complex, and decided to maintain the USMA-2006 mechanism.

Army's Reasons to Maintain the USMA-2006 Mechanism

- Adoption of a mechanism with a more involved message space was initially seen at the Army as unnecessary due to three main reasons:
 - 1. BRADSO-IC failures and detectable priority reversals have been rare in practice.
 - 2. Any BRADSO-IC failure or detectable priority reversal can be manually corrected ex-post, since each incidence only involves a cadet needlessly paying the increased price at her assigned branch.
 - While there can be additional priority reversals that cannot be manually corrected ex-post, their verification relies on cadet preferences over branch-price pairs, an information unavailable under the existing USMA-2006 message space.
- In summary, any failure of the USMA-2006 mechanism can either be manually corrected ex-post or cannot be verified with existing data.

Talent-Based Branching Program

- In 2012, the Army introduced a Talent-Based Branching (TBB) program to develop a "talent market" where additional information about each cadet influences the priority a cadet receives at a branch.
- Under the TBB program, branches rate cadets into one of three tiers: High, Medium, and Low.
 - For several years these ratings remained a pilot initiative.
 Reminder: Reforming an institution is not an easy task, even within!
 - For the Class of 2020, the Army decided to integrate them into the branching process, constructing priorities at each branch first by the tier and then by the OML within the tier.
 - BRADSO policy also changed: cadets willing to pay the increased price now received higher priority within their tier only.

USMA-2020 Mechanism

- Since the decision to integrate cadet ratings into branching process took place under an abbreviated timeline, the Army maintained the same strategy space for the new mechanism as in previous years.
- Using an adjusted priority order of cadets that takes both TBB ratings and increased-price willingness into consideration, the new mechanism used the individual-proposing deferred acceptance algorithm (Gale & Shapley 1962) to determine the branch assignments.
- The prices were then determined subsequently as follows:
 - Subject to a maximum of the number of flexible-price positions at any given branch and following the reverse-priority order of the branch, cadets who indicated willingness to pay the increased price at their assigned branch are charged the increased price, and
 - the remaining cadets who are matched are charged the base price.

Shortcomings of the USMA-2020 Mechanism

- In addition to inheriting the limitations of the USMA-2006 mechanism, the following aspect of the USMA-2020 mechanism added new challenges:
 - Even though the number of flexible-price positions at each branch was kept at 25% of the total capacity, priority upgrade due to increased-price willingness was applied for all its positions.
 - This design choice made it possible to use the vanilla version of the deferred acceptance algorithm, but it also introduced a new type of incentive compatibility failure called Strategic BRADSO.
 - Whereas indicating willingness to pay the increased price could hurt cadets due to BRADSO-IC failures, now it could also profit them with a costless priority upgrade due to Strategic BRADSO.
- The end result was a mechanism that is highly complex, and one with more widespread failures including priority reversals that cannot be manually corrected ex-post.

Concerns on the USMA-2020 Mechanism

- Root causes of the USMA-2020 mechanism's failures are same as those under the USMA-2006 mechanism: Restricted message space and lack of coordination between branch and price assignments.
- The USMA leadership immediately recognized the possibility of detectable priority reversals under the USMA-2020 mechanism due to either failure of BRADSO-IC or presence of strategic BRADSO.
- A major concern emerged as an erosion of cadets' trust in the Army's branching process.
- To address this concern, the USMA leadership decided to execute a dry run of the USMA-2020 mechanism to inform cadets of the potential cutoffs for each branch.

Concerns on the USMA-2020 Mechanism

• As emphasized in the following quote from a September 2019 U.S. Army news article, the goal of the dry run was to improve transparency and help cadets to optimize their submitted strategies:

"We're going to tell all the cadets, we're going to show all of them, here's when the branch would have went out, here's the bucket you're in, here's the branch you would have received if this were for real. You have six days to go ahead and redo your preferences and look at if you want to BRADSO or not." Sunsdahl said. "I think it's good to be transparent. I just don't know what 21-year-olds will do with that information."

• The same quote, however, also indicates that USMA leadership recognized the challenges in cadets optimizing their strategies under the USMA-2020 mechanism.

Failure Prevalence under Dry vs. Actual USMA-2020 Runs

- In each class of USMA, there are approximately 1000 cadets.
- The left side correspond to Class of 2020 cadets who are affected by the failures of the USMA-2020 mechanism in the dry-run.

The right side correspond to Class of 2020 cadets who are affected by the failures of the USMA-2020 mechanism in the actual run.



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Problems Aggravated Despite the Dry-Run

• The left side correspond to the average number cadets who are affected by the failures of the USMA-2006 mechanism for Classes of 2014-2019.

The right side correspond to Class of 2020 cadets who are affected by the failures of the USMA-2020 mechanism in the actual run



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Army's Partnership with Market Designers

- At this point, the Army reconsidered the reform proposal made earlier in Sönmez & Switzer (2013) and Sönmez (2013).
 - Reminder: The trigger of a reform is not a good alternative but rather a really bad institution in place.
- A partnership is established with Pathak and Sönmez, with Greenberg leading the reform efforts at USMA.
- Critical to forming this partnership was the Army's decision to permit cadets in the Class of 2021 to submit preferences over branch-price pairs, thus allowing to address the first root cause of the failures.
- This decision was aided by evidence from a cadet survey that mitigated concerns that ranking branch-price pairs would be overly complex or unnecessary.

Army's Partnership with Market Designers

• Indeed, some of the cadets indicated the need for a system that would allow them to rank order branch-price pairs. One cadet wrote:

"[...] I believe that DMI (Department of Military Instruction) could elicit a new type of ranking list. Within my proposed system, people could add to the list of 17 branches BRADSO slots and rank them within that list. For example: AV (Aviation) > IN (Infantry) > AV:B (Aviation with BRADSO). While this may be a transmutation of the "alternate system," I believe many cadets could utilize this system as it is the case that people view branch without ADSO and BRADSO slots are considered almost different things."

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• In the rest of this lecture, I present the formal modeling and analysis that lead to the new branching mechanism the Army adopted both for USMA and ROTC starting with the Class of 2021.

Cadets, Branches & Terms

- I: Set of cadets
 - Each in need of at most one position at a branch
- $T = \{t^0, t^+\}$: Set of possible contractual terms to acquire a position
 - Elements are totally ordered
 - t⁰: Base price
 - t⁺: Increased price
- B: Set of branches
 - q_b : # of positions at branch $b \in B$
 - q^f_b: Maximum # that can be awarded at the increased price t⁺
 Terminology: q^f_b positions are flexible-price

Cadet Preferences & Branch Baseline Priorities

- Cadet Preferences \succ_i : Linear order on $(B \times T) \cup \{\emptyset\}$
 - Assumption: For any cadet $i \in I$ and branch $b \in B$,

 $(b,t^0) \succ_i (b,t^+)$

- \succeq_i : Induced weak preference relation.
- \mathcal{Q} : Set of resulting cadet preferences
- Branch Baseline Priorities π_b : Linear order on I
 - П: Set of branch baseline priorities

Price Responsiveness Policy

- Price Responsiveness Policy ω_b(π_b): For a given b ∈ B and π_b ∈ Π, a linear order on I × T with the following two properties:
 - 1. Same as the baseline priority order π_b for any fixed contractual term.

For any $i, j \in I$ and $t \in T$,

$$(i,t) \omega_b (j,t) \iff i \pi_b j$$

2. Positively monotonic in contractual term for any given cadet. For any $i \in I$, $(i, t^+) \omega_b (i, t^0)$.

- For the Army application, also called the BRADSO policy.
- $\Omega_b(\pi_b)$: Set of resulting price responsiveness policies.
- Identifies the priority upgrade gained for the flexible-price positions by paying the increased cost. Akin to Marginal Rate of Substitution.

Examples of Price Responsiveness Policies

- Ultimate Price Responsiveness Policy
 - The increased price grants any individual higher priority over any individual who pays the base price.
 - Used at USMA for Classes of 2006-2019.
- Tiered Price Responsiveness Policy
 - Individuals are partitioned into tiers within the baseline priority order.
 - Priority upgrade due to increased price is a function of tier.
 - Ultimate price responsiveness policy is a special case with a single tier.
 - Two distinct versions used at USMA for Classes of 2020 and 2021.
- Scoring-Based Price Responsiveness Policy
 - Baseline priority order is determined with a scoring rule.
 - Increased price grants a fixed boost to total score.
 - In the past used in various Chinese cities for their public high school admissions under the ZX Policy (Ze Xiao).

Outcome: A set of Contracts

- A contract is a triple $x \equiv (i(x), b(x), t(x)) \in I \times B \times T$.
 - Interpretation: A position for cadet i(x) at branch b(x) at price t(x)
 - $\mathcal{X} \equiv I \times B \times T$: Set of all contracts
 - $\mathcal{X}_i \equiv \{x \in \mathcal{X} : i(x) = i\}$: Set of contracts that involve cadet i
 - $\mathcal{X}_b = \{x \in \mathcal{X} : \mathbf{b}(x) = b\}$: Set of contracts that involve branch b
- An allocation is a set of contracts $X \subset \mathcal{X}$, such that

 - 1. for any $i \in I$, $|\{x \in X : i(x) = i\}| \le 1$, 2. for any $b \in B$, $|\{x \in X : b(x) = b\}| \le q_b$, and 3. for any $b \in B$, $|\{x \in X : b(x) = b \text{ and } t(x) = t^+\}| \le q_b^f$.
 - A: Set of allocations

Assignment

For a given allocation X ∈ A and cadet i ∈ I, the assignment X_i of cadet i under allocation X is defined as

$$X_i = \left\{egin{array}{cc} (b,t) & ext{if } (i,b,t) \in X \ \emptyset & ext{if } X \cap \mathcal{X}_i = \emptyset. \end{array}
ight.$$

- Slight abuse of notation: $b(X_i)$ indicates the branch of assignment X_i
- A cadet $i \in I$ is unmatched under allocation $X \in A$ if $X_i = \emptyset$.

Mechanism

- A mechanism is a message space S_i for each cadet i ∈ I along with an outcome function φ : ∏_{i∈I} S_i → A that selects an allocation for each message profile.
 - $S \equiv \prod_{i \in I} S_i$: Set of message profiles
- A mechanism (S, φ) is a direct mechanism, if $S_i = Q$ for each $i \in I$.
 - As it is customary, we denote a direct mechanism with its outcome function only.

- Our approach is axiomatic. We formulate Army's policy objectives as technical axioms, and characterize the unique direct mechanism that satisfies all.
- All but one of our axioms are defined both for allocations and also for mechanisms.
 - Terminology: In those cases a mechanism satisfies the axiom if its outcome satisfies the axiom for all message profiles.

• Individual rationality: No cadet should be assigned an unacceptable branch-price pair.

Formally, an allocation $X \in A$ satisfies individual rationality if, for any $i \in I$,

$$X_i \succ_i \emptyset.$$

A mechanism (S, φ) satisfies individual rationality if the allocation φ(s) satisfies individual rationality for any message profile s ∈ S.

• Non-wastefulness: No position at a branch can be left idle while there is a cadet who is unassigned, unless she would rather remain unassigned than receive the idle position at its base price.

Formally, an allocation $X \in A$ satisfies satisfies non-wastefulness if, for any $b \in B$ and $i \in I$,

$$\frac{\left|\{x \in X : \mathsf{b}(x) = b\}\right| < q_b, \text{ and }}{X_i = \emptyset} \quad \Rightarrow \quad \emptyset \ \succ_i \ (b, t^0).$$

A mechanism (S, φ) satisfies non-wastefulness if the allocation φ(s) satisfies non-wastefulness for any message profile s ∈ S.

• No priority reversal: No cadet *i* should prefer the branch-price package (*b*, *t*) of another cadet *j* to her own assignment, even though she had a higher baseline priority for branch *b*.

Formally, an allocation $X \in A$ satisfies no priority reversals if, for any $i, j \in I$, and $b \in B$

$$\frac{\mathsf{b}(X_j) = b \text{ and }}{X_j \succ_i X_i} \bigg\} \implies j \pi_b i.$$

A mechanism (S, φ) satisfies no priority reversals if the allocation φ(s) has no priority reversals for any message profile s ∈ S.

- We next present two auxiliary definitions that highlight the intuition for our next axiom.
- Given an allocation $X \in A$ and a cadet $i \in I$ with $t(X_i) = t^+$, a cadet $j \in I \setminus \{i\}$ has a legitimate claim for a price-reduced version of cadet *i*'s assignment X_i if,

$$ig(\mathsf{b}(X_i), t^0 ig) \succ_j X_j$$
 and
 $(j, t^0) \omega_{\mathsf{b}(X_i)} (i, t^+).$

Here cadet j's claim for a position at branch b(X_i) at the base price t⁰ is legitimate, because the price responsiveness policy ω_{b(X_i)} does not overturn her higher claim for a position at branch b(X_i) in favor of cadet i even when cadet i pays the increased price.

• Given an allocation $X \in A$ and a cadet $i \in I$ with $t(X_i) = t^0$, a cadet $j \in I \setminus \{i\}$ has a legitimate claim for a price-increased version of cadet *i*'s assignment X_i if,

$$\begin{array}{c} (\mathsf{b}(X_i), t^+) \succ_j X_j \,, \\ (j, t^+) \,\omega_{\mathsf{b}(X_i)} \,(i, t^0) \,, \quad \text{and} \\ \big| \big\{ k \in I \ : \ (k, \mathsf{b}(X_i), t^+) \in X_{\mathsf{b}(X_i)} \big\} \big| < q^f_{\mathsf{b}(X_i)}. \end{array}$$

- Here cadet j's claim for a position at branch b(X_i) at the increased price t⁺ is legitimate, because, even if cadet i has a higher baseline priority at branch b(X_i),
 - 1. the price responsiveness policy $\omega_{b(X_i)}$ overturns this priority in favor of cadet j for as long as cadet j pays a higher price than cadet i, and
 - 2. awarding the position originally given to cadet *i* instead to cadet *j* albeit at a higher price t^+ is feasible and it does not result in exceeding the cap $q_{b(X_i)}^f$ for flexible-price positions at branch $b(X_i)$.

- We are ready to present the key axiom which differentiates our analytical results from earlier results in the literature.
- An allocation X ∈ A satisfies enforcement of the price responsiveness policy if, no cadet j ∈ I has a legitimate claim for either a price-reduced version or a price-increased version of the assignment X_i of another cadet i ∈ I \ {j}.
 - A mechanism (S, φ) satisfies the enforcement of the price responsiveness policy if the allocation φ(s) satisfies the enforcement of the price responsiveness policy for any message profile s ∈ S.
- Remark: Together, the axioms no priority reversal and the enforcement of the price responsiveness policy corresponds to the axiom No Justified Envy.

- Our last axiom is a highly sought-after incentive-compatibility property, defined for direct mechanisms only.
- Strategy-proofness: No cadet ever benefits from misrepresenting her preferences over branch-price pairs.

Formally, a direct mechanism φ is strategy-proof if, for any $\succ \in Q^{|I|}$, any $i \in I$, and any $\succ'_i \in Q$,

$$\left[\varphi(\succ)\right]_{i} \succeq_{i} \left[\varphi(\succ_{-i},\succ_{i}')\right]_{i}$$

Dual-Price Cumulative Offer Mechanism

- The Dual-Price Cumulative Offer (DPCO) mechanism is a direct mechanism based on the seminal cumulative offer procedure (Hatfield & Milgrom 2005) together with the following choice rule.
- Dual-Price Choice Rule C^{DP}_b: Given a branch b ∈ B and set of contracts X ∈ X_b, select (up to) q_b contracts with distinct cadets in two steps as follows:

Step 1. For the base-price positions, exclusively select base-price contracts with the highest baseline priority cadets.

Step 2. For the flexible-price positions , select the highest-priority remaining contracts based on the price responsiveness policy ω_b .

Dual-Price Cumulative Offer Mechanism

- Fix any linear order of cadets, say the OML. (This linear order does not affect the outcome by Kominers & Sönmez 2016).
- At any step ℓ of the procedure,
 - the highest-OML cadet *i*_ℓ who currently has no contract on hold offers his most-preferred previously-unrejected contract *x*_ℓ to the branch of the contract b(*x*_ℓ), and
 - considering all offers X_ℓ it has received up to (and including) Step ℓ, branch b(x_ℓ) holds the contracts in C^{DP}_{b(x_ℓ)}(X_ℓ), and rejects all others.
- The procedure terminates when either no cadet remains with an acceptable contract that has not been rejected, or when no contract is rejected. All the contracts on hold in the final step are finalized.

Main Characterization Result

Theorem (Greenberg, Pathak & Sönmez, 2021)

Fix a profile of baseline priority orders $(\pi_b)_{b\in B} \in \Pi$ and a profile of price responsiveness policies $(\omega_b)_{b\in B} \in \prod_{b\in B} \omega_b$. A direct mechanism φ satisfies

- 1. individual rationality,
- 2. non-wastefulness,
- 3. enforcement of the price responsiveness policy,
- 4. no priority reversals, and
- 5. strategy-proofness

if and only if

$$\varphi = DPCO$$

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Remark

The entire analysis, including the characterization theorem extends to multiple prices.

Technical Significance of the Characterization

- Prior to our analysis, Hirata & Kasuya (2017) and Hatfield, Kominers & Westcamp (2021) presented earlier characterizations of the cumulative offer mechanism.
 - Fundamentally different than our analysis, each institution is endowed with an exogenously given choice rule that satisfies various technical conditions in these papers.
 - In our characterization, in contrast, the dual-price choice rule emerges endogenous to the Army's policy objectives.

Technical Significance of the Characterization

- Indeed, the very concept of a choice rule is merely used in our model to describe the DPCO mechanism.
 - Not only our axioms do not rely on any structure or functional form of potential branch choice rules, even the existence of a well-defined choice rule for any given branch is not assumed in our analysis.
 - Instead, the dual-price choice rule emerges from our analysis in tandem with the cumulative offer mechanism as a collective implication of our five axioms.
 - This is why our result is a characterization of a refinement of the cumulative offer mechanism rather than another characterization of the cumulative offer mechanism itself.

Practical Significance of the Characterization

- Our axioms reflect the Army's policy objectives, and none of them are are imposed upon as technical conditions for the sake of obtaining an axiomatic characterization.
- On the contrary, the very reason the Army has initiated a collaboration with the two civilian co-authors of this paper is the design of a branching system which ideally satisfies all these axioms.
- As I emphasized earlier in the presentation, starting with 2006 the Army's adjustments in its branching mechanisms to implement its BRADSO policies have resulted in priority reversals along with incentive compatibility failures.
- In a manner of speaking, the effort to accommodate the enforcement of the price responsiveness policy axiom has resulted in an unintended consequence of the failure of two other key axioms.

Application in School Choice

- Wang and Zhou (2020): Public high school admissions in China under the ZX Policy (Ze Xiao).
 - A fraction of the seats are available with an increased tuition.
 - Baseline priorities are based on scores on a centralized exam.
 - The higher-tuition contract increases this score by a fixed amount for the ZX-eligible seats.
 - Shanghai and Tianjin both have a single ZX tuition level, making these applications completely analogous to the Army's problem.
 - In some cities there were multiple tuition levels where higher tuition levels result in higher adjustments to student score.

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 - In some cities there were multiple tuition levels where higher tuition levels result in higher adjustments to student score.
 - Discontinued after 2015.
US Army's Branching Process

- Army considered the design a success, and also adopted it for ROTC ahead of its scheduled time.
 - The decision to use DPCO mechanism for ROTC was in part due to concerns that ROTC's previous branching mechanism generated dead zones that made priority reversals particularly visible, as discussed in Sönmez (2013).
- Army has also identified additional ways to utilize the cumulative offer mechanism.
- Our model and the DPCO mechanism can be used in other applications where an agent can take a costly action in exchange for higher priority in a fraction of positions.

Minimalist Market Design

- Proof-of-concept for a new institution design paradigm.
- Especially valuable in the following settings:
 - The need for a change is not established, and the reform is merely aspired by an outsider.
 - The "intended" institution is clear, but finding it requires formalism and technical expertise.
 - The mission of the institution cannot be fully described with a single-dimension objective function.

The Importance of Theory in Policy Oriented Research

- On a broader level, the US Army's economist-guided branching reform highlights the importance of both fundamental theory and also custom-made theory in policy oriented economics research.
- Key contributions for the Army's reform (Pure Theory):
 - Gale & Shapley (1962)
 - Kelso & Crawford (1982)
 - Hatfield & Milgrom (2005)
- Unlike the more mainstream approaches in market design where the main role of theory is to provide intuition, the reform process is mainly driven by the custom-made theory in minimalist market design.
- Key contributions for the Army's reform (Custom-made Theory):
 - Sönmez & Switzer (2013) Sönmez (2013)

- Hatfield & Kojima (2010)
- Echenique (2012)