UTMD Working Paper

UTMD-009

Kick-out Voting:

Strategic Voting under the Proportional Representation and Single Non-Transferable Vote Systems

Hiroto Katsumata
University of Tokyo

Shunya Noda
University of British Columbia

June, 2021

Kick-out Voting:

Strategic Voting under the Proportional Representation and Single Non-Transferable Vote Systems*

HIROTO KATSUMATA[†] SHUNYA NODA[‡]

June 2021

Abstract

We analyze *kick-out voting*, a novel form of strategic voting induced under multi-member electoral systems, such as proportional representation. Our formal theory predicts that voters do not necessarily vote for their most preferred party/candidate who seriously competes for a seat. Instead, voters may vote for the party/candidate who would kick out their least preferred party/candidate, even if it decreases the winning probability of their most preferred. We perform empirical analyses with individual-level data of preferences and vote choices based on three data sets from diverse contexts: (i) A lab experiment in the United Kingdom, (ii) a closed-list proportional representation election in Romania, and (iii) elections under the single non-transferable vote system in Japan. Concordant with our theoretical predictions, we find empirical evidence that voters are attempting to kick out less preferred candidates. Such strategic behaviors may skew vote shares and worsen the quality of representation in these electoral systems.

(149 words)

Keywords: Strategic voting; Multi-member district; Experiment; Japan; Romania

*The authors thank Kentaro Fukumoto, Yusaku Horiuchi, Yuko Kasuya, Kenneth M. McElwain, Hirofumi Miwa, Ryo Nakai, Masaru Nishikawa, Daniel Smith, Junichi Yamasaki, and the participants of the 2021 winter meeting of the JSQPS and the 2021 annual meeting of the JAES for their helpful comments. Shogo Higashi provided superb research assistance. This work was supported by JSPS KAKENHI, Grant-in-Aid for JSPS Fellows (Grant Number 17J03508) and Grant-in-Aid for Early-Career Scientists (Grant Number 21K13225). Noda appreciates the financial support provided by the Funai Overseas Scholarship, the E. K. Potter Fellowship, and the start-up grant of Vancouver School of Economics.

[†]Graduate School of Arts and Sciences, University of Tokyo. E-mail: hrt.katsumata@gmail.com

[‡]Vancouver School of Economics, University of British Columbia. E-mail: shunya.noda@gmail.com

1 Introduction

Kick-out voting is a term we coined to describe a voting strategy that attempts to *defeat* a dispreferred candidate (or party)[1] rather than to elect a preferred candidate. This study demonstrates that kick-out voting arises as an equilibrium strategy in multi-member districts, such as the single non-transferable vote (SNTV) and closed-list proportional representation (CLPR) systems.[2] We further examine empirical and experimental data sets and validate our theory of kick-out voting.

The traditional strategic voting literature, which has mostly focused on the single-member districts (SMD), has dismissed kick-out voting. In SMD, strategic voting is quite simple: an optimal (equilibrium) voting behavior is to vote for one of the two *competing candidates* (in the sense of Duverger's law) while deserting trailing candidates who have no chance to win (Cox 1997). If a voter is a consequentialist, his vote changes his resultant utility only if the candidate for whom he votes is tied with another candidate. Since the only tie that could arise in equilibrium is a tie between these two competing candidates, kicking out a less preferred one is equivalent to electing a more preferred one. Accord-

[1]In proportional representations, voters vote for parties, not candidates (politicians). Hereafter, we simply use "candidate" when we refer to the person or party for whom votes are to be cast.

[2]Under the SNTV, a voter casts only one vote for a candidate in the multi-member district, and winners are determined by the simple plurality rule. This system is sometimes referred to differently, especially in the United States. For example, Puerto Rico elects eleven seats each for the House of Representatives and the Senate *at large*, which is the same as the SNTV (Constitution of the Commonwealth of Puerto Rico 1952, ARTICLE III, Section 3; `http://www2.pr.gov/`). Some counties and municipalities in Alabama and North Carolina use the SNTV as limited voting with one vote per voter for City and/or County Commission and/or School Board elections (Arrington and Ingalls 1998).

ingly, a strategic voter would vote sincerely (i.e., vote for the most preferred candidate) among the competing candidates.

Incentives in multi-member district elections are more complex because various types of ties could occur. Unlike in the SMD, voting sincerely for the most preferred candidate among those competing for the seat is no longer optimal in multi-member districts. Each tie occurs with a different probability, and therefore, voters need to perform complex calculations to derive an optimal voting behavior. Despite the complexity of the vote-choice problem, we (i) characterize the large-market limit of equilibria, (ii) discover that voters' asymptotically optimal behaviors can be interpreted as a simple form of kick-out voting, and (iii) empirically show that voters are indeed attempting to kick out less preferred candidates. Surprisingly, real-world voters are sensitively reacting to strategic incentives, even in a more complex environment than the simple SMD. The voting pattern discovered by this study cannot be explained by the strategic desertion of trailing candidates.

More specifically, we analyze a game-theoretic election model established by Cox (1994) and prove that kick-out voting arises as an equilibrium consequence. Our theoretical analysis predicts that kick-out voting takes either one of the two forms: (i) voting for the *runner-up* ($(M+1)$-th popular candidate in a M-seat district) to kick out one of the *leading candidates* (top M popular candidates) or (ii) voting for a less preferred leading candidate to kick out the runner-up.[3] These two predictions are testable hypotheses.

We examine three data sets from diverse contexts to analyze real-world voters' strategies empirically. The first data set is from a lab experiment conducted in the United Kingdom (Hix, Hortala-Vallve and Riambau-Armet 2017). This experimental data set contains individual-level vote choices with randomized preferences for parties, which offers inter-

[3]For proportional representation systems, the runner-up party is defined as the party which is the most likely, i.e. has the smallest vote margin, to win an additional seat and the *leading parties* are defined as the seat-winning parties other than the runner-up. Their union constitutes the *competing parties*.

nal validity to test our hypotheses. The second and third data sets are survey data for an election under the CLPR in Romania and elections under the SNTV in Japan. We examine the survey data to explore kick-out voting in real-world elections. The wide variety of these data sets enhances the external validity of our study.

The results of our empirical analyses support our theoretical predictions. First, we find that, with all the three data sets, if voters dislike some of the leading candidates more strongly, then they are more likely to vote for the runner-up. Our theory predicts this tendency because such voters want to kick out some leading candidates by supporting the runner-up. Second, with the UK experiment and the Romanian election, we find that if voters dislike the runner-up more strongly, then they are less likely to vote for their most preferred candidate. This behavior can also be interpreted as kick-out voting because such voters want to support a leading candidate who is more likely to be tied with the runner-up. These two phenomena never happen if voters sincerely vote for their most preferred candidate or if voters support their favorite candidate among competing ones (as in strategic voting in the SMD).

The SNTV and CLPR are not only ideal for the analysis of voters' strategic behaviors but also important in and of themselves. Notably, the proportional representation (PR) system with low-magnitude districts is known as the "electoral sweet-spot", which enhances the broad representation of political preferences while simplifying government coalition and fostering clear accountability (Carey and Hix 2011). While the SNTV is infamous for its pork-barrel politics (Catalinac 2016; Rosenbluth and Thies 2010), it has an advantage in facilitating minority representation (Lijphart, Pintor and Sone 1986).[4]

[4] In 2009, a United States District Judge ordered a school district board of education to use limited voting, with only one vote per voter (i.e., the same electoral system as the SNTV), to afford minorities a meaningful opportunity to elect their preferred candidates (United States v. Euclid City School District Board of Education, No. 1:08-cv-02832-KMO (N.D. Ohio Dec. 2, 2008)).

While early studies conjectured that strategic voting is absent or weak in PR systems (Duverger 1954; Sartori 1968), later works argue that voters desert weak parties in PR systems (Abramson et al. 2010; Artabe and Gardeazabal 2014; Cox 1997). Some existing studies also argue that multi-member district electoral systems induce a different type of strategic incentives than the SMD. In a foundational work, Cox (1994) theorized that voters desert candidates who are "too strong" as well as candidates who are "too weak" under the SNTV system because excess votes for strong candidates do not change electoral results. Under PR systems with low-magnitude districts, the integer constraint on the seat allocation does not allow seats to be allocated completely proportionally to votes among parties. Hence, increasing or decreasing a few votes for some of the parties does not have a chance to change the seat allocation at the district level (Cox 1997; Cox and Shugart 1996). Consequently, voters would avoid wasted votes by shifting votes from non-marginal (too strong or too weak) parties to marginal ones competing for the last seat in a district.[5]

This paper brings to light the unique role of runners-up, which has been overlooked in the previous studies. The most effective attempt to kick out a leading candidate is to vote for a runner-up. Therefore, the runner-up would be supported by *all* the voters who dislike *some* of the leading candidates. Consequently, runners-up are favored under kick-out voting: runners-up tends to obtain a larger vote share than under sincere voting. When voters' preferences are diverse, this share tends to be large because various leading candidates could be dispreferred. Consequently, multi-member districts often suffer from the

[5] Hix, Hortala-Vallve and Riambau-Armet (2017) shows that although voters tend to vote for parties that maximize their expected utility in multi-member districts, sincere voting increases as the district magnitude increases. In addition, other studies argue that voters conduct coalition-oriented strategic voting in anticipation of likely coalition-formation and policy negotiations under PR systems (Bargsted and Kedar 2009; Blais et al. 2006; Bowler, Karp and Donovan 2010; Cho 2014; Duch, May and Armstrong 2010).

indeterminacy of the voting outcome: as the runner-up is favored by the equilibrium feature, all competing candidates (including the runner-up) tend to obtain a similar voting share in equilibrium. Accordingly, even in the limit of infinitely many voters, the SNTV and CLPR may fail to elect the M most "popular" candidates deterministically.

The rest of the article is organized as follows. Section 2 provides a formal theory of kick-out voting to derive our empirical hypotheses. Section 3 turns to the empirical evaluation of our theory with individual-level data of preferences and vote choices in diverse contexts. Section 4 points out the shortcoming of the SNTV and CLPR caused by kick-out voting. Section 5 concludes and suggests possibilities for future research.

2 Formal Theory of Kick-out Voting

2.1 Model

This section analyzes the equilibrium under SNTV theoretically. (Later, we discuss that SNTV and CLPR are similar in a large market.) We study the environment formulated by Cox (1994) and follow the notations of that paper. There are K candidates, indexed by $k \in \mathcal{K} := \{1, 2, \ldots, K\}$, competing for $2 \leq M < K$ seats. Candidates take no action, and the preferences of the voters are exogenously endowed. Each voter votes for exactly one candidate, and seats are filled by the M candidates with the most votes.

Each voter i has an additive separable utility on the candidates and can thus be represented by a von Neumann–Morgenstern utility vector $u^i = (u^i_1, u^i_2, \ldots, u^i_K)$. When the candidates $S \subset \mathcal{K}$ are elected, their payoff is $\sum_{k \in S} u^i_k$. Without loss of generality, we rescale the voter's utility in such a way that his most preferred candidate yields a utility of 1, and his least preferred candidate yields a utility 0. Then, we can define

$$\mathcal{U} := \left\{ u \in \mathbb{R}^K : \max_{k \in \mathcal{K}} u_k = 1, \max_{k \in \mathcal{K}} u_k = 0, u_j \neq u_k \text{ for } j \neq k \right\}$$

as the domain of agents' preference vectors.

The voters' preferences are distributed according to a continuous distribution function F on $\mathcal{U}$, and n randomly picked voters will participate in the election. Although F and n are common knowledge, each voter does not observe the realized preference vectors of the other voters. Hence, each voter's strategy is a mapping from the voter's preference to a candidate to be voted.

Our equilibrium concept is the Bayesian Nash equilibrium: in equilibrium, (i) each voter maximizes his payoff according to his expectation (belief), and (ii) the beliefs are consistent with the strategies that the other agents are actually taking. Let $\pi = (\pi_1, \ldots, \pi_K)$ be the probability vector such that each π_k represents the probability that a randomly selected voter will vote for candidate k. Voter i's own preference vector u^i, the belief π, and the number of voters n specify voter i's decision problem. For each $u^i \in \mathcal{U}$, we denote the candidate for whom voter i votes by $V(u^i; \pi, n)$. Let $H_k(\pi; n) := \{u \in \mathcal{U} : k \in V(u; \pi, n)\}$ be the set of voters who vote for k given π and n. When voters' belief is π, the vector of expected shares is given by $g_k(\pi; n) := F(H_k(\pi; n))$. In equilibrium, we must have $\pi = g(\pi; n)$. We say that π is *rational* given n if $\pi = g(\pi; n)$.

We focus our attention on the case with sufficiently many voters, i.e., the limit of $n \to \infty$. We say that π is a *limit of rational expectations* if for every $\epsilon > 0$, there exists an integer N and a sequence $\pi(n)$ of rational expectations such that for all k and $n > N$, $|\pi_k(n) - \pi_k| < \epsilon$. Note that at the limit of rational expectations, π_k also represents the equilibrium share of candidate k due to the law of large numbers. Accordingly, if each of the top M candidates gets strictly larger shares than candidate $M + 1$, the outcome of the election is asymptotically deterministic: the winning probability of top M candidates converges to one as $n \to \infty$.

2.2 Discriminating Equilibria

Without loss of generality, we sort the candidates by the ex ante probability of receiving votes, i.e., assume $\pi_1 \geq \pi_2 \geq \cdots \geq \pi_K$. In this paper, we focus on *Duvergerian equilibria*, in which exactly $M+1$ candidates obtain positive shares, i.e., $\pi_{M+2} = \cdots = \pi_K = 0$, at the limit of $n \to \infty$.[6] That is, we restrict our attention to the case in which we have exactly $M+1$ competing candidates. Preferences for the other candidates are irrelevant in such equilibria. From now, we put the assumption for $K = M+1$, for notational simplicity.

Among such Duvergerian equilibria, we say that π is *discriminating* if $\pi_M > \pi_{M+1}$. Recall that π represents the share of each candidate at the limit of $n \to \infty$. Accordingly, if the limit of rational expectations is discriminating and there are sufficiently many voters, the top M candidates will be in office with a large probability. One of the main reasons to have an election is to select the M most "popular" candidates (with respect to a preference distribution F). If a voting system has a discriminating equilibrium, then it can select the set of popular candidates deterministically.

First, we introduce the main result of Cox (1994) as a preliminary.

Theorem 1 (Cox, 1994)**.** *If π is a limit of rational expectations, then $\pi_1 = \pi_2 = \cdots = \pi_M$.*

Theorem 1 implies that all the leading candidates (which do not include the runner-up) obtain the same voting share in the limit of rational expectations. The intuition of Theorem 1 is as follows. A vote changes the set of elected candidates only when some candidates are in a tie, i.e., the voter is pivotal. However, if π_k where $k < M$ is strictly larger than π_M, by the law of large numbers, each voter knows that candidate k surely gets a larger share than candidate M, given that there are sufficiently many voters. Accordingly, voting for k is asymptotically equivalent to abstaining. Since each voter has a strict preference over candidates, even if candidate k is his favorite, it is better to vote for the

[6] In our model setting, (i) a Duvergerian equilibrium always exists, and (ii) non-Duvergerian equilibria are unstable.

marginal candidates. Hence, (almost) no one wants to vote for candidates $1, 2, \ldots, M-1$, and therefore, the belief $\pi_k > \pi_M$ cannot be a rational expectation. Consequently, in a limit of rational expectations, $\pi_1 = \cdots = \pi_M$ must be the case.

While Theorem 1 is an interesting observation, Cox (1994) does not provide a prediction for the share of the runner-up (candidate $M+1$). To investigate the existence of discriminating equilibria, we need to figure out whether $\pi_M > \pi_{M+1}$ is achievable or we always have $\pi_M = \pi_{M+1}$ in all equilibria. If we regard the election as a procedure for selecting candidates, the share of the runner-up is even more important than the share among winning candidates because it may change the set of elected candidates.

We will propose a way to decide whether or not a discriminating π can be a limit of rational expectation for each F. We denote the probability that candidates j and k are in a tie for Mth place with n voters by $T_{j,k}(n)$. Once we specify $\pi(n)$ and n, $T_{j,k}(n)$ can be straightforwardly calculated. A voter (with the preference vector u) prefers to vote for j over l if and only if[7]

$$\sum_{k=1}^{M+1} T_{j,k}(n)(u_j - u_k) \geq \sum_{k=1}^{M+1} T_{l,k}(n)(u_l - u_k).$$

In order to analyze voters' optimal strategy in a large-market limit, we study the conditional tie probability in the limit. Define

$$A(n) := \sum_{k=1}^{M} \sum_{l \neq k} T_{k,l}(n).$$

$A(n)$ represents the total probability that some candidates are in a tie for Mth place with

[7]To be more precise, we need to assume that (i) the probability of r-way ties for $r > 2$ is negligible, and (ii) the probability that "k is in Mth place, tied with j" equals the probability that "k is in Mth place, one vote ahead of j" to obtain this inequality.

n voters. Then,

$$t_{k,l}(n) := \frac{T_{k,l}(n)}{A(n)}$$

represents the probability that k and l are in a tie, conditional on the event that a two-way tie happens. For notational convenience, define $t_{k,k} = 0$ for all k. For all n, $t := (t_{k,l})_{k,l}$ is a $(M+1)^2$-dimensional probability vector. Since $\Delta^{(M+1)^2}$ is a compact subset of $\mathbb{R}^{(M+1)^2}$, there exists a convergent subsequence, $(t(n_m))_{m=1}^{\infty}$. We define the *limiting conditional tie probability* τ as this limit of this sequence.

$$\lim_{m\to\infty} t(n_m) =: \tau \in \Delta^{(M+1)^2}.$$

To study the behavior of the limiting conditional tie probability, we introduce the following lemma.

Lemma 1. *Suppoise that $k, l \leq M$ and π is discriminating (i.e., $\pi_M > \pi_{M+1}$). Then, we have*

$$\lim_{n\to\infty} \frac{T_{k,l}(n)}{T_{k,M+1}(n)} = 0. \tag{1}$$

Proof. We use a useful lemma proved by Cox (1994).

Lemma 2 (Lemma 2.1 of Cox 1994)**.** *Let a, b and c be elements of $\mathcal{K}$ such that $a < M$ and $a < b \leq M+1$ and $(b = M+1 \Rightarrow c \leq M)$ and $\pi_a > \pi_b$. Then,* $\lim_{n\to\infty} T_{c,a}(n)/T_{c,b}(n) = 0$.

Take $a = k$, $b = l$, and $c = M+1$. By applying Lemma 2, we obtain the desired result. □

The intuition for Lemma 1 is as follows. If k and l are in a tie for Mth place, the rest of the candidates, including candidate $M+1$, who has a strictly smaller probability to be voted than the others, will be in the first to $(M-1)$th places. However, as $n \to \infty$, the probability of such events rapidly gets small relative to the probability that "one of the top M candidates and candidate $M+1$ are competing for the last seat" because candidate

$M+1$ is strictly weaker than the other candidates (i.e., $\pi_{M+1} < \pi_M$). Accordingly, the tie probability of k and l for $k, l \le M$ is much smaller than the tie probability of k and $M+1$ at large n.

Lemma 1 implies that the probability that two winning candidates ($k, l \le M$) are tied vanishes more rapidly than the probability that $M+1$ is involved in a tie. Formally, (1) implies

$$\tau_{k,l} := \lim_{n\to\infty} \frac{T_{k,l}(n)}{A(n)} \le \lim_{n\to\infty} \frac{T_{k,l}(n)}{T_{k,M+1}(n)} = 0 \quad \text{for all } k, l \le M.$$

Hence, $\tau_{k,l} = 0$ for all $k, l \le M$. In the limit of $n \to \infty$, conditional on the event that there are tied candidates, the runner-up (candidate $M+1$) is involved in the tie with probability one.

The observation above indicates that many entries of τ are zero: only M entries, $(\tau_{1,M+1}, \tau_{2,M+1}, \ldots, \tau_{M,M+1})$ take positive values. To simplify the notation, we define $\alpha_k := \tau_{k,M+1}$ for $k \le M$. Clearly, $\alpha \in \Delta^M$. To summarize, in a large-market limit, when a tie happens, it must be a tie between candidate $M+1$ and one of the leading candidates $k \in \{1, \ldots, M\}$, and α_k represents the conditional probability that candidate k and $M+1$ are tied.

Now, we consider each voter's behavior in the limit. Since $A(n) > 0$ for all n, for each n, a voter with the preference vector u prefers to vote for a candidate who is in

$$\operatorname*{argmax}_{j=1,\ldots,M+1} \sum_{k=1}^{M+1} \frac{T_{j,k}(n)}{A(n)} (u_j - u_k).$$

Taking the limit of $n \to \infty$, we have

$$\sum_{k=1}^{M+1} \frac{T_{j,k}(n)}{A(n)} (u_j - u_k) \to \alpha_j (u_j - u_{M+1}) \quad \text{as } n \to \infty \quad \text{for } j = 1, \ldots, M,$$

$$\sum_{k=1}^{M+1} \frac{T_{M+1,k}(n)}{A(n)} (u_{M+1} - u_k) \to \sum_{k=1}^{M} \alpha_k (u_{M+1} - u_k) \quad \text{as } n \to \infty.$$

Hence, α characterizes the voter's behavior in the limit. When each candidate's share induced by α is consistent with the expectation, π is achieved in equilibrium.

Theorem 2. *π is discriminating and a limit of rational expectations only if there exists $\alpha \in \Delta^M$ such that*

$$V(u;\pi) := \operatorname{argmax}\left\{\alpha_1(u_1 - u_{M+1}), \ldots, \alpha_M(u_M - u_{M+1}), \sum_{k=1}^{M} \alpha_k(u_{M+1} - u_k)\right\},$$

$$H_j(\pi) := \{u \in \mathcal{U} : j \in V(u;\pi)\},$$

$$g_j(\pi) := F(H_j(\pi)) > \frac{1}{M+1} \quad \textit{for all } k = 1, \ldots, M,$$

$$g_{M+1}(\pi) := F(H_{M+1}(\pi)) < \frac{1}{M+1}, \textit{ and}$$

$$\pi = g(\pi).$$

The probability that $k, l \leq M$ are in a tie rapidly vanishes as $n \to \infty$. Hence, at large n, voting for $k \leq M$ does not decrease the probability that another candidate $l \leq M$ is elected. In this sense, voting for $k \leq M$ means "supporting candidate k against candidate $M+1$." In contrast, candidate $M+1$ can possibly be in a tie with all the other candidates. Hence, the voter can "simultaneously support candidate $M+1$ against all the other candidates, $1, 2, \ldots, M$" by voting for $M+1$. A voter becomes much more influential when he votes for candidate $M+1$: conditional on the event that a two-way tie occurs, the probability that the voter is pivotal is $\sum_{k=1}^{M} \alpha_k = 1$ if he votes for $M+1$ while it is only $\alpha_l \leq 1$ if he votes for $l \leq M$.

This equilibrium feature favors candidate $M+1$. Importantly, a voter may vote for candidate $M+1$ even when he does not particularly prefer $M+1$. This is because voting for candidate $M+1$ is the most effective way to increase the probability of kicking out a dispreferred candidate from the office. If a voter strongly dislikes a leading candidate $j \leq M$ (i.e., u_j is very small), then $\sum_{k=1}^{M} \alpha_k(u_{M+1} - u_k)$ would be large even if u_{M+1} is smaller than u_k for any $k \neq j$.

For such a discriminating equilibrium to exist, candidate $M+1$ must be sufficiently unpopular than the other candidates (with respect to the preference distribution F). Otherwise, candidate $M+1$ would obtain a large share ($> 1/(M+1)$) because he is favored by the equilibrium feature, which contradicts the assumption that candidate $M+1$ is a runner-up. Consequently, SNTV fails to elect M most popular candidates deterministically for a nondegenerate set of voter distributions F. This issue will be discussed further in Section 4.

Remark 1 (CLPR). Under CLPR, each voter votes for a party, not a candidate. Each party pre-decides the ordered list of candidates, and the candidates positioned high on the list tend to always get a seat. In the limit of infinite voters, this situation is equivalent to SNTV. As the number of voters increases, the probability that an upset occurs decreases. Accordingly, conditional on the event that a tie occurs, each party has at most one candidate who could be involved in a tie with positive probability, asymptotically. From the perspective of voters, voting for a party is strategically equivalent to voting for the marginal candidate of the party, who could be tied with another candidate. This situation is isomorphic to SNTV, and therefore, SNTV and CLPR have a similar equilibrium structure.

2.3 Example

In this subsection, we introduce an example that demonstrates how kick-out voting arises as an equilibrium behavior of SNTV. We consider an environment where there are $M=2$ seats and three competing candidates, A, B, and C. Each voter's preference is specified by a three dimensional vector $u = (u_A, u_B, u_C)$. Since we normalize $\max_{k \in \{A,B,C\}} u_k = 1$ and $\min_{k \in \{A,B,C\}} u_k = 0$, the domain of preference vectors u is on a regular hexagon whose vertices are $(1,0,0)$, $(1,1,0)$, $(0,1,0)$, $(0,1,1)$, $(0,0,1)$, and $(1,0,1)$ (as depicted in

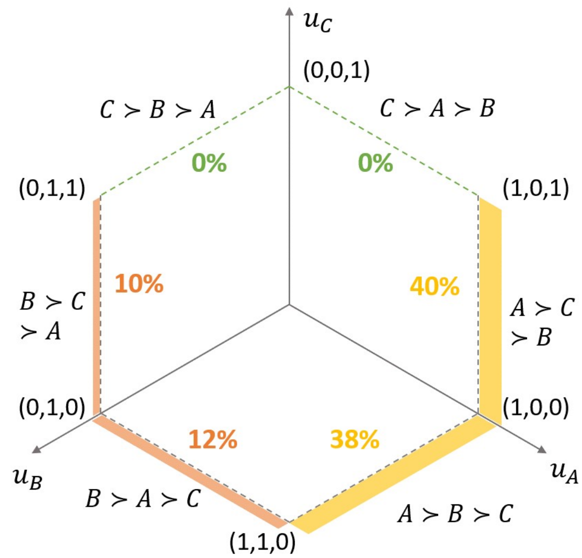


Figure 1: The preference distribution and the vote share under sincere voting.

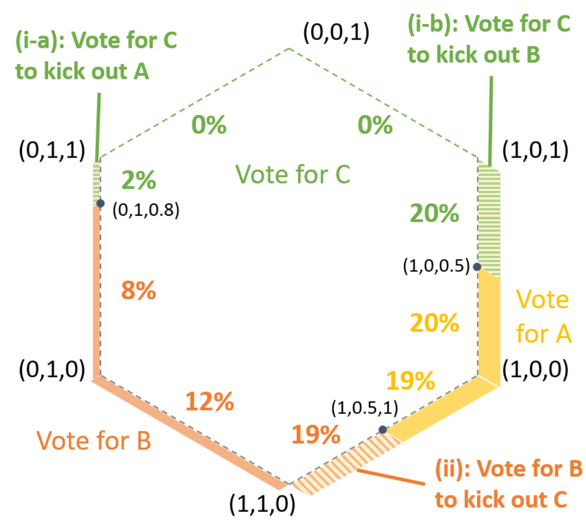


Figure 2: Voters' strategies in the discriminating equilibrium.

Figure 1).[8] Voters are uniformly distributed on each edge, while their density is unequal across edges. Specifically, the density f of voters' preference distribution function F is specified as

$$f(u) = \begin{cases} 0.40 & \text{if } u \in \{(1,0,u_C) : u_C \in [0,1]\}, \\ 0.38 & \text{if } u \in \{(1,u_B,0) : u_B \in [0,1]\}, \\ 0.12 & \text{if } u \in \{(u_A,1,0) : u_A \in [0,1]\}, \\ 0.10 & \text{if } u \in \{(0,1,u_C) : u_C \in [0,1]\}, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Under sincere voting, these three candidates have very different voting shares: Candidate A obtains 78% of total votes, candidate B obtains 22%, and candidate C obtains no vote (since there is no voter who prefers candidate C the most). In this sense, candidate

[8]Precisely speaking, these six vertices are not on the same plane. Thus, the domain appears as a regular hexagon only when it is projected on the plane specified by $x + y + z = c$ for some c. Figures 1 and 2 are drawn in such a way.

A is overwhelmingly popular, and candidate C is very minor.

Applying Theorem 2 to the preference distribution (2), we can obtain the equilibrium voting strategy. In equilibrium, the leading candidates (candidates A and B) obtain the same voting share of 39%, and candidate C also obtains a significant share, 22%. A more popular candidate is less likely to be involved in a tie: $\alpha_A = 1/3$ and $\alpha_B = 2/3$ (α_A and α_B are determined in such a way that the two leading candidates A and B obtain the same voting share), and therefore, given that a tie occurs, candidate A, B, and C are involved with probability 1/3, 2/3, and 1, respectively. This feature induces voters to vote for a less popular and less preferred candidate, and the equilibrium vote share becomes more equable than the popularity. The voters' strategies are illustrated in Figure 2.

There are three groups of voters who do not vote sincerely and vote for the second-most preferred candidate instead. All the voters but these three groups vote for their most preferred candidate.

(i) This group votes for the runner-up C to kick out one of the leading candidates, A or B.

  (i-a) This group has a preference $B \succ C \succ A$ and $u_C \geq 0.5$. Since this group is nearly indifferent between B and C, their primary interest is to kick out A. Since A and B are tied with probability zero, voting for C is the only way of kicking out A (conditional on the event that a tie occurs, A and B are tied with probability zero, but A and C are tied with probability 1/3). While there is a chance of kicking out the most preferred candidate B, the gain from kicking out A is overwhelming.

  (i-b) This group has a preference $A \succ C \succ B$ and $u_C \geq 0.8$. While they are attempting to kick out B (instead of A), the runner-up (C) is still the candidate for whom this group votes, and their motivation is parallel to case (i-a).

(ii) This group has a preference $A \succ B \succ C$ and $u_B \geq 0.5$. This group votes for the leading candidate B to kick out the runner-up C. Since this group is nearly indifferent between A and B, their primary interest is to kick out C. Since candidate B is less popular than A, B is more likely to be involved in a tie (conditional on the event that a tie occurs, A and C are tied with probability $1/3$, and B and C are tied with probability $2/3$). Accordingly, this group votes for B to maximize the chance of kicking out the runner-up, C.

2.4 Testable Hypotheses

Our theoretical analysis shows that kick-out voting could arise as an equilibrium voting behavior. In this subsection, we state two hypotheses on kick-out voting, which can be tested with empirical/experimental data.[9] All of our hypotheses are verified if (some) voters attempt to kick out less preferred candidates and are falsified if they vote sincerely (among the competing candidates).

Hypothesis 1 (Kicking Out a Leading Candidate)**.** If voters dislike a leading candidate more strongly, then they are more likely to vote for a runner-up.

Hypothesis 1 corresponds to Case (i) of the example in Subsection 2.3. In a discriminating equilibrium, (a) a payoff from voting for leading candidate k is $\alpha_k(u_k - u_{M+1})$, and (b) a payoff from voting for the runner-up $M+1$ is $\sum_{l=1}^{M} \alpha_l(u_{M+1} - u_l)$. For any $k \leq M$, if u_k decreases, then the former term decreases and the latter term increases. Accordingly, if some of the leading candidates are dispreferred more strongly, the voter should shift to vote for the runner-up though the payoff from voting for the most preferred is unchanged. Intuitively, this is because voting for runners-up is the most effective way of kicking out a dispreferred leading candidate. By contrast, if voters vote sincerely (i.e.,

[9] We present and test an additional hypothesis derived from two hypotheses presented in this section in Appendix E.

vote for the most preferred candidate) among the competing candidates, the preference for less preferred candidates does not influence the voting behavior.

Hypothesis 2 (Kicking Out the Runner-up)**.** Suppose that a voter prefers a leading candidate the most. If he dislikes the runner-up more strongly, then he is less likely to vote for the first-most preferred candidate. Instead, he shifts to vote for a less preferred leading candidate.

Hypothesis 2 corresponds to Case (ii) of the example in Subsection 2.3. Suppose that a voter prefers a leading candidate k^* the most, and given his utility vector $u = (u_k)_{k=1}^{M+1}$ and the conditional tie probability $\alpha = (\alpha_k)_{k=1}^{M}$, he would vote for candidate k^*. Suppose also that there exist $k^{**} \leq M$ such that $\alpha_{k^{**}} > \alpha_{k^*}$, i.e., candidate k^{**} is a leading candidate who is less popular than candidate k^*, and therefore is more likely to be tied with the runner-up than candidate k^*.[10] The fact that this voter votes for candidate k^* implies that

$$\alpha_{k^*}(u_{k^*} - u_{M+1}) > \alpha_{k^{**}}(u_{k^{**}} - u_{M+1}). \tag{3}$$

Even when $\alpha_{k^{**}} > \alpha_{k^*}$, (3) could hold when this voter prefers candidate k^* over k^{**} by much: $u_{k^*} \gg u_{k^{**}}$. However, this situation could be overturned if this voter were to dislike the runner-up more strongly: as u_{M+1} decreases by one, the payoff from voting for k^* increases by α^*, while the payoff from voting for k^{**} increases by α^{**} $(> \alpha^*)$. Accordingly, if u_{M+1} becomes sufficiently small, the voter would shift to vote for candidate k^*. Put another way, if a voter strongly dislikes the runner-up, the voter would support the leading candidate who is the most likely to be tied with the runner-up. By contrast, if voters vote sincerely (among the competing candidates), they always vote for the most preferred competing candidate, and their voting behavior is not influenced by their attitude toward the runner-up (as long as the runner-up is not the most preferred).

[10]When α_{k^*} is also the largest, the voter always votes for candidate k^* regardless of the value of u_{M+1}.

Remark 2. Some studies on strategic voting adopt an approach that tests whether or not a voter votes to maximize her expected utility (Hix, Hortala-Vallve and Riambau-Armet 2017). For a conservative evaluation, however, we do not use this approach because it leads to an overestimation of kick-out voting. Since we cannot distinguish strategic voting from sincere voting for voters whose most preferred candidate also maximizes their expected utility, sincere voters may be mistakenly treated as strategic in this approach.

3 Empirical Evidence of Kick-out Voting

To test our hypotheses, we examine three data sets from diverse contexts. The first one is from a lab experiment in the United Kingdom, which offers complete information about voters' preference for the competing parties. This experimental data is unique in the sense that the multi-member district experiment is conducted in a country well-known for the SMD election, which is suitable for examining an aspect of the generalizability of our theory. The second and third data sets are from a CLPR election in Romania and elections under the SNTV system in Japan, which enable us to assess the validity of our theory with individual-level data in real-world contexts.

With these data sets, we focus on situations in which our theory of kick-out voting and the traditional strategic-voting theory provide different predictions. Specifically, by verifying Hypotheses 1 and 2, we demonstrate that there is voting patterns predicted by our kick-out voting theory but is explained by neither sincere voting nor strategic desertion of the "too weak" or "too strong " candidates.

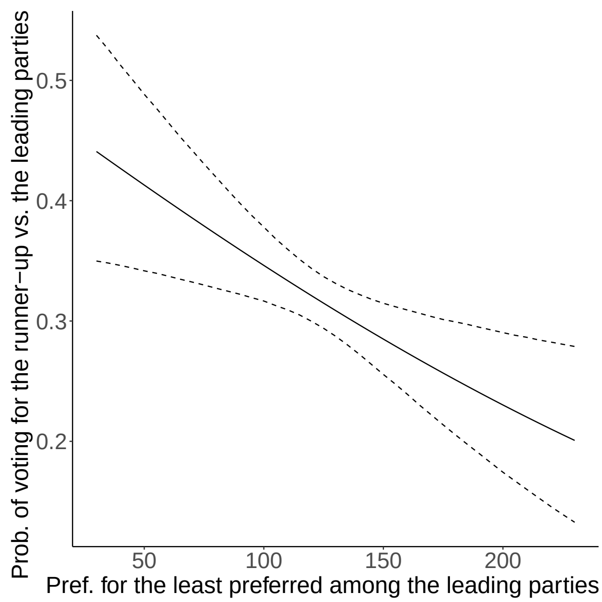
3.1 Lab Experiment in the United Kingdom

Hix, Hortala-Vallve and Riambau-Armet (2017) conducted a lab experiment to evaluate the effects of district magnitude on voting behavior under the CLPR. To this end, their experimental design controls the voters' preferences and the number of parties. This experimental setting is ideal for our purpose to investigate kick-out voting because (i) we have complete information on the randomly generated utility of the study participants for each party, and (ii) the participants are informed of which parties are leading parties and which is the runner-up (Fey 1997; Hix, Hortala-Vallve and Riambau-Armet 2017). Furthermore, the participants' payoffs are the same as what our model assumes.

We use the data of 120 participants, who voted either in two- or three-member district elections. Each participant voted 60 times in elections with the same district magnitude, casting a single vote for one of the five parties under the CLPR with the Sainte-Laguë divisor method. There are two groups for the two-member district elections, consisting of 25 participants each, and three groups for the three-member district elections, consisting of 22, 24, and 24 participants (the number of participants varies due to no-show) as the constituency. Every five periods, preferences are randomly redrawn and participants are informed of their preferences over parties privately. After each election, the aggregate vote distribution is publicly announced. We exclude the first election of every consecutive five elections from the analysis because participants have no information to make the belief about the expected results of the election. Participants are students recruited through the online recruitment system, and the experiment took place on networked personal computers at Nuffield College, Oxford, in November 2011.

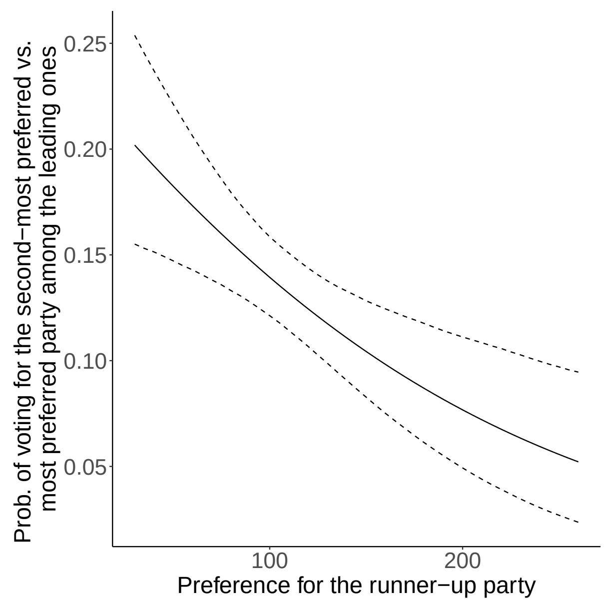
In the analysis, we exclude elections with only two competing parties for a conservative evaluation because kick-out voting cannot be distinguished from sincere voting in the two-party competition. Thus, we focus on the two-member district elections with three competing parties and the three-member district elections with three or four competing parties. Our analysis of the three-member district elections includes the number of competing parties as covariates.[11] Since participants' preferences for each party are fixed during the consecutive five elections, we cluster standard errors within the five consecu-

[11]The number of the competing parties may vary election-by-election within each group depending on the results for the last election.



Notes: The solid curve shows the point estimates of the probability of voting for the runner-up party when the preference for the least preferred party among the top $M = 2$ ones changes in two-member districts. The dashed curves show their 95% confidence intervals.

Figure 3: Kicking out the Leading Parties in the Lab



Notes: The solid curve shows the point estimates of the probability of voting for the second-most preferred against the most preferred among the top $M = 3$ parties when the preference for the runner-up party changes in three-member districts. The dashed curves show their 95% confidence intervals.

Figure 4: Kicking out the Runner-up Party in the Lab

tive elections for each participant.

3.2 Results of the Analysis of the Lab Experiment

First, we test Hypothesis 1. We use the probit regression to examine how the preference for the least preferred leading party influences the probability of voting for the runner-up. We focus on the voters who voted for one of the competing parties, and the dependent variable takes one when voters vote for the runner-up and zero when they vote for one of the leading parties. We control the effects from preferences for the runner-up and the most and the second-most preferred leading parties.

Observation 1. In the lab experiment, when the preference for the least preferred leading party decreases from the third quantile (156) to the first one (104), the probability of voting for the runner-up against one of the leading party increases by 6.3% (the 95% confidence interval is from 2.0% to 10.6%) from 27.8% to 34.1% in the two-member district elections. This observation is concordant with Hypothesis 1.

Figure 3 depicts how the probability of voting for the runner-up party against one of the leading parties changes when the preference for the least preferred leading party changes in the two-member district elections. Table 1 in Appendix D provides the details of the estimation results.

Next, we test Hypothesis 2. We use the probit regression to examine how the preference for the runner-up party influences the probability of voting for the most and the second-most preferred leading parties. To this end, we focus on the voters who voted for either one of these two parties. The dependent variable takes one when voters vote for the second-most preferred leading party and zero when they vote for the most preferred leading party. We control the effects from the preferences for the most and the second-most preferred leading candidates.

Observation 2. In the lab experiment, when the preference for the runner-up party decreases from the third quantile (130) to the first one (79), the probability of voting for the second-most preferred against the most preferred leading party increases by 3.9% (the 95% confidence interval is from 1.6% to 6.3%) from 11.7% to 15.7% in the three-member district elections. This observation is concordant with Hypothesis 2.

Figure 4 depicts how the probability of voting for the second-most preferred against the most preferred leading party changes when the preference for the runner-up party changes in the three-member district elections, and Table 2 in Appendix D provides the details of the estimation results.

These results imply that voters tend to kick out dispreferred leading candidates as predicted in Hypothesis 1 in the two-member districts, and they tend to kick out runner-ups as predicted in Hypothesis 2 in the three-member districts. Although the strength of the voters' response to the different incentives varies depending on the district magnitude, the results on the Hypothesis 3 in the Appendix E suggest that kick-out voting is generally observed irrespective of the district magnitude.

3.3 Closed-list Proportional Representation Election in Romania

The general election held in Romania on November 28, 2004, adopted the closed-list proportional representation method with small district magnitudes for the Chamber of Deputies (Marian and King 2010). There are 41 districts (constituencies) allocated four to twelve seats, other than the City of Bucharest, which is allocated 28 seats. Each voter cast a vote for a party in a district, and seats were allocated according to a variant of the Hare method (see Appendix A for the details); the resulting seat allocation was exactly the same as the Hare method in 38 out of the 41 districts.

After this election, a face-to-face survey was conducted between December 14, 2004, and January 7, 2005, as a part of the Comparative Study of Electoral Systems (Comparative Study of Electoral Systems 2015). The stratified random sampling was used and 1,913 responses were recorded (the response rate was 70%). In this survey, respondents were asked how much they (dis)like each party on a 0–10 scale. We use their responses as their preferences for each party (see Appendix B for the specific question wording). The survey also includes the question of the respondent's vote choice, which is utilized as the outcome variable in our analysis.

There are only four major parties/alliances competing for legislative seats because the high national electoral threshold prevents minor parties from winning a seat. We also regard a party as non-competing in each district when it received less than one-fifth of the Hare Quota, which results in three- or four-party competition in every district.

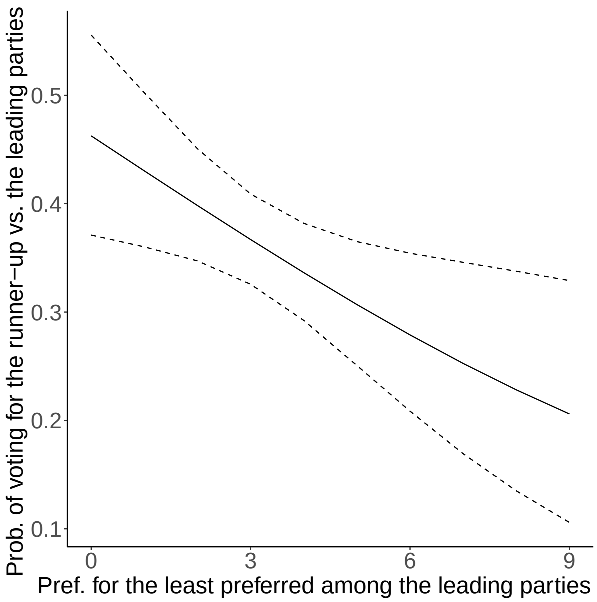
Since the formal procedure and actual election results are quite similar to the Hare method, we assume that a voter has a belief that the seat allocation among parties is the same as the Hare method in the districts.[12] To avoid confusion, we use respondents in those 38 districts where the seat allocation was the same as the Hare method.

3.4 Results of the Analysis of the CLPR Election

First, we test Hypothesis 1. We use the probit regression to examine how the preference for the least preferred leading party influences the probability of voting for the runner-up. We focus on the voters who voted for one of the competing parties, and the dependent variable takes one when voters vote for the runner-up and zero when they vote for one of the leading parties. We control district fixed effects, which eliminates the bias from all the unobserved district-level covariates, such as the number of competing parties and their popularity and policy positions. We also include control variables of the preference for the runner-up party and the most and the second-most preferred leading parties.

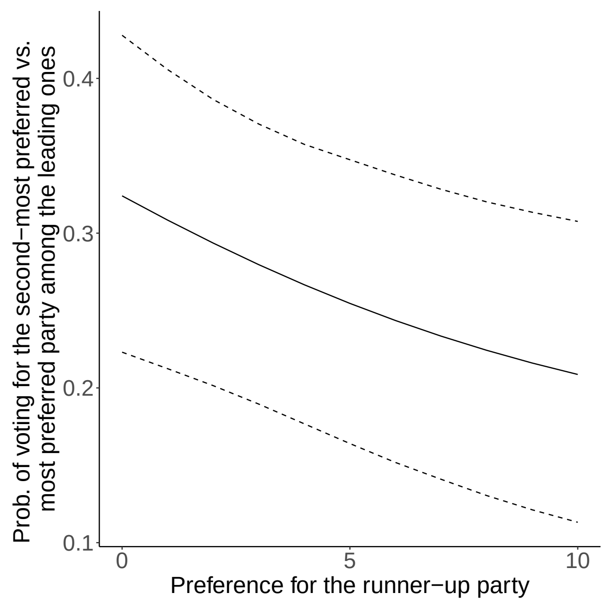
Observation 3. In the CLPR election in Romania, when the preference for the least preferred leading party decreases from the third quantile (5.0) to the first one (1.0), the prob-

[12] There is some evidence that voters can access pre-election poll results. First, newspaper stories mention opinion poll results predicting the outcome of the elections frequently. Banducci et al. (2014) shows that 6.4% of the newspapers contain opinion poll results on average in Romania, which is 18th out of 27 EU member countries and comparable to Austria (8.1%), Germany (7.4%), the Czech Republic (6.1%), and the Netherlands (5.8%). Considering that these data were collected during the European Parliament election campaign period in 2009, voters have sufficient information on pre-election poll results for national elections, which attract higher interest. Second, although anecdotal, pre-election poll results were reported during the 2004 national election campaign period in Romania (For example, see Gabriel Partos, "Close race for Romania poll rivals," *BBC News*, November 26, 2004, available at `http://news.bbc.co.uk/2/hi/europe/4042675.stm` (last accessed on March 10, 2021)).



Notes: The solid curve shows the point estimates of the probability of voting for the runner-up party when the preference for the least preferred among the top M parties changes. The dashed curves show their 95% confidence intervals.

Figure 5: Kicking out the Leading Parties in Romania



Notes: The solid curve shows the point estimates of the probability of voting for the second-most preferred against the most preferred among the top M parties when the preference for the runner-up party changes. The dashed curves show their 95% confidence intervals.

Figure 6: Kicking out the Runner-up in Romania

ability of voting for the runner-up against one of the leading party increases by 12.4% (the 95% confidence interval is from 2.7% to 22.0%) from 30.7% to 43.1%. This observation is concordant with Hypothesis 1.

Figure 5 depicts how the probability of voting for the runner-up party against one of the leading parties changes when the preference for the least preferred leading party changes. Table 3 in Appendix D provides the details of the estimation results.

Next, we test Hypothesis 2. We use the probit regression to examine how the preference for the runner-up party influences the probability of voting for the second-most preferred and the most preferred party. We focus on the voters who voted for either one of these two parties, and the binary dependent variable takes one when voters vote for the second-most preferred leading party and zero when they vote for the most preferred leading party. We control district fixed effects and effects of the preference for the most and the second-most preferred leading parties. We also include covariates for respondents' age, gender, and ethnicity.

Observation 4. In the CLPR election in Romania, when the preference for the runner-up party decreases from the third quantile (5.5) to the first one (1.0), the probability of voting for the second-most preferred against the most preferred leading party increases by 6.0% (the 95% confidence interval is from 1.3% to 10.8%) from 24.9% to 30.9%. This observation is concordant with Hypothesis 2.

Figure 6 depicts how the probability of voting for the second-most preferred against the most preferred leading party changes when the preference for the runner-up party changes. Table 4 in Appendix D provides the details of the estimation results.

3.5 Elections under the Single Non-transferable Vote System in Japan

Japanese Upper House elections adopt the SNTV system for the district election part and the open-list system with the single national district for the PR part. This study focuses on the district election part in the 2001 and 2010 elections, where each of the 47 prefectures constitutes a district and elects 1–5 Councilors depending on its population.

The 2001 and 2010 elections saw interesting competition between two major parties, the Liberal Democratic Party (LDP) and the Democratic Party of Japan (DPJ). In these elections, the larger party, the LDP in 2001 and the DPJ in 2010, ran two candidates in most of the two-member districts. We call the larger party (the LDP in 2001 and the DPJ in 2010) the *double-candidate party* and the smaller party (the DPJ in 2010 and the LDP in 2010) the *single-candidate party*. While the double-candidate party was more popular than the single-candidate party in the relevant election, the double-party candidate rarely won both of the two seats because it was difficult to have two candidates to beat the single-candidate party. Indeed, no party has won both of the two seats in the two-member districts since the 2001 election. The strongest candidate of the double-candidate party (whom we call the *strong candidate*) and the candidate of the single-candidate party won the seats and can be regarded as leading candidates. The second candidate of the double-candidate party (whom we call the *weak candidate*) is the runner-up. We expect that voters who strongly disprefer the candidate of the single-candidate party should vote for the weak candidate of the double-candidate party to kick out the candidate of the single-candidate party.

For each of these elections, pre- and post-election surveys were conducted. The pre-election face-to-face survey was conducted between July 19 and 28 and the post-election telephone survey was conducted between August 1 and 5 for the 2001 election held on July 29 as waves of the "Nation-wide Longitudinal Survey Study on Voting Behavior in the Early 21st Century" (JES III Research Project 2007). These surveys used stratified two-stage random sampling and collected 2,061 and 1,253 responses, respectively, where response rates were 68.7% and 60.8%, respectively. For the 2010 election held on July 11, the pre-election face-to-face survey was conducted between June 30 and July 10, and the post-election face-to-face survey was conducted between July 12 and August 4 as waves of the "Nation-wide Longitudinal Survey Study on Voting Behavior in an Age of Political Change" (JES IV Research Project 2016). These surveys used stratified two-stage random sampling and collected 1,767 and 1,707 responses, respectively, where response rates were 55.5% and 82.2%, respectively. In the pre-election surveys, respondents were asked to report their preferences for each candidate in their district and each of the major parties, and they were also asked to report their vote choices in the post-election surveys (see Appendix C for the specific question wording).

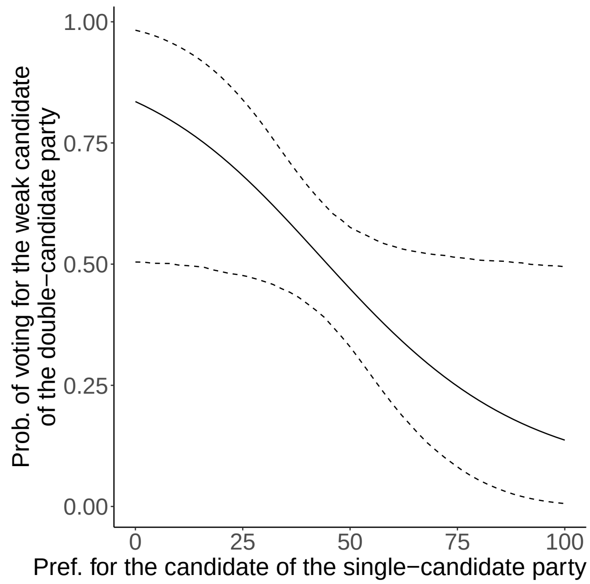
As explained above, most of the two-member districts in these SNTV elections saw a three-candidate competition of the two major parties, i.e., the strong and weak candidates of the double-candidate party and the candidate of the single-candidate party. We exploit this situation to test Hypothesis 1, i.e., whether a voter who dislikes the candidate of the single-candidate party votes for the weak candidate of the double-candidate party (the runner-up).

Unfortunately, this situation is not suitable for testing Hypothesis 2, i.e., the more strongly voters disprefer the runner-up, the more likely they vote for the second-most preferred candidate instead of the most preferred candidate. This is because voters who strongly disprefer the runner-up are supporters of the single-candidate party, and they are unlikely to vote for the strong candidate of the double-candidate party to kick out the weak candidate of that party.

We exclude districts where the two major parties did not run three candidates in total (eight districts in the 2001 election) and a candidate of a party other than the two major parties competed as one of the top three candidates (one district in the 2001 election and three districts in the 2010 election). Since voters need to know which candidate is stronger than the other in the double-candidate party, we exclude Miyagi district in the 2001 election and Gifu district in the 2010 election, where two candidates were closely competitive (the vote share ratio of the strong and weak candidates of the double-candidate party in each district was more than 0.9). We use the remaining five districts in the 2001 election and the seven districts in the 2010 election.

3.6 Results of the Analysis of the Elections under the SNTV

To test Hypothesis 1, we use the probit regression to examine how the preference for the candidate of the single-candidate party influences the probability of voting for a weaker candidate of the double-candidate party. We focus on the voters who voted for either the stronger or weaker candidate of the double-candidate party, and the dependent variable takes one when voters vote for the weak candidate and zero when they vote for the strong candidate. We control district fixed effects and effects of the preference for each of the strong and weak candidates of the double-candidate party and the preference for the



Notes: The solid curve shows the point estimates of the probability of voting for the weak against the strong candidate of the double-candidate party when the preference for the candidate of the single-candidate party changes. The dashed curves show their 95% confidence intervals.

Figure 7: Kicking out the Candidate of the Rival Party in Japan

double-candidate party itself.[13]

Observation 5. In the SNTV election in Japan, when the preference for the candidate of the single-candidate party decreases from the third quantile (50) to the first one (25), the probability of voting for the weak against the strong candidate of the double-candidate party increases by 24.5% (the 95% confidence interval is from 0.5% to 44.3%) from 43.7% to 68.2%. This observation is concordant with Hypothesis 1.

Figure 7 illustrates how the probability of voting for the weak against the strong candidate of the double-candidate party changes when the preference for the candidate of the single-candidate party changes, and the details of the estimation results are presented in Table 5 in Appendix D.

[13]For robustness checks, we also include covariates for respondents' age, gender, and political ideology, and election fixed effects, and the main results hold. See Appendix D for the detail.

For the analysis in this subsection, only a small sample was available because collecting information on voters' preference for all of the three candidates in the district was difficult: many respondents did not provide their complete preferences over candidates. To address this issue, we use the preference for the two major parties instead of that for the candidates in the robustness checks. We expect that respondents are more likely to vote for the weak candidate of the double-candidate party when they dislike the single-candidate party more strongly. Table 6 and 7 in Appendix D provide the results of the probit regression for the vote choice between the two candidates of the double-candidate party with district fixed effects. We use the preference for the single-candidate party as a proxy for the preference for the candidate of the single-candidate party in Table 6 and further drop the preference for each of the candidates of the double-candidate party in Table 7. For robustness checks, we include covariates for respondents' age, gender, and political ideology. In addition, we also incorporate election fixed effects. The main results hold the same.

3.7 Summary of the Empirical Analysis

The empirical evidence with individual-level data of voters' preferences and vote choices from various contexts is concordant with Hypotheses 1 and 2. This indicates that real-world voters are attempting to kick out less preferred candidates. The lab experiment in the United Kingdom provides detailed information on controlled electoral competition, which supports the internal validity of our theory of kick-out voting. The consistent results from the real-world elections under the CLPR in Romania and the SNTV system in Japan offer external validity of our theory.

4 Indeterminacy

In this section, we investigate the property of F that admits discriminating π as a limit of rational expectations. We show that, with a wide range of parameters, SNTV has no discriminating equilibrium. If voters believe that a discriminating equilibrium occurs, then they are tempted to vote for the runner-up to kick out a dispreferred candidate. This motivation increases the share of the runner-up, and therefore, the belief cannot be consistent with the actual voting strategies. The nonexistence of discriminating equilibria is undesirable because it implies SNTV cannot elect M most popular candidates deterministically.

We start from studying the case of the uniform distribution as a benchmark. Assume that F is a uniform distribution over $[0,1]^{M+1}$, i.e., $f(u) = 1$ for all $u \in [0,1]^{M+1}$ where f is the probability density function of F.

Since the uniform distribution is symmetric (i.e., all $M+1$ candidates are "equally strong"), we naturally expect that there is no discriminating equilibrium with such a preference distribution. We will further show that the set of preference distributions that does *not* admit discriminating equilibria is non-degenerate.

At the limit of discriminating equilibria, a voter votes for candidate $M+1$ if and only if

$$\sum_{k=1}^{M} \alpha_k (u_{M+1} - u_k) \geq \alpha_j (u_j - u_{M+1}) \quad \text{for } j = 1, \dots, M,$$

or equivalently,

$$u_{M+1} \geq \frac{\sum_{k=1}^{M} \alpha_k u_k + \alpha_j u_j}{1 + \alpha_j} \quad \text{for } j = 1, \dots, M. \tag{4}$$

When F is the uniform distribution, $\alpha_1 = \cdots = \alpha_M (= 1/M)$ is necessary and sufficient for having $\pi_1 = \cdots = \pi_M$. Using this fact, (4) reduces to

$$u_{M+1} \geq \frac{1}{M+1} \cdot \left(\sum_{k=1}^{M} u_k + u_j \right) \quad \text{for } j = 1, \dots, M.$$

Hence, at the limit of $n \to \infty$, the share of candidate $M+1$ is

$$\Pr\left(u_{M+1} \geq \frac{1}{M+1} \cdot \left(\sum_{k=1}^{M} u_k + u_j\right) \text{ for } j = 1, \dots, M\right). \tag{5}$$

Let $\bar{u} := \max_{j\in\{1,2,\dots,M\}} u_j$. Since $\bar{u}$ is the first order statistic of $u_1, \dots, u_M$, which are uniformly distributed, the probability density function of $\bar{u}$ is $f^{(1)}(\bar{u}) = M\bar{u}^{M-1}$. Then, (5) is equal to

$$\int_0^1 \Pr\left(u_{M+1} \geq \frac{1}{M+1} \cdot \left(\sum_{k=2}^{M} u_k + 2\bar{u}\right)\right) \cdot M\bar{u}^{M-1} d\bar{u}. \tag{6}$$

Conditional on $u_j \leq \bar{u}$ for all j ($2 \leq j \leq M$), u_j follows the uniform distribution over $[0, \bar{u}]$, i.i.d. across j. Accordingly, (6) is equal to

$$\int_0^1 \left(1 - \frac{(M-1)\bar{u}/2 + 2\bar{u}}{M+1}\right) \cdot M\bar{u}^{M-1} d\bar{u} = \frac{M^2+M+2}{2(M+1)^2}. \tag{7}$$

For candidate $M+1$ to be a runner-up, her share, (7), must be smaller than $1/(M+1)$; i.e., we must have

$$\frac{M^2+M+2}{2(M+1)^2} - \frac{1}{M+1} = \frac{M(M-1)}{2(M+1)^2} < 0. \tag{8}$$

However, a direct calculation shows that the left hand side of (8) is increasing in M, and for $M \geq 2$, the left hand side of (8) takes a minimum value of $1/9$ when $M = 2$. Hence, (8) does not hold because the left-hand side of (8) is positive and bounded away from 0 whenever the district has multiple seats.

This indicates the following two results. First, if F is a uniform distribution, there is no discriminating equilibria at the limit of $n \to \infty$. Second, even if we perturb F slightly, discriminating equilibria would not arise because candidate $(M+1)$'s share from the uniform distribution is bounded below by $1/(M+1)$. Therefore, the set of preference distributions (F) that admit no discriminating equilibria is not degenerate. Accordingly, SNTV (and CLPR) fails to elect M most popular candidates in equilibrium, even if there are infinitely many voters.

5 Conclusions

We theorize and empirically investigate a novel type of strategic voting in multi-member district electoral systems. Our theory predicts that voters do not necessarily vote for their most preferred competing candidate. Instead, voters sometimes vote for a candidate who is likely to be tied with the candidates they disprefer in order to kick them out. This kick-out voting sometimes contradicts sincere voting and even decreases the winning probability of the voter's most preferred candidate.

We test empirical hypotheses derived for examining our theory of kick-out voting with individual-level data of preferences and vote choice based on three data sets from diverse contexts: (i) a lab experiment in the United Kingdom, (ii) a closed-list proportional representation election in Romania, and (iii) elections under the single non-transferable vote system in Japan. The results confirm our theory of kick-out voting by showing that real-world voters follow some voting patterns that are predicted by our theory but cannot be explained by sincere voting among competing candidates.

Our findings imply that runners-up get more support under kick-out voting than sincere voting. All the voters who strongly dislike some of the leading candidates may vote for the runner-up to kick out the dispreferred candidates. Accordingly, the vote bonus for runners-up tends to be large when voters' preferences are diverse because various leading candidates could be dispreferred.

This equilibrium feature may affect the quality of representation in multi-member district electoral systems in the following two senses. First, it may affect representation by changing who is represented well (Eggers and Vivyan 2020). For minority representation, in addition to lowering the hurdle by increasing the district magnitude, kick-out voting boosts support for minority candidates if they can be runners-up. In addition, kick-out voting would foster third-party entry in multi-member districts, although such districts foster third-party entry even without the presence of kick-out voting. As demonstrated in Subsection 2.3, even if a candidate is not popular at all, she might obtain a significant fraction of the vote share because voters may vote for her to kick out some dispreferred leading candidates. This may partly explain why in recent years, new challenger parties that politicize new issues that cross-cut traditional party alignment have risen successfully in Europe (De Vries and Hobolt 2020).

Second, kick-out voting may skew vote shares among candidates and possibly change the winners by increasing the vote share of the runner-up while also decreasing those of the strong leading candidates. This equalizing effect may result in the indeterminacy of winners. It also implies that kick-out voting undermines the validity of the conventional measure of electoral (dis)proportionality between seat and vote shares, such as the Gallagher index (Gallagher 1991). Instead of measuring indirectly by parties' vote shares, we may need to measure voters' preferences directly through survey questions.

One of the limitations of this study is that we fix voters' preference over candidates and do not consider their endogenous strategy to seek electoral support in our theoretical model. However, the asymmetric competition between leading candidates and the runner-up highlighted in this study suggests that their strategies may also diverge. As Myerson (1993) shows, for elections under sincere voting, the best strategy for the leading candidates under kick-out voting should be cultivating narrow but strong support. By contrast, we expect that the best strategy for the runner-up would be enhancing weak but broad support because she would get votes from various voters who dislike some of the leading competitors. We also expect that it would become similar to that of the leading competitors as the runner-up increases their popularity. We are currently developing a formal theory of the endogenous candidate strategy under kick-out voting and testing hypotheses to examine it empirically.

References

Abramson, Paul R., John H. Aldrich, André Blais, Matthew Diamond, Abraham Diskin, Indridi H. Indridason, Daniel J. Lee and Renan Levine. 2010. "Comparing Strategic Voting Under FPTP and PR." *Comparative Political Studies* 43(1):61–90.

Arrington, Theodore S. and Gerald L. Ingalls. 1998. "The Limited Vote Alternative to Affirmative Districting." *Political Geography* 17(6):701–728.

Artabe, Alaitz and Javier Gardeazabal. 2014. "Strategic Votes and Sincere Counterfactuals." *Political Analysis* 22(2):243–257.

Banducci, Susan, Claes H. de Vreese, Holli A. Semetko, Hajo G. Boomgarden, Maarja Luhiste, Jochen Peter, Andreas Schuck and Georgios Xezonakis. 2014. "European Parliament Election Study, Longitudinal Media Study 1999, 2004, 2009." GESIS Data Archive, Cologne. ZA5178 Data file Version 1.0.0, https://doi.org/10.4232/1.5178.

Bargsted, Matias A. and Orit Kedar. 2009. "Coalition-Targeted Duvergerian Voting: How Expectations Affect Voter Choice under Proportional Representation." *American Journal of Political Science* 53(2):307–323.

Blais, André, John H. Aldrich, Indridi H. Indridason and Renan Levine. 2006. "Do Voters Vote For Government Coalitions?: Testing Downs' Pessimistic Conclusion." *Party Politics* 12(6):691–705.

Bowler, Shaun, Jeffrey A. Karp and Todd Donovan. 2010. "Strategic Coalition Voting: Evidence from New Zealand." *Electoral Studies* 29(3):350–357.

Carey, John M and Simon Hix. 2011. "The Electoral Sweet Spot: Low-Magnitude Proportional Electoral Systems." *American Journal of Political Science* 55(2):383–397.

Catalinac, Amy. 2016. "From Pork to Policy: The Rise of Programmatic Campaigning in Japanese Elections." *Journal of Politics* 78(1):1–18.

Cho, Seok-Ju. 2014. "Voting Equilibria Under Proportional Representation." *American Political Science Review* 108(2):281–296.

Comparative Study of Electoral Systems. 2015. "CSES Module 2 Full Release." https://doi.org/10.7804/cses.module1.2015-12-15.

Cox, Gary W. 1994. "Strategic Voting Equilibria Under the Single Nontransferable Vote." *American Political Science Review* 88(3):608–621.

Cox, Gary W. 1997. *Making Votes Count: Strategic Coordination in the World's Electoral Systems*. Cambridge: Cambridge University Press.

Cox, Gary W. and Matthew Soberg Shugart. 1996. "Strategic Voting under Proportional Representation." *Journal of Law, Economics, & Organization* 12(2):299–324.

De Vries, Catherine E. and Sara B. Hobolt. 2020. *Political Entrepreneurs: The Rise of Challenger Parties in Europe*. Princeton University Press.

Duch, Raymond M., Jeff May and David A. Armstrong. 2010. "Coalition-directed Voting in Multiparty Democracies." *American Political Science Review* 104(4):698–719.

Duverger, Maurice. 1954. *Political Parties : Their Organization and Activity in the Modern State*. New York: Wiley.

Eggers, Andrew C. and Nick Vivyan. 2020. "Who Votes More Strategically?" *American Political Science Review* 114(2):470–485.

Fey, Mark. 1997. "Stability and Coordination in Duverger's Law: A Formal Model of Preelection Polls and Strategic Voting." *American Political Science Review* 91(1):135–147.

Gallagher, Michael. 1991. "Proportionality, Disproportionality and Electoral Systems." *Electoral Studies* 10(1):33–51.

Hix, Simon, Rafael Hortala-Vallve and Guillem Riambau-Armet. 2017. "The Effects of District Magnitude on Voting Behavior." *Journal of Politics* 79(1):356–361.

JES III Research Project. 2007. "Nation-wide Longitudinal Survey Study on Voting Behavior in the Early 21st Century (JES III SSJDA version), 2001–2005.".

JES IV Research Project. 2016. "Nation-wide Longitudinal Survey Study on Voting Behavior in an Age of Political Change (JES IV SSJDA version), 2007–2011.".

Lijphart, Arend, Rafael López Pintor and Yasunori Sone. 1986. The Limited Vote and the Single Nontransferable Vote: Lessons from the Japanese and Spanish Examples. In *Electoral Laws and Their Political Consequences*, ed. Bernard Grofman and Arend Lijphart. New York: Agathon Press pp. 154–169.

Marian, Cosmin Gabriel and Ronald F. King. 2010. "Plus ça Change: Electoral Law Reform and the 2008 Romanian Parliamentary Elections." *Communist and Post-Communist Studies* 43(1):7–18.

Myerson, Roger B. 1993. "Incentives to Cultivate Favored Minorities Under Alternative Electoral Systems." *American Political Science Review* 87(4):856–869.

Rosenbluth, Frances McCall and Michael F. Thies. 2010. *Japan Transformed: Political Change and Economic Restructuring*. Princeton University Press.

Sartori, Giovanni. 1968. Political Development and Political Engineering. In *Public Policy*, ed. J. D. Montgomery and A. O. Hirschman. Cambridge: Harvard University Press pp. 261–298.

Appendix

A Details of Seat Allocation in the 2004 Romanian Election

In the 2004 Romanian general election, seat calculation proceeds in the district stage with the Hare method and the national stage with the d'Hondt method. Suppose that party i gets v_{ij} votes in district j where s_j seats are allocated. First, a Hare Quota q_j is calculated in each district by dividing the sum of valid votes for qualified parties/alliances achieving the national electoral threshold by the number of seats allocated to the district ($q_j = s_j / \sum_{i \in l_{ij}} v_{ij}$, where l_{ij} is a set of valid parties in district j). Then, the valid votes for each party are divided by this Hare Quota (v_{ij}/q_j), producing integer and remainder parts. Each party wins the number of seats equal to its integer s_{ij} and the remainder of votes $r_{ij} = v_{ij} - q_j s_{ij}$ are aggregated at the national level.

At the national level, aggregated unused votes for each party $r_i = \sum_j r_{ij}$ are divided sequentially by $1, 2, \ldots, s'$, where s' is the total number of the remaining seats ($s' = \sum_j s_j - \sum_i \sum_j s_{ij}$). Then, the party with the highest unawarded quotient is awarded one additional seat successively until all the remaining seats are allocated, resulting in s'_i seats for party i.

After the calculation of the total remaining seats allocated to each party, they are allocated to district candidates to determine which candidates of the party are elected. In this calculation, the ranking score x_{ij} is used, which is calculated as the number of remaining seats for party i multiplied by unused votes for the party in district j divided by aggregated unused votes of the party $x_{ij} = s'_i r_{ij} / r_i$, which represents the ranking of the priority to allocating seats to party i in district j. In descending order of the score, a seat is allocated to party i in district j if the remaining seats for the party s'_i and the remaining seats for district j remain, where x_{ij} is skipped when either of these two remaining seats is filled up and the allocation is finished when both of these two are filled-up.

As the formal characteristics of the d'Hondt method, we, and voters in the election, expect that the seat allocation is *proportional* to the remaining votes, thereby the seat-vote ratios s'_i/r_i are almost the same for the major parties (in the 2004 election, they are 5.15, 5.02, and 5.30 for three major parties and 4.00 for the smallest major party). This implies that the ranking score is almost proportional to the remaining vote $x_{ij} \approx r_{ij}k$, where k is a constant representing the seat-vote ratio s'_i/r_i. Thus, seats are allocated in district j in descending order of the remaining votes r_{ij} almost surely, which in turn is the same mechanism as the Hare method.

B Question Wording in the Election Survey in Romania

Preference for each party "I'd like to know what you think about each of our political parties. After I read the name of a political party, please rate it on a scale from 0 to 10, where 0 means you strongly dislike that party and 10 means that you strongly like that party. If I come to a party you haven't heard of or you feel you do not know enough about, just say so. The first party is PARTY A."

Party list

PARTY A Social Democratic Party (PSD)

PARTY B National Liberal Party (PNL)

PARTY C Democratic Party (PD)

PARTY D Greater Romania Party (PRM)

PARTY E Democratic Alliance of Hungarians in Romania (UDMR)

PARTY F Humanist Party of Romania (PUR)

PARTY G New Generation Party (PNG)

PARTY H Christian Democratic National Peasants' Party (PNTCD)

- National Alliance: Social Democratic Party + Humanist Party of Romania
- Truth and Justice Alliance: National Liberal Party + Democratic Party

Preferences for electoral alliances are calculated as the mean preferences for member parties. Respondents who responded as "haven't heard of" or "don't know" or refused to respond are re-coded as missing.

C Question Wording in the Election Survey in Japan

Preference for each candidate First, I would like to ask you some questions concerning those people who field their candidature in your electoral district.

1. Are there any names on this list that you know?
   - Yes
   - No
2. How much do you know about this candidate?
   - Know a lot
   - Know a little
   - Know only the candidate's name
3. (Ask to those who answered: "know a lot" or "know a little") Please express your feeling toward the candidate in terms of temperature (50–100°C).

Preference for each party What are your feelings toward people and political parties that are influential in politics? Please express your likes and dislikes in terms of temperature. If you feel completely neutral about the person or the political party on the list, indicate this neutral feeling by giving them a rating of 50°C. If you like the person or political party listed, rate them between 50 to 100°C depending on how positively you feel. Conversely, if you dislike the person or party, rate them 0 to 50°C depending on how negatively you feel.

1. Liberal Democratic Party
2. Democratic Party of Japan

D Details of the Results for the Empirical Analyses

Table 1: Kicking out the Least Preferred Leading Parties in the Lab

	Model C1	Model C2	Model C3	Model C4
District magnitude (M)	2	3	2	3
Pref. for the runner-up party	2.096	2.188	2.057	2.285
	(0.194)	(0.132)	(0.201)	(0.159)
Pref. for the most preferred among the leading parties	-1.652	-1.783	-1.381	-1.716
	(0.210)	(0.201)	(0.232)	(0.225)
Pref. for the second-most preferred among the leading parties		-0.376		-0.334
		(0.314)		(0.342)
Pref. for the least preferred among the leading parties	**-0.566**	**0.149**	**-0.672**	**-0.094**
	(0.203)	**(0.285)**	**(0.223)**	**(0.310)**
Number of observations	1200	2270	797	1656
Number of clusters	552	799	464	739

Notes: Coefficients and standard errors are multiplied by 100. The outcome variable is voting for the runner-up against one of the leading parties excluding the runner-up. The mth-most preferred party among the leading ones excludes the runner-up. The models for $M = 3$ include the number of competing parties as covariates. The samples for Models C3 and C4 exclude voters who vote for the Mth-place party. Cluster-robust standard errors in parentheses.

Table 2: Kicking out the Runner-up Party in the Lab

	Model B1	Model B2
District magnitude (M)	2	3
Pref. for the most preferred among the leading parties	-1.691	-2.373
	(0.303)	(0.279)
Pref. for the second-most preferred among the leading parties	0.976	1.829
	(0.281)	(0.280)
Pref. for the runner-up party	**-0.086**	**-0.389**
	(0.203)	**(0.117)**
Nmuber of observations	818	1636
Number of clusters	470	719

Notes: Coefficients and standard errors are multiplied by 100. The outcome variable is voting for the second-most preferred against most preferred party among the leading ones. The model for $M = 3$ includes the number of competing parties as covariates. Cluster-robust standard errors in parentheses.

Table 3: Kicking out the Least Preferred Leading Parties in Romania

	Model C1	Model C2	Model C3	Model C4
Pref. for the runner-up party	5.757	5.551	5.661	5.439
	(0.455)	(0.468)	(0.502)	(0.521)
Pref. for the most preferred among the leading parties	-4.306	-4.255	-3.839	-3.871
	(0.482)	(0.495)	(0.552)	(0.589)
Pref. for the second-most preferred among the leading parties	0.574	0.730	-0.088	0.144
	(0.993)	(1.047)	(1.048)	(1.129)
Pref. for the least preferred among the leading parties	**-2.460**	**-2.499**	**-2.269**	**-2.330**
	(0.970)	**(1.031)**	**(0.993)**	**(1.082)**
District fixed effects	✓	✓	✓	✓
Covariates		✓		✓
Number of observations	903	861	691	663

Notes: Coefficients and standard errors are multiplied by 100. The outcome variable is voting for the runner-up against one of the leading parties excluding the runner-up. The mth-most preferred party among the leading ones excludes the runner-up. The samples for Models C3 and C4 exclude voters who vote for the Mth-place party. Covariates include respondents' age, gender, and ethnicity. Standard errors in parentheses.

Table 4: Kicking out the Runner-up Party in Romania

	Model B1	Model B2
Pref. for the most preferred among the leading parties	-2.517 (0.668)	-2.575 (0.747)
Pref. for the second-most preferred among the leading parties	3.831 (0.659)	4.196 (0.759)
Pref. for the runner-up party	**-0.996** **(0.395)**	**-0.895** **(0.442)**
District fixed effects	✓	✓
Covariates		✓
Number of observations	594	561

Notes: Coefficients and standard errors are multiplied by 100. The outcome variable is voting for the second-most preferred against most preferred party among the leading ones. Covariates include respondents' age, gender, and ethnicity. Standard errors in parentheses.

Table 5: Kicking out the Candidate of the Rival Party in Japan

	Model 1	Model 2	Model 3	Model 4
Pref. for the candidate of the single-candidate party	**-3.929** **(1.937)**	**-4.072** **(2.309)**	**-3.824** **(1.932)**	**-4.043** **(2.351)**
Pref. for the strong candidate of the double-candidate party	1.031 (1.537)	2.508 (1.816)	1.116 (1.552)	2.694 (1.850)
Pref. for the weak candidate of the double-candidate party	3.894 (1.547)	4.403 (2.192)	3.864 (1.552)	4.313 (2.209)
Pref. for the double-candidate party	-2.608 (1.504)	-3.901 (1.956)	-2.562 (1.504)	-3.665 (1.953)
District fixed effects	✓	✓	✓	✓
Election fixed effects			✓	✓
Covariates		✓		✓
Number of observations	46	42	46	42

Notes: Coefficients and standard errors are multiplied by 100. The sample consists of voters who vote for either of the candidates of the double-candidate party. Covariates include respondents' age, gender, and political ideology. Standard errors in parentheses.

Table 6: Kicking out the Candidate of the Rival Party in Japan: Robustness Checks 1

	Model 5	Model 6	Model 7	Model 8
Pref. for the single-candidate party	**-3.321**	**-3.374**	**-3.295**	**-3.423**
	(1.298)	**(1.480)**	**(1.303)**	**(1.488)**
Pref. for the strong candidate	0.281	1.719	0.372	1.911
of the double-candidate party	(1.296)	(1.506)	(1.302)	(1.523)
Pref. for the weak candidate	3.425	3.116	3.440	3.117
of the double-candidate party	(1.378)	(1.546)	(1.398)	(1.565)
Pref. for the double-candidate party	-1.767	-2.334	-1.706	-2.103
	(1.201)	(1.515)	(1.2)	(1.512)
District fixed effects	✓	✓	✓	✓
Election fixed effects			✓	✓
Covariates		✓		✓
Number of observations	61	57	61	57

Notes: Coefficients and standard errors are multiplied by 100. The sample consists of voters who vote for either of the candidates of the double-candidate party. Covariates include respondents' age, gender, and political ideology. Standard errors in parentheses.

Table 7: Kicking out the Candidate of the Rival Party in Japan: Robustness Checks 2

	Model 9	Model 10	Model 11	Model 12
Pref. for the single-candidate party	**-1.204**	**-1.069**	**-1.127**	**-0.986**
	(0.577)	**(0.603)**	**(0.581)**	**(0.608)**
Pref. for the double-candidate party	0.264	0.412	0.289	0.487
	(0.523)	(0.578)	(0.524)	(0.581)
District fixed effects	✓	✓	✓	✓
Election fixed effects			✓	✓
Covariates		✓		✓
Number of observations	184	175	184	175

Notes: Coefficients and standard errors are multiplied by 100. The sample consists of voters who vote for either of the candidates of the double-candidate party. Covariates include respondents' age, gender, and political ideology. Standard errors in parentheses.

E Additional Hypothesis Testing

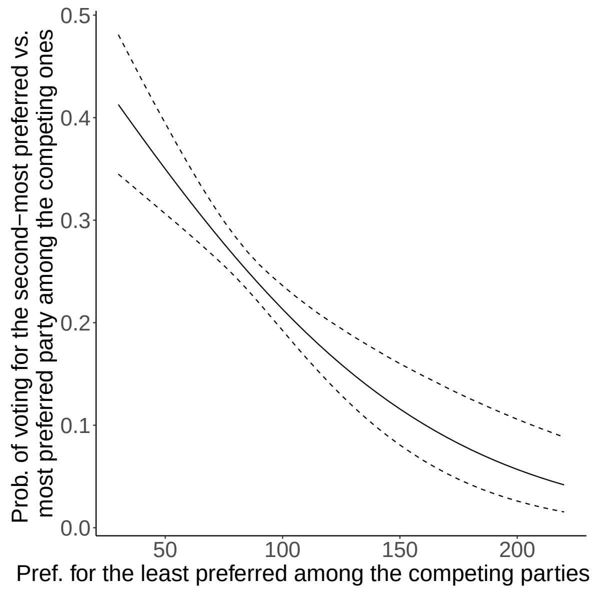
In this section, we test an additional hypothesis derived from Hypotheses 1 and 2. Hypothesis 1 is about the vote choice between the leading candidates and the runner-up and Hypothesis 2 is about the vote choice among the leading candidates. The hypothesis examined in this section is about the vote choice among competing candidates, which consist of the leading candidates and the runner-up.

Hypothesis 3 (Kicking Out a Competing Candidate)**.** If voters dislike some of the competing candidates more strongly, then they are more likely to vote for the second-most preferred candidate.

Like hypotheses tested in the main text, this hypothesis is also validated if (some) voters attempt to kick out less preferred candidates and rejected if they vote sincerely (among the competing candidates). In a sense, testing this hypothesis is redundant because we have already tested two more detailed hypotheses. Nevertheless, an additional hypothesis testing is still valuable in the sense that Hypothesis 3 can be derived from weaker assumptions on the voters' belief on the ranking of the competing candidates. To test this hypothesis, we only need to assume that voters are able to distinguish the competing candidates from the trailing candidates, which is easier than distinguishing the runner-up from the leading candidates. Thus, confirming this hypothesis with weaker assumptions lends additional support to our theory of kick-out voting. We test this hypothesis with the lab experiment data set from the United Kingdom and survey data from Romania.

E.1 Lab Experiment in the United Kingdom

We use the probit regression to examine how the preference for the runner-up party influences the vote choice over the most and second-most preferred competing parties. We focus on the voters who voted for either one of these two parties, and the dependent variable takes one when voters vote for the second-most preferred competing party and



Notes: The solid curve shows the point estimates of the probability of voting for the second-most preferred against the most preferred among the competing parties when the preference for the least preferred party changes in three-member districts and the dashed curves show their 95% confidence intervals.

Figure 8: Kicking out the Least Preferred Competing Parties in the Lab

zero when they vote for the most preferred competing party. We control the effects of the preference for the most and the second-most preferred competing parties.

Observation 6. In the lab experiment, when the preference for the least preferred party decreases from the third quantile (106) to the first one (60), the probability of voting for the second-most preferred against the most preferred party increases by 12.0% (the 95% confidence interval is from 7.4% to 16.4%) from 19.9% to 32.0% in the three-member district elections. This observation is concordant with Hypothesis 3.

Figure 8 depicts how the probability of voting for the second-most preferred against the most preferred competing party changes when the preference for the least preferred party changes in the three-member district elections. The details of the estimation results are provided in Table 8, which also shows that effects are similar in both two- and three-member districts.

Table 8: Kicking out the Least Preferred Competing Parties in the Lab

	Model A1	Model A2
District magnitude (M)	2	3
Pref. for the most preferred	-1.546	-0.636
among the competing parties	(0.239)	(0.096)
Pref. for the second-most preferred	1.782	2.231
among the comepting parties	(0.283)	(0.243)
Pref. for the least preferred	**-0.680**	**-0.968**
among the competing parties	**(0.214)**	**(0.202)**
Number of observations	1152	2092
Number of clusters	541	787

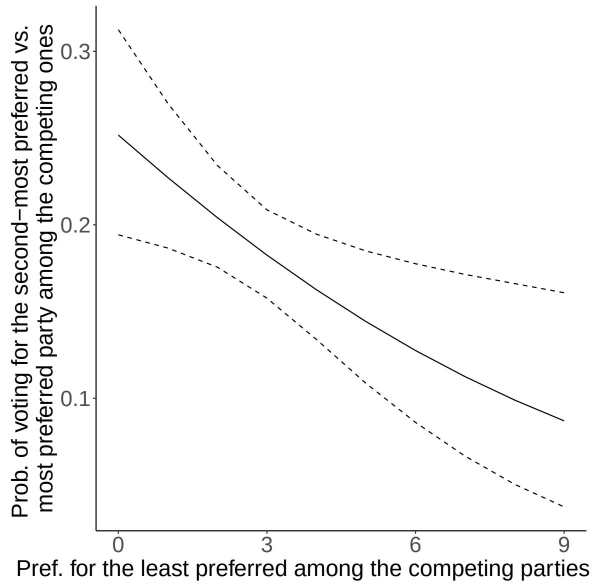
Notes: Coefficients and standard errors are multiplied by 100. The outcome variable is voting for the second-most preferred against most preferred party among the competing ones. The model for $M = 3$ includes the number of competing parties as covariates. Cluster-robust standard errors in parentheses.

E.2 CLPR in Romania

We use the probit regression to examine how the preference for the runner-up party influences the vote choice over the most and second-most preferred competing parties. We focus on the voters who voted for either one of these two competing parties, and the dependent variable takes one when voters vote for the second-most preferred competing party and zero when they vote for the most preferred competing party. We control for the district fixed effects and the preference for the most and the second-most preferred competing parties and respondents' age, gender, and ethnicity.

Observation 7. In the CLPR election in Romania, when the preference for the least preferred party decreases from the third quantile (4) to the first one (0), the probability of voting for the second-most preferred against the most preferred party increases by 9.0% (the 95% confidence interval is from 2.1% to 16.2%) from 16.2% to 25.2%. This observation is concordant with Hypothesis 3.

Table 9 provides the details of the estimation results, and Figure 9 depicts how the



Notes: The solid curve shows the point estimates of the probability of voting for the second-most preferred party against the most preferred one among the competing ones when the preference for the least preferred party changes and the dashed curves show their 95% confidence intervals.

Figure 9: Kicking out the Least Preferred Competing Parties in Romania

Table 9: Kicking out the Least Preferred Competing Parties in Romania

	Model A1	Model A2
Pref. for the most preferred	-2.912	-2.813
among the competing parties	(0.457)	(0.470)
Pref. for the second-most preferred	3.348	3.448
among the competing parties	(0.488)	(0.505)
Pref. for the least preferred	**-0.980**	**-1.085**
among the competing parties	**(0.383)**	**(0.394)**
District fixed effects	✓	✓
Covariates		✓
Number of observations	817	776

Notes: Coefficients and standard errors are multiplied by 100. The outcome variable is voting for the second-most preferred against most preferred party among the competing ones. Covariates include respondents' age, gender, and ethnicity. Standard errors in parentheses.

probability of voting for the second-most preferred against the most preferred competing party changes when the preference for the least preferred party changes.