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Assignments with Ethical Concerns

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Abstract

This study investigates an axiomatic approach to in-kind assignment problems with a single-unit demand such as triage. We consider multiple ethical criteria regarding which agents should be assigned that conflict with one another. To make compromises between criteria, we introduce two methods for configuring social choice rules that map from various problems to agents who are assigned slots: the method of procedure and the method of aggregation. From inter-problem regularities, we demonstrate characterization results, implying that the method of procedure emphasizes consistent respect for individual criteria across problems, whereas the method of aggregation emphasizes consistent respect for individual agents across problems. These methods are incompatible because only ethical dictatorships are induced by both methods. We show that the method of aggregation is superior when we can utilize detailed information about ethical concerns such as cardinality and comparability, while the method of procedure is superior when there are severe informational limitations.

JEL Classification Codes: D30, D45, D63, D71

Keywords: Triage, Multiple Criteria, Ethical Social Choice Theory, Procedure and Aggregation, Ethical Dictatorship, Informational Basis.

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1. Introduction

This study investigates assignment problems, where there exist multiple homogeneous slots and multiple agents (participants), and the number of available slots is less than that of the participants. We assume a single-unit demand in that each agent prefers a single slot to nothing, but they[3] do not need more slots. We demonstrate an axiomatic approach to characterize social choice rules (SCRs) that determine which agents are assigned slots in various assignment problems that are associated with different numbers of slots and participants, as a manifestation of distributive justice in the community.

Centrally, we consider multiple conflicting ethical concerns about who should be prioritized in slot assignment and reflect them in configuring an SCR. In particular, we even investigate situations in which there may be some need to interfere with consumer sovereignty. The quality of assignments that are involved in life, dignity, health, citizenship, poverty, discrimination, agriculture, environment, endangered species, culture, and art cannot be evaluated solely by individual agents' selfish willingness to pay. In this case, to determine priorities over agents in slot assignment, we must consider social, ethical, and sacred merits that cannot be replaced by monetary transfers and reduced to willingness to pay in an easy-to-understand manner. This study focuses on in-kind assignments *without* side payments. Matsushima (2021), which is a companion paper of this study, investigates the problems in which side payments are permitted.

In a broad sense, the perspectives and concepts related to this study are impartial observers (Smith, 1749/1969), ethical preferences (Harsanyi, 1955), merit goods (Musgrave, 1957, 1987), primary goods (Vickrey, 1960; Rawls, 1971, 1988), community preferences (Colm, 1965), specific egalitarianism (Tobin, 1970), random allocations (Weitzman, 1977), commitment (Sen, 1977), libertarian paternalism (Thaler and Sunstein, 2003, 2008), social common capital (Uzawa, 2008), and more secular concepts such as externalities and commons. Related issues that are growing concerns include measures against global warming (Schelling, 1995; Stern, 2007; Uzawa, 2008; Tirole, 2017; Sunstein and Reisch, 2013; Weizman, 2016), allocation of scarce resources (triage)

[3] To avoid gendered language altogether, this study uses "they" instead of she or he.

during wartime or pandemics (Pathak et al., 2020), affirmative action in educational opportunities (Abdulkadiroglu and Sonmez, 2003; Kamada and Kojima, 2015; Aygun and Bo, 2020), and refugee resettlement (Delacretaz et al., 2019).

This study includes issues of efficiency-equity tradeoffs where monetary transfers are restricted for some exogenous reasons and individual agents have asymmetric welfare weights (Diamond and Mirrlees, 1971; Saez and Stantcheva, 2016; Dworczak et al., 2020). As a result of considering various factors such as age, health, poverty, and potential social contribution, we might inevitably have multiple criteria (views) about how to weight between agents that conflict with one another as they would prioritize agents differently. Hence, the central issue in assignment is to make a reasonable compromise between these criteria and link it to a persuasive social decision on which participants can be assigned slots.

One important motivation for writing this paper was the recent heated debate in COVID-19 pandemic on who to preferentially assign scarce resources such as vaccines and ventilators. In real-world situations biased assignments are likely to occur, because of the tendency of a community to taboo the prioritization itself (triage in Japan), and also because of the inability of a society to agree on a single priority criterion. Therefore, it is important to elucidate the cause of such bias in axiomatic approach and use the knowledge obtained to solve a wide range of social problems. For this purpose, we consider a suitable abstract model limited to the multiunit assignment problems with a single-unit demand.

For example, suppose that the central planner wants to preferentially allocate scarce resources (or recycled materials) to producers who do not cause environmental load as much as possible. However, there are disagreements over what criteria should be used to evaluate the environmental load, such as lack of consensus on how to evaluate the impact on the future generation. We can analyze such environmental issues with the same model as pandemic issues. Another example to keep in mind is the early days of the COVID-19 pandemic, in which, the Japanese government provided all citizens with sanitary masks in-kind. It was a policy to deal with the situation in which the willingness to pay of the poor was below the market price of the mask, which has been controversial with Japanese scholars who advocate consumer sovereignty. A further example is the coexistence of different evaluation criteria for personnel affairs in a university faculty that consists of different disciplines. Since it is inappropriate for the faculty meeting to integrate these criteria into one by force, it is hoped that the faculty has an attitude of mutual respect for such potentially conflicting criteria.

Based on these background of research motives, this study focuses on whether we have consistent assignments across different assignment problems. For example, we require that the same agents are assigned slots when the number of available slots increases (Axiom 1) and when the number of participants decreases (Axiom 2). Therefore, this study defines a social choice rule (SCR) as a mapping from various assignment problems to agents who are assigned slots, and then characterizes a class of reasonable SCRs by requiring additional axioms concerning inter-problem regularities and intra-problem regularities.

To make a reasonable compromise, we consider the possibility of respecting individual criteria as they are without careless processing. Specifically, we introduce an axiom termed "*fair justification*" (Axiom 3), which requires that a SCR has three properties concerning: "which criterion is utilized to justify why an agent is assigned a slot (*respecting priorities*)," "whether an agent is justified by the same criterion even if the assignment problem changes (*invariance across problems*)," and "whether the number of assigned agents who are justified by a criterion does not change even if the assignment problem changes (*diversity in justification*)." Simply put, Axiom 3 emphasizes consistent respect for individual criteria across various assignment problems without aggregating them for convenience.

To clarify SCRs that satisfy Axioms 1, 2, and 3, this study introduces a method for compromising between criteria, which we term "*the method of procedure.*" We define a procedure as a priority order over the criteria. According to a predetermined procedure and the following multiple steps, we specify an SCR. In the first step, a slot is assigned to the top-ranked agent (participant) at the criterion that has the highest rank in the procedure. Recursively, at each step, a slot is assigned to the agent who has the highest rank among the remaining agents at the criterion that has the corresponding rank in the procedure. At each step, only a single agent was selected and assigned. We permit the same criterion to appear many times during these steps. This study shows a characterization result in that an SCR satisfies Axioms 1, 2, and 3 if and only if there exists a procedure that specifies this SCR, implying that it is induced by the method of procedure (Theorem 2).

This characterization is related to Pathak et al. (2020) but with substantial differences. Pathak et al. (2020) investigated in-kind assignment problems in triage and considered multiple ethical criteria. Adding ethical preferences of patients over the criteria for convenience and requiring property (iii) in Axiom 3 (respecting priorities), they characterized stable matching between patients and criteria as the equivalence with cut-off price equilibrium and as the equivalence with deferred acceptance algorithm. Both Pathak et al. (2020) and this study consider multiple ethical criteria and show their respective ways to find compromises between them without aggregation. However, unlike this study, Pathak et al. (2020) did not consider inter-problem regularities such as properties (i) and (ii) in Axiom 3 (invariance across problems, and diversity in justification). The consideration of such regularities is the main theme of this study.

Pathak et al. (2020) introduced the reserve system that we regard as a special case of the method of procedure. The reserve system assigns all reserves for each criterion at once according to a pre-specified priority order over the criteria. However, if we apply this system to various assignment problems, we will inevitably face some unfairness between the criteria regarding reserve achievement and eligibility constraints. Since the method of procedure is a more general concept than the reserve system, we can solve this by setting a procedure more carefully.

The method of procedure is excellent in that any of these criteria can be reflected indiscriminately across various assignment problems. Conversely, this method is inferior as it does not significantly respect individual agents. To clarify this flaw, we introduce "*agent consistency*" (Axiom 4), implying that the same agents can obtain slots when the set of participants becomes larger and the same number of slots as this increase are added. Unlike Axiom 3, Axiom 4 emphasizes consistent respect for individual agents across problems in the sense that a change of problem does not greatly affect which agents are assigned slots. We show that Axioms 3 and 4 are incompatible: only SCRs that satisfy both axioms (and Axioms 1 and 2) are *ethical-dictatorial* in the sense that slot assignment is determined according to a single criterion alone and the other criteria are always ignored (Theorem 4). Hence, there is a dilemma regarding which method to use, i.e., "respect criteria? or respect agents?"

To address this dilemma, we investigate the informational basis of SCRs by considering the state of the world that includes detailed information about the criteria concerning their cardinal aspects and inter-criterion comparability. We permit an SCR to depend on the state. We argue that the method of procedure is not suitable for making good use of such detailed information. Specifically, we introduce "*comparability*" (Axiom 6), implying that if an agent is outstandingly highly evaluated by a particular criterion, they are assigned a slot. We show an impossibility result that there exists no SCR that is induced by the method of procedure and satisfies this comparability (Theorem 5).

Because of this failure, we introduce an alternative method that we term "*the method of aggregation.*" According to this, priority orders over agents implied by various conflicting criteria are aggregated into a single artificially created priority order over agents. This method specifies an SCR by assigning slots to the higher-ranked agents (participants) for this artificially created priority order over agents in every assignment problem. The method of aggregation is analogous to social choice theory, where conflicting individual agents' preferences are aggregated into a single social preference, and this social preference solves social decisions in various situations.

The method of aggregation is incompatible with the method of procedure: any SCR that is induced by the method of aggregation satisfies Axiom 4 but not Axiom 3, except for ethical dictatorships. Unlike the method of procedure, however, the method of aggregation can successfully induce nontrivial SCRs that satisfy Axioms 4 and 6 (agent consistency, and comparability). Conversely, if we cannot use detailed information such as cardinality and comparability, the method of aggregation fails to induce nontrivial SCRs. In fact, only SCRs that can be induced by the method of aggregation are ethical-dictatorial. This negative result can be proved in the same manner as Arrow's impossibility theorem (Arrow, 1951).

To summarize:

1) This study investigates in-kind multiunit assignment problems with single-unit demand.
2) We consider multiple conflicting ethical criteria concerning priority over agents.
3) We introduce two methods to compromise between conflicting criteria and configure SCRs: the method of procedure and the method of aggregation.

4) We axiomatize these methods from the viewpoint of inter-problem regularities. Our characterization results imply that the method of procedure emphasizes consistent respect for individual criteria across problems, while the method of aggregation emphasizes consistent respect for individual agents across problems.

5) These methods are incompatible. Only ethical dictatorships can be induced by both methods.

6) The method of aggregation is superior to the method of procedure when we can utilize detailed information such as cardinality and comparability, while the method of procedure is superior when there are severe informational limitations.

The remainder of this paper is organized as follows. Section 2 presents the basic model for assignment problems. Section 3 shows that with Axioms 1 and 2, any SCR is induced by an artificially created procedure (Theorem 1). Section 3 demonstrates the method of procedure and shows a characterization theorem (Theorem 2) by requiring Axiom 3 (fair justification). Section 3 further discusses the eligibility constraints, where we introduce two perspectives on how to generalize SCRs and show the importance of provisional engagement in slot assignment.

Section 4 describes the method of aggregation. By introducing Axiom 4 (agent consistency), we characterize the SCRs induced by the method of aggregation (Theorem 3). Section 4 explains the incompatibility between the two methods and shows an impossibility theorem implying ethical dictatorship (Theorem 4).

Section 5 introduces the state space and investigates the informational basis of state-dependent social choice rules (SSCRs). We show that there exist nontrivial SSCRs that are induced by the method of aggregation and satisfy comparability, while any SSCR that is induced by the method of procedure fails to satisfy comparability. Section 6 concludes.

2. Assignment Problem

Let $N = \{1, ..., n\}$ denote a finite set of agents, where $n \geq 3$. Let a nonempty subset of agents $I \subset N$ denote the set of participants. Let a positive integer q denote the number of available slots (i.e., units of a single commodity). The *assignment problem* is defined as (I,q), where $q \le |I|$. Let X denote the set of all the assignment problems.

The SCR is defined as $C: X \to 2^N$, where $C(I,q) \subset I$ and $|C(I,q)| = q$. The central planner must prepare an SCR as a countermeasure before the assignment problem actually occurs. An SCR C determines which participants are assigned slots in various assignment problems: any agent in $C(I,q)$ obtains a single slot, whereas any agent who is not in $C(I,q)$ obtains nothing. We introduce two basic axioms for an SCR C.

Axiom 1: For every $(I,q) \in X$ and $i \in I$,

$$[\, i \in C(I,q) \,] \Rightarrow [\, i \in C(I,q+1) \,].$$

Axiom 2: For every $(I,q) \in X$, $i \in I$, and $j \in I \setminus \{i\}$,

$$[\, i \in C(I,q) \,] \Rightarrow [\, i \in C(I \setminus \{j\}, q) \,].$$

Axiom 1 implies that the same agents can obtain slots when the number of available slots increases. Axiom 2 implies that the same agents can obtain slots when the set of participants becomes smaller. Since both axioms are quite reasonable, this study will focus on SCRs that satisfy Axioms 1 and 2.[4]

We denote a set of *criteria* as $D = \{1, 2, ..., \bar{d}\}$. We denote a *priority order over agents at each criterion* $d \in D$ using a one-to-one mapping $\pi_d : \{1,...,n\} \to N$. Some examples are orders of willingness to pay multiplied by welfare weights, income order, age order, orders of degree of diseases, hybrids of these orders, and others. For each $h \in \{1,...,n\}$, agent $\pi_d(h) \in N$ has the h^{th} rank of criterion $d \in D$. For simplicity, we assume strict ordering over agents to eliminate tie-breaking cases.[5] We denote the profile of these priority orders as $\pi = (\pi_d)_{d \in D}$.

[4] Axiom 2 excludes SCRs such as the Borda rule (Borda, 1781; Maskin, 2020). The Borda rule uses each participant's priority order over potential agents, instead of over actual participants, which causes the contradiction with Axiom 2.

[5] Because of this assumption, we do not handle some categories such as gender differences without spectrum considerations. However, this study does not depend on it;

3. Method of Procedure

We first demonstrate a basic characterization of SCRs that satisfy Axioms 1 and 2. We then introduce the *method of procedure* to configure SCRs $\boldsymbol{C}$ that are associated with a pre-existing combination of a set of criteria and a profile of priority orders over agents $(\boldsymbol{D},\boldsymbol{\pi})$.

3.1. Basic Characterization

We denote a priority order over the criteria by $\gamma:\{1,2,...,z\}\to \boldsymbol{D}$, where $z\geq \boldsymbol{n}$. (In all parts of this study, except for Subsection 3.3, we consider the case in which $z=\boldsymbol{n}$.) We define a *procedure* in this subsection as a combination of a set of criteria, a profile of priority orders over agents, and a priority order over criteria, which is denoted by $\Gamma=(\boldsymbol{D},\boldsymbol{\pi},\gamma)$.

A procedure Γ uniquely determines an SCR, which is denoted by $\boldsymbol{C}^{\Gamma}$, according to the following steps. Consider an arbitrary assignment problem $(\boldsymbol{I},\boldsymbol{q})\in \boldsymbol{X}$. In step 1, the top-ranked agent at criterion $\gamma(1)\in \boldsymbol{D}$ among $\boldsymbol{I}$ is selected. This agent is denoted by $\boldsymbol{i}(1)\in \boldsymbol{I}$. At each step $\boldsymbol{k}\in\{2,...,\boldsymbol{q}\}$, the top-ranked agent at criterion $\gamma(\boldsymbol{k})\in \boldsymbol{D}$ among the set of remaining participants $\boldsymbol{I}\setminus\{\boldsymbol{i}(1),...,\boldsymbol{i}(\boldsymbol{k}-1)\}$ is selected. This agent is denoted by $\boldsymbol{i}(\boldsymbol{k})\in \boldsymbol{I}\setminus\{\boldsymbol{i}(1),...,\boldsymbol{i}(\boldsymbol{k}-1)\}$. We then define $\boldsymbol{C}^{\Gamma}$ by:

$$\boldsymbol{C}^{\Gamma}(\boldsymbol{I},\boldsymbol{q})=\{\boldsymbol{i}(1),...,\boldsymbol{i}(\boldsymbol{q})\}\quad\text{for all}\quad(\boldsymbol{I},\boldsymbol{q})\in \boldsymbol{X}.$$

Note that for each $\boldsymbol{k}\in\{1,...,\boldsymbol{n}\}$, the corresponding agent $\boldsymbol{i}(\boldsymbol{k})\in \boldsymbol{N}$ is selected and assigned a slot based on the corresponding criterion $\gamma(\boldsymbol{k})\in \boldsymbol{D}$ in any assignment problem $(\boldsymbol{I},\boldsymbol{q})\in \boldsymbol{X}$, provided that $\boldsymbol{i}\in \boldsymbol{I}$ and $\boldsymbol{k}\leq \boldsymbol{q}$. We interpret a priority order over

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we can simply add arbitrary strict orderings (age order, for example), which eliminates tie-breaking cases.

criteria γ as a device to minimize biases regarding which criteria are used to justify assigned agents across various problems.[6]

The following theorem states that an SCR satisfies Axioms 1 and 2 if and only if it can be induced by an artificially created procedure.

Theorem 1: *An SCR* C *satisfies Axioms 1 and 2 if and only if there exists* $\Gamma = (D, \pi, \gamma)$ *such that* $C = C^{\Gamma}$.

Proof: Consider an arbitrary procedure $\Gamma = (D, \pi, \gamma)$. Clearly, C^{Γ} satisfies Axiom 1: for each $I \subset N$ and $h \in \{1, ..., n\}$, the same agent $i(h) \in I$ is assigned the h^{th} slot in an assignment problem (I, q) whenever $q \geq h$. The SCR C^{Γ} also satisfies Axiom 2: any agent who has a better rank than agent $i(h)$ at criterion $\gamma(h)$ is either absent or assigned a slot before agent $i(h)$. Hence, the fact that agent $i(h)$ has the highest rank at criterion $\gamma(h)$ at the h^{th} step is unchanged after eliminating agent j, irrespective of whether agent j is assigned a slot before agent $i(h)$ or not.

Next, consider an arbitrary SCR C that satisfies Axioms 1 and 2. Let $\bar{d} = n$, i.e.,

$$D = \{1, ..., n\},$$

and specify π_1 as follows:

$$\{\pi_1(1)\} = C(N, 1),$$

and recursively, for each $k \in \{2, ..., n\}$,

$$\{\pi_1(k)\} = C(N \setminus \{\pi_1(1), ..., \pi_1(k-1)\}, 1).$$

From Axiom 1, for each $d \in \{2, ..., n\}$, we can recursively specify π_d as follows:

$$\pi_d(1) = \pi_1(1),\ \ \pi_d(2) = \pi_2(2),\ \ldots,\ \ \pi_d(d-1) = \pi_{d-1}(d-1),$$

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[6] What the specification of C^{Γ} and the serial dictatorship (Luce and Raiffa, 1957; Satterthwaite and Sonnenschein, 1981; Abdulkadiroglu and Sonmez, 1998; Piccione and Rubinstein, 2007) have in common is that priority is given in order, but these are essentially different. Unlike the serial dictatorship, the steps for specifying C^{Γ} are not avaricious: these steps give each criterion chances of priority many times, but in each chance only the priority to select one agent is allowed. We have the same point of difference from the reserve system in Pathak et al. (2020). See Subsection 3. 3.

$$\{\pi_d(d)\} = C(N,d) \setminus \{\pi_1(1),...,\pi_{d-1}(d-1)\},$$

and for each $k \in \{d+1,...,n\}$,

$$\{\pi_d(k)\} = C(N \setminus \{\pi_d(d),...,\pi_d(k-1)\},d) \setminus \{\pi_1(1),...,\pi_{d-1}(d-1)\}.$$

Based on the specified (D,π), we specify γ to satisfy

$$\gamma(d) = d \quad \text{for all} \quad d \in \{1,...,n\}.$$

Hence, we have specified $\Gamma = (D,\pi,\gamma)$.

We show $C = C^{\Gamma}$ as follows. Suppose that $C \neq C^{\Gamma}$. From Axiom 1, there exist $(I,q) \in X$ and $(k,k') \in \{q,...,n-1\}^2$ such that $k' < k$, $\pi_q(k') \in I$, $\pi_q(k') \notin C(I,q)$, and $\pi_q(k) \in C(I,q)$. From Axiom 2, we have

$$C(C(I,q) \cup \{\pi_q(k')\},q) = C(I,q),$$

that is,

$$\pi_q(k') \notin C(C(I,q) \cup \{\pi_q(k')\},q).$$

However, from the specification of π and Axiom 2, we have

$$\pi_q(k') \in C(N \setminus \{\pi_q(q),...,\pi_q(k'-1)\},q).$$

Since

$$C(I,q) \cup \{\pi_q(k')\} \subset N \setminus \{\pi_q(q),...,\pi_q(k'-1)\},$$

it follows from Axiom 2 that

$$\pi_q(k') \in C(C(I,q) \cup \{\pi_q(k')\},q),$$

which is a contradiction.

Q.E.D.

Any SCR that satisfies Axioms 1 and 2 can be induced by a common priority order over criteria γ for various problems, by making up multiple criteria and their priority orders over agents (D,π) appropriately, and artificially. This following example is helpful to understand Theorem 1.

Example 1: Assume $n = 3$. Consider an SCR C given by

$$C(\{1,2,3\},1) = \{3\}, \quad C(\{1,2,3\},2) = \{2,3\}, \quad C(\{1,2\},1) = \{1\},$$

$$C(\{1,3\},1)=\{3\}, \text{ and } C(\{2,3\},1)=\{3\}.$$

See Figure 1.a. Clearly, C satisfies Axioms 1 and 2. However, C lacks transitivity, because not agent 1 but agent 2 is assigned in $(\{1,2,3\},2)$, while not agent 2 but agent 1 is assigned in $(\{1,2\},1)$. Let us specify (D,π) as follows:

$$D=\{1,2\},$$

$$\{\pi_1(1)\}=C(\{1,2,3\},1)=\{3\},\ \{\pi_1(2)\}=C(\{1,2\},1)=\{1\},$$

$$\pi_2(1)=\pi_1(1)=3, \text{ and } \{\pi_2(2)\}=C(\{1,2,3\},2)\setminus\{3\}=\{2\}.$$

See Figure 1.b. We then specify γ as

$$\gamma(1)=1,\ \gamma(2)=2.$$

Clearly, we have $C=C^{\Gamma}$, that is, we can rationalize such a nontransitive SCR, where we denote $\Gamma=(D,\pi,\gamma)$.

Figure 1.a: C

	{1,2,3}	{1,2}	{1,3}	{2,3}
1	{3}	{1}	{3}	{3}
2	{2,3}			

Figure 1.b: π

π_1	3	1	2
π_2	3	2	1

3.2. Justification

The procedure in the proof of Theorem 1 was artificially created to explain how an SCR can be configured. Hereafter, we shall fix (D,π) and then investigate the SCRs

$\boldsymbol{C}$ that are associated with this pre-existing $(\boldsymbol{D},\boldsymbol{\pi})$. Since $(\boldsymbol{D},\boldsymbol{\pi})$ pre-exists, we shall regard a priority order over criteria $\boldsymbol{\gamma}$, instead of $\Gamma=(\boldsymbol{D},\boldsymbol{\pi},\boldsymbol{\gamma})$, as a *procedure*. We also denote the SCR induced Γ by $\boldsymbol{C}^{\gamma}$ instead of $\boldsymbol{C}^{\Gamma}$. We term the method to configure an SCR by specifying a procedure $\boldsymbol{\gamma}$ as *the method of procedure*.[7]

To associate an SCR $\boldsymbol{C}$ with the pre-existing $(\boldsymbol{D},\boldsymbol{\pi})$, we introduce a *justification* as $\boldsymbol{\delta}=(\boldsymbol{\delta}(\boldsymbol{I},\boldsymbol{q}))_{(\boldsymbol{I},\boldsymbol{q})\in \boldsymbol{X}}$, where $\boldsymbol{\delta}(\boldsymbol{I},\boldsymbol{q}):\boldsymbol{C}(\boldsymbol{I},\boldsymbol{q})\to \boldsymbol{D}$ for each $(\boldsymbol{I},\boldsymbol{q})\in \boldsymbol{X}$. Each agent $\boldsymbol{i}\in \boldsymbol{C}(\boldsymbol{I},\boldsymbol{q})$ uses a criterion $\boldsymbol{d}=\boldsymbol{\delta}(\boldsymbol{I},\boldsymbol{q})(\boldsymbol{i})\in \boldsymbol{D}$ to explain why they are assigned in an assignment problem $(\boldsymbol{I},\boldsymbol{q})\in \boldsymbol{X}$. We introduce an axiom on $(\boldsymbol{C},\boldsymbol{D},\boldsymbol{\pi})$ regarding fairness in justification from three points of view: *respecting priorities, diversity in justification*, and *invariance across problems*.

Axiom 3 (Fair Justification): There exists a justification $\boldsymbol{\delta}$ that satisfies the following three properties:

Property (i) (Priority): For every $(\boldsymbol{I},\boldsymbol{q})\in \boldsymbol{X}$, $\boldsymbol{i}\in \boldsymbol{C}(\boldsymbol{I},\boldsymbol{q})$, and $\boldsymbol{j}\in \boldsymbol{I}\setminus \boldsymbol{C}(\boldsymbol{I},\boldsymbol{q})$,

$$\boldsymbol{\pi}^{-1}_{\boldsymbol{\delta}(\boldsymbol{I},\boldsymbol{q})(\boldsymbol{i})}(\boldsymbol{i})<\boldsymbol{\pi}^{-1}_{\boldsymbol{\delta}(\boldsymbol{I},\boldsymbol{q})(\boldsymbol{i})}(\boldsymbol{j}).$$

Property (ii) (Diversity): For every $(\boldsymbol{I},\boldsymbol{q})\in \boldsymbol{X}$ and $\boldsymbol{d}\in \boldsymbol{D}$,

$$\left|\left\{\boldsymbol{i}\in \boldsymbol{C}(\boldsymbol{I},\boldsymbol{q})\mid \boldsymbol{\delta}(\boldsymbol{I},\boldsymbol{q})(\boldsymbol{i})=\boldsymbol{d}\right\}\right|=\left|\left\{\boldsymbol{i}\in \boldsymbol{C}(\boldsymbol{N},\boldsymbol{q})\mid \boldsymbol{\delta}(\boldsymbol{N},\boldsymbol{q})(\boldsymbol{i})=\boldsymbol{d}\right\}\right|.$$

Property (iii) (Invariance): For every $(\boldsymbol{I},\boldsymbol{q})\in \boldsymbol{X}$ and $\boldsymbol{i}\in \boldsymbol{C}(\boldsymbol{I},\boldsymbol{q})$,

$$\boldsymbol{\delta}(\boldsymbol{I},\boldsymbol{q})(\boldsymbol{i})=\boldsymbol{\delta}(\boldsymbol{I},\boldsymbol{q}+1)(\boldsymbol{i}).$$

To justify why an agent is assigned to push others away, Axiom 3 recommends coherently using a single criterion that is appropriate for this agent, rather than using a makeshift mixture of conflicting criteria. Property (i) implies respecting priorities in a way that any assigned agent $\boldsymbol{i}\in \boldsymbol{C}(\boldsymbol{I},\boldsymbol{q})$ can explain why agent $\boldsymbol{i}$ was assigned to push any unassigned agent away by using criterion $\boldsymbol{\delta}(\boldsymbol{I},\boldsymbol{q})(\boldsymbol{i})$. Hence, any unassigned agent

[7] This study focuses on deterministic social choice rules because we consider avoiding frustration after the assignment is determined. As an extension of this study, we can consider a method of randomizing among multiple SCRs that this study evaluates as meaningful. An example is a randomization of the ethical dictatorships that will be addressed in Subsection 4.2, i.e., random ethical dictatorship.

$j \in I \setminus C(I,q)$ has a worse rank than the assigned agent i for this criterion $\delta(I,q)(i)$.[8]

Properties (ii) and (iii) are the main components of Axiom 3, which concern regularities across different problems. Property (ii) implies diversity in justification, that is, emphasizes a consistent respect for individual criteria across various problems, in that for each criterion $d \in D$, the number of agents who are assigned slots and justified by d is unaffected by who actually participate in the problem. This reflects that the procedure γ avoids biases due to a change in who participate and maintains the same diversity concerning which criteria are used for justification.[9] Property (ii) plays a particularly important role in this study. However, in cases where eligibility is considered, how to reconcile property (ii) with eligibility will be an important issue. We will carefully discuss this issue in Subsection 3.3.

Property (iii) implies invariance across problems, that is, another aspect of consistent respect for individual criteria across problems, in that the criterion $\delta(I,q)(i)$ that justifies an assigned agent $i \in C(I,q)$ is unchanged as the number of slots q increases. (Note from Axiom 2 that this agent i is still assigned a slot.)

From property (i), each criterion must justify the assignment in order from the highest rank (intra-problem regularity). From properties (ii) and (iii), which agents to be assigned ($C(I,q) \subset I$) must be determined by a common priority order over criteria ($\gamma : \{1,...,n\} \to N$) for different problems ($(I,q) \in X$). Owing to these properties, the following theorem states that any SCR C that satisfies Axioms 1, 2, and 3 can be induced by a procedure γ associated with the pre-existing (D,π).

Theorem 2: *An SCR C satisfies Axioms 1, 2, and 3 if and only if γ exists such that $C = C^{\gamma}$.*

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[8] Property (i) corresponds to the third definition of Pathak et al. (2020).

[9] Property (ii) permits who are assigned and justified by criterion d to be affected by who participate. See Section 4.

Proof: For every procedure γ, the corresponding SCR C^{γ} $(=C^{\Gamma})$ satisfies Axiom 3. According to the steps explained in Subsection 3.1, we can specify a justification δ^{γ} so that for each $(I,q)\in X$,

$$\delta^{\gamma}(I,q)(i(k))=\gamma(k) \text{ for all } k\in\{1,...,q\}.$$

Clearly, C^{γ} satisfies Axiom 3, where we set $\delta=\delta^{\gamma}$. From Theorem 1, C^{γ} satisfies Axioms 1 and 2.

Suppose that C satisfies Axioms 1, 2, and 3. We set an arbitrary justification δ to satisfy Axiom 3. We then specify γ as follows. From Axiom 1, we can define $i(k)\in N$ for each $k\in\{1,...,n\}$ in a recursive manner:

$$\{i(1)\}=C(N,1),$$

and for each $k\in\{2,...,n\}$,

$$\{i(k)\}=C(N,k)\setminus\{i(1),...,i(k-1)\}.$$

From Axiom 3, for each $k\in\{1,...,n\}$, we select

$$\gamma(k)=\delta(N,k)(i(k))\in D.$$

Hence, we have specified γ.

We show $C=C^{\gamma}$ as follows. Note

$$C^{\gamma}(N,k)=C(N,k) \text{ for all } k\in\{1,...,n\}.$$

Consider any arbitrary $(I,q)\in X$, where $I\neq N$. Suppose that

$$C^{\gamma}(I,q)\neq C(I,q).$$

From properties (i) and (ii) in Axiom 3, $q\geq 2$ must hold. Without loss of generality, we assume that

$$C^{\gamma}(I,q-1)=C(I,q-1).$$

From Axiom 2, we have

$$C^{\gamma}(I,q-1)\subset C^{\gamma}(I,q) \text{ and } C(I,q-1)\subset C(I,q).$$

From properties (ii) and (iii) in Axiom 3, the added agent must be justified by thecriterion $\gamma(q)$. From property (i) in Axiom 3, they must be the top-ranked agent among the set of the remaining participants at the criterion $\gamma(q)$. Hence, both $C(I,q)$ and $C^{\gamma}(I,q)$

must include the same agent in addition to $C^{\gamma}(I,q-1)=C(I,q-1)$. This is a contradiction.

Q.E.D.

Remark 1: An example is the *reserve procedure* γ^{*}, defined as follows. We regard criterion 1 as a baseline, for example, the order of willingness to pay. Any other criterion $d\in\{2,...,\bar{d}\}$ has a reserve $v_d\in\{1,...,n\}$. The reserve procedure γ^{*} secures these reserves in an equal manner irrespective of the number of available slots:

$$\gamma^{*}(1)=2,\ \gamma^{*}(2)=3,\ \ldots,\ \gamma^{*}(\bar{d}-1)=\bar{d},$$

$$\gamma^{*}(\bar{d})=2,\ \gamma^{*}(\bar{d}+1)=3,\ \ldots.,$$

where, once the reserve is filled for a criterion $d\in\{2,...,\bar{d}\}$, this criterion is excluded, and the steps continue without it. Once the reserves are filled for all criteria except the baseline (criterion 1), the steps continue to select the baseline until all slots are assigned.

Remark 2: We have multiplicities of procedure and justification for an SCR to satisfy Axiom 3. Consider (D,π) and C addressed in Example 1. We specify a justification δ by:

$$\delta(\{1,2,3\},1)(3)=1,\ \delta(\{1,2,3\},2)(2)=2,\ \delta(\{1,2\},1)(2)=2,$$

$$\delta(\{1,3\},1)(3)=1, \text{ and } \delta(\{2,3\},1)(3)=1.$$

Thus, C satisfies Axiom 3. According to the proof of Theorem 2, we specify the procedure γ by

$$\gamma(1)=\delta(\{1,2,3\},1)(3)=1 \text{ and } \gamma(2)=\delta(\{1,2,3\},2)(2)=2.$$

Clearly, we have $C=C^{\gamma}$. Alternatively, we can specify another justification $\tilde{\delta}$ by

$$\tilde{\delta}(\{1,2,3\},1)(3)=2,\ \tilde{\delta}(\{1,2,3\},2)(2)=2,\ \tilde{\delta}(\{1,2\},1)(2)=2,$$

$$\tilde{\delta}(\{1,3\},1)(3)=2, \text{ and } \tilde{\delta}(\{2,3\},1)(3)=2.$$

According to the proof of Theorem 2, we can specify another procedure $\tilde{\gamma}$ by

$$\tilde{\gamma}(1)=\tilde{\delta}(\{1,2,3\},1)(3)=2 \text{ and } \tilde{\gamma}(2)=\tilde{\delta}(\{1,2,3\},2)(2)=2.$$

Clearly, we have $C=C^{\gamma}=C^{\tilde{\gamma}}$, even if $\tilde{\delta}\neq\delta$ and $\tilde{\gamma}\neq\gamma$.

Remark 3: If we do not require properties (ii) and (iii) (inter-problem regularities), we can give a different characterization as follows. Consider an arbitrary SCR C that satisfies Axioms 1 and 2. Consider an arbitrary problem $(I,q) \in X$. For each criterion $d \in D$, we can uniquely define the cut-off price $h_d = h_d(I,q) \in \{1,...,n\}$ as the lowest rank such that for every $i \in I$,

$$i \in C(I,q) \text{ whenever } \pi_d^{-1}(i) \le h_d .$$

The SCR C satisfies property (i) in Axiom 3 if and only if for every $i \in C(I,q)$, $d \in D$ exists such that $\pi_d^{-1}(i) \le h_d$. Importantly, we can select any criterion d for agent i's justification where $\pi_d^{-1}(i) \le h_d$. This observation corresponds to Theorem 1 in Pathak et al. (2020), which characterizes the slot assignment by the cut-off price equilibrium. However, this degree of freedom in what to use for justification is not consistent with properties (ii) and (iii).

3.3. Eligibility

We define an *eligibility constraint* as $r = (r_d)_{d\in D} \in \{1,...,n\}^{\bar{d}}$ where, for each $d \in D$, r_d implies the lowest rank at criterion d such that only participants of this rank or higher deserve to be justified by criterion d. An agent $i \in N$ is said to be *eligible for criterion* $d \in D$ if

$$\pi_d^{-1}(i) \le r_d .$$

An agent is considered *eligible* if there exists a criterion for which they are eligible. This subsection considers the case in which there exist participants who are not eligible and, therefore, should not be given preferential treatment over the other agents.

For each $r \in \{1,...,n\}^{\bar{d}}$, we define a *social choice rule with eligibility* (SCRE) as $C(r): X \to 2^N$, where the slots are assigned to as many agents as possible but only eligible agents are assigned; for each $(I,q) \in X$, any agent $i \in C(r)(I,q)$ must be eligible and

$$|C(r)(I,q)| = \min\,[\,q\,,\,|\{\,i \in I \mid \text{agent } i \text{ is eligible}\}|\,].$$

We must note that it is impossible for an SCRE to satisfy property (ii) in Axiom 3. The number of assigned agents who are justified by a criterion crucially depends on the number of participants who are eligible for this criterion. Hence, to reconcile eligibility with property (ii), this subsection first specifies an SCR without eligibility constraints and then shows how to extend it to an SCR with eligibility constraints (i.e., an SCRE) reasonably. (The SCREs discussed in this subsection will satisfy Axioms 1 and 2, and Properties (i) and (iii) in Axiom 3.)

We introduce two perspectives on how to extend a given SCR to an SCRE, which we call the perspective of *priority over eligible agents* and the perspective of *compatibility between justification and eligibility.* We argue that these perspectives have their respective advantages but are incompatible with one another.

The first perspective requires an SCRE to satisfy that the priority over eligible agents adheres to the original SCR as much as possible. In particular, we specify an SCRE $C(r)$ as follows. For each $(I,q)\in X$, we select the smallest number $\tilde{q}\geq q$ satisfying

$$|C(I,\tilde{q})\bigcup\{i\in I \mid \text{agent } i \text{ is eligible}\}|$$
$$=\min[q, |\{i\in I \mid \text{agent } i \text{ is eligible}\}|],$$

and specify

$$C(r)(I,q)=C(I,\tilde{q})\bigcup\{i\in I \mid \text{agent } i \text{ is eligible}\}.$$

More specifically, we consider an arbitrary SCR that satisfies Axioms 1, 2, and 3, that is, $C=C^{\gamma}$ for some procedures γ. The corresponding SCRE $C(r)=C^{\gamma}(r)$ exists uniquely and satisfies property (i) in Axiom 3 (respecting priorities) by using the same justification as C^{γ}, that is, δ^{γ}. Hence, for every $i\in C^{\gamma}(r)(I,q)$ and $j\in I\setminus C^{\gamma}(r)(I,q)$, we have

$$\pi^{-1}_{\delta^{\gamma}(I,\tilde{q})(i)}(i)<\pi^{-1}_{\delta^{\gamma}(I,\tilde{q})(i)}(j).$$

However, this perspective has a drawback: an assigned agent i is surely eligible, but they are not necessarily eligible for $\delta^{\gamma}(I,\tilde{q})(i)$. To overcome this drawback, we introduce the second perspective, that is, *compatibility between justification and eligibility*, by constructing another SCRE $C^{\gamma\dagger}(r)$ according to the following steps. We extend γ to an arbitrary function from $\{1,...,n\bar{d}\}$ to D. Consider an arbitrary assignment problem $(I,q) \in X$. We set the *dummy* agent ϕ as the $(n+1)^{th}$ rank agent for every criterion. (We assume that the dummy agent ϕ is not eligible.) In step 1, the top-ranked agent among I at criterion $\gamma(1) \in D$ is selected. This agent is denoted by $i(1) \in I$. If agent $i(1)$ is eligible for $\gamma(1)$, then they are assigned a slot and justified by

$$\delta^{\gamma\dagger}(I,q)(i(1)) = \gamma(1).$$

If they are not eligible for $\gamma(1)$, they are regarded as a *provisionally selected agent*.

Recursively, at each step $k \geq 2$, the top-ranked agent at criterion $\gamma(k) \in D$ among the set of remaining participants, including the dummy agent ϕ, that is, $I \setminus \{i(1),...,i(k-1)\} \cup \{\phi\}$, is selected. This agent is denoted by $i(k)$. Any provisionally selected agent i is assigned a slot if they have a better rank than agent $i(k)$ at the criterion $\gamma(k)$ and is eligible for it. This agent i is justified by

$$\delta^{\gamma\dagger}(I,q)(i) = \gamma(k).$$

If the selected agent $i(k)$ is eligible for $\gamma(k)$, they are assigned a slot and is justified by

$$\delta^{\gamma\dagger}(I,q)(i(k)) = \gamma(k).$$

If they are not eligible for $\gamma(k)$, they are regarded as a provisionally selected agent.

We continue these steps until the number of agents with assigned slots equals

$$\min\,[q\,, |\{\,i \in I \mid \text{agent } i \text{ is eligible}\}|].$$ [10]

We then specify $C^{\gamma\dagger}(r)(I,q)$ as the set of all agents assigned slots through the above steps.

Clearly, the newly specified SCRE $C^{\gamma\dagger}(r)$ overcomes the above-mentioned drawback, that is, any assigned agent i is eligible for criterion $\delta^{\gamma\dagger}(I,q)(i)$ and even justified by it: for every $i \in C^{\gamma\dagger}(r)(I,q)$, we have

$$\pi^{-1}_{\delta^{\gamma\dagger}(I,q)(i)}(i) \leq r_{\delta^{\gamma\dagger}(I,q)(i)},$$

and

[10] Note that the dummy agent ϕ may be selected many times but is never assigned, because that agent is not eligible.

$$\pi^{-1}_{\delta^{\gamma\dagger}(I,q)(i)}(i) < \pi^{-1}_{\delta^{\gamma\dagger}(I,q)(i)}(j) \text{ for all } j \in I \setminus C^{\gamma\dagger}(I,q).$$

These two perspectives are incompatible, because in general, $C^{\gamma\dagger}(r) \neq C^{\gamma}(r)$.

The following example is helpful to understand these perspectives.

Example 2: Consider $n = 3$,

$\bar{d} = 3$,

$\pi_1(1) = 1$, $\pi_1(1) = 2$,

$\pi_2(1) = 1$, $\pi_2(2) = 3$, and

$\pi_3(1) = 2$, $\pi_3(2) = 3$.

See Figure 2.a. We specify a procedure as

$\gamma(1) = 1$, $\gamma(2) = 2$, and $\gamma(3) = 3$,

where the associated SCR $C = C^{\gamma}$ is given by

$C(\{1,2,3\},1) = \{1\}$, $C(\{1,2,3\},2) = \{1,3\}$, $C(\{1,2\},1) = \{1\}$,

$C(\{1,3\},1) = \{1\}$, and $C(\{2,3\},1) = \{3\}$.

See Figure 2.b. We specify a justification $\delta = \delta^{\gamma}$ by

$\delta(\{1,2,3\},1)(1) = \gamma(1) = 1$, $\delta(\{1,2,3\},2)(3) = \gamma(2) = 2$,

$\delta(\{1,2\},1)(1) = \gamma(1) = 1$, $\delta(\{1,3\},1)(1) = \gamma(1) = 1$, and

$\delta(\{2,3\},1)(3) = \gamma(1) = 1$.

Clearly, this specification is consistent with Axiom 3 (fair justification). We introduce an eligibility constraint by

$r_1 = 1$, $r_2 = 1$, and $r_3 = 2$.

Note that the corresponding SCRE $C^{\gamma}(r)$ from the perspective of priority over eligible agents was the same as that of the original SCR C. However, in the assignment problem $(\{1,2,3\},2)$, the assigned agent 3 has no criterion by which they are justified and for which they are eligible: agent 3 is eligible only for criterion 3, and is justified only by criterion 2. On the other hand, the modified SCRE $C^{\gamma\dagger}(r)$ from the perspective of compatibility between justification and eligibility is given by

$C^{\gamma\dagger}(r)(\{1,2,3\},1) = \{1\}$, $C^{\gamma\dagger}(r)(\{1,2,3\},2) = \{1,2\}$,

$$C^{\gamma\dagger}(r)(\{1,2\},1)=\{1\},\ \ C^{\gamma\dagger}(r)(\{1,3\},1)=\{1\}, \text{ and}$$
$$C^{\gamma\dagger}(r)(\{2,3\},1)=\{2\}.$$

See Figure 2.c. Note that $C^{\gamma\dagger}(r)$ is different from the original SCR C, because agent 2 was assigned instead of agent 3 in the problem $(\{2,3\},1)$. However, the assigned agent 2 is successfully justified by, and is also eligible for, criterion 3.

Figure 2.a: π

π_1	1	3	3
π_2	1	3	2
π_3	2	3	1

Figure 2.b: C^{γ} $(=C^{\gamma}(r))$

	{1,2,3}	{1,2}	{1,3}	{2,3}
1	{1}	{1}	{1}	{3}
2	{1,3}			

Figure 2.c: $C^{\gamma\dagger}(r)$

	{1,2,3}	{1,2}	{1,3}	{2,3}
1	{1}	{1}	{1}	{2}
2	{1,2}			

Remark 4: The manner of specifying $C^{\gamma\dagger}(r)$ and the deferred acceptance (DA) algorithm (Gale and Shapley, 1962) have an essential similarity: like the DA algorithm, the steps for specifying $C^{\gamma\dagger}(r)$ incorporate a device in which agents are designated as provisional candidates and the final decision on whether they are actually assigned a slot is deferred. This provisional engagement achieves a better match between justification and eligibility than without it ($C^{\gamma}(r)$).

Remark 5: Pathak et al. (2020) considered the reserve system with eligibility that corresponds to the following procedure, denoted by $\tilde{\gamma}:\{1,...,n\bar{d}\}\to D$, such that for each $d\in D$,

$$\tilde{\gamma}(k)=d \quad \text{for all} \quad k\in\{(d-1)n+1,...,dn\}.$$

According to the corresponding $C^{\tilde{\gamma}\dagger}(r)$, the slots are preferentially assigned to all participants who are eligible for criterion 1; the remaining slots are preferentially assigned to all the remaining participants who are eligible for criterion 2, and so on. These steps continue until all slots are assigned. Compared with the reserve procedure γ^* in Remark 1, the reserve system $\tilde{\gamma}$ in this remark is inappropriate because of the following asymmetries between the criteria in terms of reserve achievement and eligibility constraints. That is, a higher prioritized criterion has an advantage from the viewpoint of reserve achievement when the number of slots is limited. Conversely, when there are plenty of slots, a lower prioritized criterion has an advantage from the viewpoint of eligibility: many agents who have higher ranks at this criterion have already been assigned on the basis of the other, more prioritized criteria, and therefore, relatively low-rank agents can be assigned by using this criterion as a justification. In contrast, the reserve procedure in Remark 1 does not cause any such imbalance.

4. Method of Aggregation

As an alternative to the method of procedure, we demonstrate the *method of aggregation* and show that these methods are incompatible with one another.

4.1. Agent Consistency

We introduce an axiom on C, which is more restrictive than Axiom 1 but is still irrelevant of (D,π), as follows:

Axiom 4 (Agent Consistency): For every $(I,q)\in X$, $i\in I$, and $j\in I\setminus\{i\}$,

$$[\boldsymbol{i} \in \boldsymbol{C}(\boldsymbol{I},\boldsymbol{q})] \Rightarrow [\boldsymbol{i} \in \boldsymbol{C}(\boldsymbol{I} \cup \{\boldsymbol{j}\}, \boldsymbol{q}+\mathbf{1})].$$

Axiom 4 implies that the same agents can obtain slots when the set of participants becomes larger and the same number of slots as this increase of participants are added. Axiom 4 guarantees that a change in the problem does not greatly affect which agents are assigned slots, implying a consistent respect for individual agents across problems.

We define an *aggregation* as a one-to-one mapping (priority order over agents), which is denoted by $\boldsymbol{f}:\{\mathbf{1},\mathbf{2},...,\boldsymbol{n}\} \to \boldsymbol{N}$. We then define an SCR induced by an aggregation $\boldsymbol{f}$, denoted by $\boldsymbol{C}=\boldsymbol{C}^{f}$: for every $(\boldsymbol{I},\boldsymbol{q}) \in \boldsymbol{X}$,

$$\boldsymbol{C}(\boldsymbol{I},\boldsymbol{q}) = \{\boldsymbol{i} \in \boldsymbol{I} \mid \boldsymbol{f}^{-1}(\boldsymbol{i}) \le \boldsymbol{q}\}.$$

According to $\boldsymbol{C}^{f}$, an agent with a higher rank in $\boldsymbol{f}$ always has a higher priority in slot assignment irrespective of the problem. We can also regard $\boldsymbol{C}^{f}$ as an SCR that is induced by an artificially created procedure $\Gamma = (\boldsymbol{D},\boldsymbol{\pi},\boldsymbol{\gamma})$ with a single criterion, where $\bar{\boldsymbol{d}} = \mathbf{1}$, $\boldsymbol{\pi}_1 = \boldsymbol{f}$, and $\boldsymbol{\gamma}(\boldsymbol{k}) = \mathbf{1}$ for all $\boldsymbol{k} \in \{\mathbf{1},...,\boldsymbol{n}\}$.

Axiom 4 requires an SCR that satisfies Axioms 1 and 2 to be induced by a common priority order over agents ($\boldsymbol{f}:\{\mathbf{1},\mathbf{2},...,\boldsymbol{n}\} \to \boldsymbol{N}$) for different problems. Hence, as the following theorem shows, any SCR that satisfies Axioms 1, 2, and 4 must be in the form of $\boldsymbol{C}^{f}$.

Theorem 3: *An SCR* $\boldsymbol{C}$ *satisfies Axioms 2 and 4 if and only if there exists* $\boldsymbol{f}$ *such that* $\boldsymbol{C}=\boldsymbol{C}^{f}$.

Proof: From Theorem 1, $\boldsymbol{C}^{f}$ satisfies Axiom 2. We can show that $\boldsymbol{C}^{f}$ satisfies Axiom 4: if agent $\boldsymbol{i}$ has a better rank than $\boldsymbol{q}+\mathbf{1}$ in $\boldsymbol{f}$ in the problem $(\boldsymbol{I},\boldsymbol{q})$, they have a better rank than $\boldsymbol{q}+\mathbf{2}$ in $\boldsymbol{f}$ in the problem $(\boldsymbol{I} \cup \{\boldsymbol{j}\}, \boldsymbol{q}+\mathbf{1})$, implying Axiom 4.

Suppose that $\boldsymbol{C}$ satisfies Axioms 2 and 4. We specify the aggregation $\boldsymbol{f}$ as follows:

$$\{\boldsymbol{f}(\mathbf{1})\} = \boldsymbol{C}(\boldsymbol{N},\mathbf{1}),$$

and, recursively, for each $k \in \{2,...,n\}$,

$$\{f(k)\} = C(N \setminus \{f(1),...,f(k-1)\},1).$$

Clearly, the specified f is a one-to-one mapping. Suppose that there exist $(I,q) \in X$, $i \in I$, and $j \in I \setminus \{i\}$, such that

$$i \in C(I,q),\ \ j \notin C(I,q), \text{ and } \ f^{-1}(i) > f^{-1}(j).$$

From Axiom 2, we have

$$j \notin C(C(I,q) \cup \{j\},q).$$

From Axiom 4, we have

$$C(\{i,j\},1) = \{i\}.$$

However, from Axiom 4,

$$C(N \setminus \{f(1),...,f(f(j)-1)\},1) = \{j\}, \text{ and}$$

$$i \in N \setminus \{f(1),...,f(\gamma(i)-1)\}.$$

Hence, from Axiom 2, we have

$$C(\{i,j\},1) = \{j\},$$

which is a contradiction. Hence, we have $C = C^f$.

Q.E.D.

The aggregation f in the proof of Theorem 3 was artificially created to configure an SCR that satisfies Axioms 2 and 4. How an aggregation f should be related to the pre-existing (D,π) is explained in Subsection 5.2.

4.2. Ethical Dictatorship

Axiom 3 emphasizes a consistent respect for individual criteria and plays a central role in configuring SCRs through the method of procedure. Axiom 4 emphasizes a consistent respect for individual agents and plays a central role in configuring SCRs through the method of aggregation. This subsection shows that Axioms 3 and 4 are incompatible with one another.

Consider the pre-existing (D,π). An SCR C is said to be *ethical-dictatorial* for a criterion $d \in D$ if for every $(I,q) \in X$,

$$C(I,q) = \{i \in I \mid \pi_d^{-1}(i) \le q\}.$$

An SCR C is said to be *ethical-dictatorial* if there exists $d \in D$ for which C is ethical-dictatorial. An ethical-dictatorial SCR always determines the slot assignments according to a single criterion. For example, only the order of willingness to pay is used for slot assignment even if there are good reasons to prioritize the poor with low willingness to pay. Clearly, any ethical-dictatorial SCR satisfies Axioms 3 and 4, as well as Axioms 1 and 2.

The following theorem states that any SCR that satisfies Axioms 2, 3, and 4 must be ethical-dictatorial; hence, the two methods are incompatible with each other. Axiom 3 requires that a procedure (i.e., a priority order over criteria) γ exists. Axiom 4 requires that an aggregation (i.e., a priority order over agents) f exists. For an SCR to satisfy both axioms, the aggregation f and the first-step priority order over agents $\gamma(1)$ must be equivalent: for every $k \in \{2,...,n\}$, if $f(k') \notin I$ for all $k' \in \{1,...,k-1\}$ and $f(k) \in I$, then $\pi_{\gamma(1)}(k) = f(k)$ must hold.

Theorem 4: *An SCR* C *satisfies Axioms 2, 3, and 4 if and only if it is ethical-dictatorial.*

Proof: Suppose that C satisfies Axioms 2, 3, and 4. From Theorem 2, γ exists such that $C = C^{\gamma}$. Without loss of generality, we assume $\gamma(1) = 1$. From Theorem 3, f exists such that

$$[\, i \in C(I,q),\ j \notin C(I,q), \text{ and } j \in I \,] \Rightarrow [\, f^{-1}(i) < f^{-1}(j) \,].$$

We show $\pi_1 = f$ as follows. Clearly, we have $\pi_1(1) = f(1)$. Consider an arbitrary $k \in \{2,...,n\}$. Suppose that $\pi_1(k') = f(k')$ for all $k' \in \{1,...,k-1\}$. Since

$$C(N \setminus \{\pi_1(1),...,\pi_1(k-1)\},1) = \pi_1(k),$$

it follows from Axiom 3 that

$$C(N,k) = \{\pi_1(1),...,\pi_1(k-1),\pi_1(k)\},$$

which implies $\pi_1(k) = f(k)$. Hence, (C, D, π) is ethical-dictatorial, where $C = C^\gamma$, and $\gamma(h) = 1$ for all $h \in \{1, ..., n\}$. **Q.E.D.**

The method of procedure emphasizes a consistent respect for individual criteria across problems. Conversely, the method of aggregation emphasizes a consistent respect for individual agents across problems. The method of aggregation avoids losing the right of an individual agent to be assigned a slot owing to a slight change of the problem. The method of aggregation does not necessarily prioritize a single criterion and neglect other criteria. However, by applying both methods together, it is inevitable to prioritize a single criterion, such as willingness to pay, and neglect other criteria.

5. State-Dependent Social Choice Rule

Let Ω denote the set of *states*. This section considers how an SCR depends on the state $\omega \in \Omega$. We denote the *state-dependent social choice rule* (SSCR) as $C = C(\omega) = C(\cdot;\omega)$. We write $C(I, q) = C(I, q; \omega)$.

Consider an arbitrary set of criteria $D = \{1, ..., \bar{d}\}$. Specifically, we denote a state by $\omega = (\omega_d)_{d \in D}$, where for each $d \in D$,

$$\omega_d : N \to R \text{ for each } d \in D,$$

and

$$\omega_d(i) \neq \omega_d(j) \text{ for all } d \in D,\ i \in N, \text{ and } j \in N \setminus \{i\}.$$

We define Ω as the set of all such possible states ω. We refer to $\omega_d(i) \in R$ the *evaluation of the agent* $i \in N$ *at the criterion* d. We denote the evaluation list for each agent i by $\omega(i) = (\omega_d(i))_{d \in D} \in R^{\bar{d}}$.

We regard a state ω as detailed information about the cardinal aspect of each criterion and the comparability across the criteria. Associated with each state $\omega \in \Omega$, we denote by $\pi = \pi(\omega)$ the corresponding ordinal aspect of the criteria; for every $d \in D$ and $h \in \{1, ..., n-1\}$,

$$\omega_d(\pi_d(h)) > \omega_d(\pi_d(h+1)).$$

An agent i has higher rank than agent j at a criterion $d \in D$, that is,

$$\pi_d^{-1}(i) < \pi_d^{-1}(j),$$

if and only if agent $i's$ evaluation is greater than agent $j's$ evaluation at d, that is,

$$\omega_d(i) > \omega_d(j).$$

This specificity excludes the case in which the state includes information about which assignment problem to be solved and both γ and f depend on this information. This exclusion makes the question of how to configure an SSCR essential.

This section investigates how an SCR utilizes detailed information about the evaluations. We write $\gamma = \gamma(\omega)$, $\Gamma = \Gamma(\omega) = (D, \pi(\omega), \gamma(\omega))$, $f = f(\omega)$, $C^\gamma = C^{\gamma(\omega)}$, $C^f = C^{f(\omega)}$, and so on. We regard all the previous axioms, that is, Axioms 1, 2, 3, and 4, as the properties for all $\omega \in \Omega$. We introduce two axioms for an SSCR C as follows:

Axiom 5 (Independence): For every $\omega \in \Omega$, $\omega' \in \Omega$, and $(I, q) \in X$,

$$[\, \omega(i) = \omega'(i) \text{ for all } i \in I \,] \Rightarrow [\, C(I, q; \omega) = C(I, q; \omega') \,].$$

Axiom 6 (Comparability): For every $\omega \in \Omega$, $d \in D$, and $i \in N$, if

$$\pi_d(\omega_d)(1) = i,$$

then a sufficiently large real number $l > 0$ exists such that for every $\omega' \in \Omega$,

$$[\, \omega'_d(i) \geq \omega_d(i) + l,$$

$$\omega'_d(j) = \omega_d(j) \text{ for all } j \neq i, \text{ and}$$

$$\omega'_{d'} = \omega_{d'} \text{ for all } d' \in D \setminus \{d\} \,]$$

$$\Rightarrow [\, C(N, 1; \omega') = \{i\} \,].$$

Axiom 5 implies that the assignment choice does not depend on the evaluation of non-participants. Axiom 6 implies that if an agent is evaluated outstandingly high by a particular criterion, they are assigned a slot. Axioms 5 and 6 play a central role in associating the method of aggregation with the pre-existing (D, π) (see Subsection 5.2.)

5.1. Method of Procedure

According to Axiom 6, if a criterion suggests that an agent should be given an exceptional priority, the central planner must disregard whether this criterion has a low or high priority and give this agent priority over anyone else. This, however, clearly contradicts property (ii) in Axiom 3, which, for any problem, and for any criterion, requires a certain number of assigned agents to be justified by this criterion. Hence, as the following theorem shows, we have an impossibility result in the method of procedure: the method of procedure is incompatible with Axiom 5 (independence) and Axiom 6 (comparability).

Theorem 5: *There exists no SSCR* $\boldsymbol{C}$ *that satisfies Axioms 1, 2, 3, 5, and 6.*

Proof: From Axioms 1, 2, and 3, and from Theorem 2, a procedure $\gamma(\omega)$ exists such that $C(\omega) = C^{\gamma(\omega)}(\omega)$. We select two large positive real numbers, l' and l''. Consider $\omega \in \Omega$, $\omega' \in \Omega$, and $\omega'' \in \Omega$, where we assume that

$$\omega(i) = \omega'(i) = \omega''(i) \quad \text{for all} \quad i \neq 1,$$

$$\omega_1'(1) = \omega_1(1) + l',$$

$$\omega_d'(1) = \omega_d(1) \quad \text{for all} \quad d \neq 1,$$

$$\omega_2''(1) = \omega_2(1) + l'',$$

$$\omega_d''(1) = \omega_d(1) \quad \text{for all} \quad d \neq 2,$$

and

$$\pi_1(\omega')(2) \neq \pi_2(\omega'')(2).$$

Since l' and l'' are selected as large, from Axiom 6, $\gamma(\omega')(1) = 1$ and $\gamma(\omega'')(1) = 2$ must hold. Hence, we have

$$C(N \setminus \{1\}, 1; \omega') = \{\pi_1(\omega')(2)\} \quad \text{and}$$

$$C(N \setminus \{1\}, 1; \omega'') = \{\pi_2(\omega'')(2)\}.$$

However, from Axiom 5, we have

$$\pi_1(\omega')(2) = \pi_2(\omega'')(2),$$

which is a contradiction.

Q.E.D.

The following example is helpful in understanding Theorem 5.

Example 3: Consider $n = 3$, $\bar{d} = 2$, and two states, ω and ω', which are given by

$$\omega_1(1) = 1000,\ \omega_1(2) = 10,\ \omega_1(3) = 5,$$
$$\omega_2(1) = 0,\ \omega_2(2) = 5,\ \omega_2(3) = 10,$$
$$\omega_1'(1) = 0,\ \omega_1'(2) = 10,\ \omega_1'(3) = 5,$$
$$\omega_2'(1) = 1000,\ \omega_2'(2) = 5, \text{ and } \omega_2'(3) = 10.$$

See Figures 3.a and 3.b. Consider an arbitrary procedure γ. Suppose that C^{γ} satisfies Axiom 6. Since we can regard 1000 as a sufficiently large number, from Axiom 6, it must hold that

$$C^{\gamma(\omega)}(\{1,2,3\},1;\omega) = \{1\},\ \gamma(\omega)(1) = 1,$$
$$C^{\gamma(\omega')}(\{1,2,3\},1;\omega') = \{1\}, \text{ and } \gamma(\omega')(1) = 2.$$

Hence,

$$C^{\gamma(\omega)}(\{2,3\},1;\omega) = \{2\} \text{ and } C^{\gamma(\omega')}(\{2,3\},1;\omega') = \{3\},$$

which contradicts Axiom 5, because $\omega_1(2) = \omega_1'(2)$ and $\omega_1(3) = \omega_1'(3)$.

Figure 3.a: ω

	1	2	3
ω_1	1000	10	5
ω_2	0	5	10

Figure 3.b: ω'

	1	2	3
ω_1'	0	10	5
ω_2'	1000	5	10

5.2. Method of Aggregation

In contrast to the method of procedure, we can demonstrate SSCRs induced by the method of aggregation that satisfy Axioms 5 and 6 and are reasonable to some extent, as follows. Consider an arbitrary increasing function $\boldsymbol{m}: \boldsymbol{R}^{\bar{d}} \to \boldsymbol{R}$ as a *priority point system*. We specify a state-dependent aggregation $f = f^{m}$ as follows: for each $\omega \in \Omega$, $i \in N$, and $j \in N \setminus \{i\}$,

$$[\, f^{m}(\omega)(i) < f^{m}(\omega)(j)\,] \Leftrightarrow [\, m(\omega(i)) > m(\omega(j))\,].$$

According to $f^{m}(\omega)$, each agent's $i's$ evaluations at various criteria are aggregated into a single value $m(\omega(i)) \in \boldsymbol{R}$. Any agent whose aggregate value is greater has a higher priority in slot assignments. Clearly, the corresponding SSCR $C^{f^{m}}$ satisfies Axioms 5 and 6: the method of aggregation is compatible with independence and comparability.

A special case of a priority point system is the *weighted sum point system*. We fix an arbitrary $\bar{d}$ -dimensional vector, $w = (w_d)_{d \in D} \in \boldsymbol{R}_+^{\bar{d}}$, and specify $m = m^*$ by

$$m^*(v) = \sum_{d \in D} w_d v_d \quad \text{for all} \quad v = (v_d)_{d \in D} \in \boldsymbol{R}_+^{\bar{d}}.$$

According to $f^{m^*}(\omega)$, each agent $i's$ evaluations are aggregated into the weighted sum of these evaluations $\sum_{d \in D} w_d \omega_d(i)$.

More specifically, we can interpret the weighted sum point system as follows.[11] We fix an arbitrary state $\omega \in \Omega$. We denote agent $i's$ willingness to pay by $u_i(\omega) \in \boldsymbol{R}_+$. The evaluation of agent i at a criterion d is given by

$$\omega_d(i) = \rho_d(i) u_i(\omega),$$

where $\rho_d(i)$ denotes the welfare weight that the criterion d gives agent $i's$ willingness to pay. Hence, $\sum_{d \in D} w_d \rho_d(i)$ implies the weighted sum welfare weight of agent $i's$ willingness to pay. Each agent i is prioritized according to the size of

[11] See Pathak et al. (2021). In triage, this point system was quite popular, but it became controversial through comparison with the reserve system.

willingness to pay $u_i(\omega)$ multiplied by weighted sum welfare weight $\sum_{d\in D} w_d \rho_d(i)$, that is,

$$\sum_{d\in D} w_d \omega_d(i) = \{\sum_{d\in D} w_d \rho_d(i)\} u_i(\omega).$$ [12]

We can extend $f^{m^*}(\omega)$ to the case of eligibility. Consider the zero evaluation as the threshold for each criterion, such that for every $d \in D$ and $\omega \in \Omega$, each agent i is regarded as eligible for the criterion d if and only if

$$\omega_d(i) \geq 0.$$

Hence, the eligibility constraint at the state $\omega \in \Omega$, $r = r(\omega)$, is given by

$$r_d(\omega) = \arg\max\{h \in \{1,...,n\} \mid \omega_d(\pi_d(h)) \geq 0\} \text{ for each } d \in D.$$ [13]

We then specify $m = m^{**}$ as

$$m^{**}(v) = \sum_{d\in D(v)} (w_d v_d + f_d),$$

where $D(v) \subset D$ denotes the set of all criteria d such that $v_d \geq 0$, and

$$\omega_d \in R_+ \text{ and } f_d \in R_+ \text{ for each } d \in D.$$

The positivity of f_d implies a discontinuity in the evaluation between eligible agents and ineligible agents at criterion d. The corresponding state-dependent social choice function with eligibility (SSCRE), denoted by $C^{f^{m^{**}}}(r(\omega))$, excludes all ineligible agents without contradicting Axioms 5 and 6.

5.3. Informational Basis

The arguments in Subsections 5.1 and 5.2 indicate that the method of procedure is inferior to the method of aggregation from the viewpoint of the effective use of ethical information. However, it may not make much sense to quantify and compare different

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12 It is important to note that the welfare evaluation is not the expression of the equivalent monetary value. The central planner's concern is the relative importance of each agent's assignment in the welfare evaluation. For more detailed explanations, see the companion paper by Matsushima (2021).

13 If there exists no such h, we assume $r_d(\omega) = 0$.

criteria with each other. There may also be arbitrariness in quantifying and comparing different criteria.

This subsection argues that under limited informational bases, the method of procedure is still valid compared with the method of aggregation. We introduce an axiom for an SSCR $\boldsymbol{C}$ as follows:

Axiom 7 (Ethical Pareto): For every $\boldsymbol{\omega} \in \Omega$, $(\boldsymbol{I}, \boldsymbol{q}) \in \boldsymbol{X}$, $\boldsymbol{i} \in \boldsymbol{I}$, and $\boldsymbol{j} \in \boldsymbol{I} \setminus \{\boldsymbol{i}\}$,

$$[\, \boldsymbol{i} \in \boldsymbol{C}(\boldsymbol{I}, \boldsymbol{q}; \boldsymbol{\omega}) \text{ and } \boldsymbol{\pi}^{-1}(\boldsymbol{\omega})(\boldsymbol{i}) > \boldsymbol{\pi}^{-1}(\boldsymbol{\omega})(\boldsymbol{j}) \,]$$
$$\Rightarrow [\, \boldsymbol{j} \in \boldsymbol{C}(\boldsymbol{I}, \boldsymbol{q}; \boldsymbol{\omega}) \,],$$

where we denote $\boldsymbol{\pi}^{-1}(\boldsymbol{\omega})(\boldsymbol{i}) \equiv (\boldsymbol{\pi}_d^{-1}(\boldsymbol{\omega})(\boldsymbol{i}))_{\boldsymbol{d} \in \boldsymbol{D}}$.

Axiom 7 implies that if agent $\boldsymbol{i}$ has a better rank than agent $\boldsymbol{j}$ at all criteria, then agent $\boldsymbol{i}$ has precedence over agent $\boldsymbol{j}$. Axiom 7 is a basic axiom: in fact, any SSCR induced by the method of procedure (that satisfies property (i) in Axiom 3) automatically satisfies Axiom 7, and the SSCR considered in Subsection 5.2, which was induced by the method of aggregation, also satisfies Axiom 7.

We introduce an axiom for an SSCR $\boldsymbol{C}$ regarding limitations on comparability.

Axiom 8 (Ordinality without Comparability): For every $\boldsymbol{\omega} \in \Omega$ and $\boldsymbol{\omega}' \in \Omega$,

$$[\, \boldsymbol{\pi}(\boldsymbol{\omega}) = \boldsymbol{\pi}(\boldsymbol{\omega}') \,] \Rightarrow [\, \boldsymbol{C}(\boldsymbol{\omega}) = \boldsymbol{C}(\boldsymbol{\omega}') \,].$$

Axiom 8 implies that an SSCR $\boldsymbol{C}(\boldsymbol{\omega})$ depends on the state $\boldsymbol{\omega}$ only through its ordinal aspect $\boldsymbol{\pi}(\boldsymbol{\omega})$. Importantly, even under such limited informational bases, the method of procedure can still adopt a wide range of SSCRs. For example, suppose that a procedure $\boldsymbol{\gamma}(\boldsymbol{\omega})$ is fixed arbitrarily and independently of the state $\boldsymbol{\omega}$. It still can induce the state-dependent SCR that meaningfully reflects the multiple criteria.

In contrast, Axiom 8 severely limits the effectiveness of the method of aggregation. Consider an SSCR induced by an aggregation $\boldsymbol{f}$, that is, $\boldsymbol{C}^{\boldsymbol{f}}$. Suppose that $\boldsymbol{C}^{\boldsymbol{f}}$ satisfies Axioms 7 and 8. Then, the aggregation $\boldsymbol{f}$ must satisfy that for every $\boldsymbol{\omega} \in \Omega$ and $\boldsymbol{\omega}' \in \Omega$,

$$[\pi(\omega) = \pi(\omega')] \Rightarrow [f(\omega) = f(\omega')],$$

and for every $\omega \in \Omega$, $i \in N$, and $j \in N \setminus \{i\}$,

$$[\pi^{-1}(\omega)(i) > \pi^{-1}(\omega)(j)] \Rightarrow [f(\omega)(i) > f(\omega)(j)].$$

In this case, according to Arrow's (1951) impossibility theorem, we can prove that there must exist a criterion $d \in D$, such that

$$f(\omega) = \pi_d(\omega) \text{ for all } \omega \in \Omega.$$

Hence, for every $\omega \in \Omega$, $(I,q) \in X$, $i \in I$, and $j \in I$,

$$[i \in C^{f(\omega)}(I,q;\omega) \text{ and } j \notin C^{f(\omega)}(I,q;\omega)]$$
$$\Rightarrow [\pi_d^{-1}(\omega)(i) < \pi_d^{-1}(\omega)(j)].$$

This implies that irrespective of the state $\omega \in \Omega$, the corresponding SCR $C^{f(\omega)}(\omega)$ is ethical-dictatorial for the criterion d. Hence, we proved the following impossibility theorem:

Theorem 6: *An SSCR C satisfies Axioms 2, 4, 7, and 8 if and only if there exists $d \in D$ for which $C(\omega)$ is ethical-dictatorial in all states.*[14]

Even if we weaken Axiom 8, we still have impossibility results in the method of aggregation. For instance, we introduce a weaker version of Axiom 8 as follows:

Axiom $8^{\dagger}$ (Cardinality without Comparability): For every $\omega \in \Omega$ and $\omega' \in \Omega$, if for each $d \in D$, there exists a positive linear transformation ρ_i such that $\omega'_d = \rho_d \circ \omega_d$, then $C(\omega) = C(\omega')$.

According to Sen (1970), we can generalize Theorem 6: an SSCR C satisfies Axioms 2, 4, 7, and $8^{\dagger}$ if and only if there exists $d \in D$ for which $C(\omega)$ is ethical-dictatorial in all states.

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[14] Compared with Theorem 6, Theorem 4 is more positive in that the criterion for which an SCR is ethical-dictatorial can depend on the state.

Axiom 8 (and Axiom $\mathbf{8}^{\dagger}$) is restrictive because it excludes the possibility of considering eligibility constraints. Hence, we weaken Axiom 8 differently by permitting partial comparability, as follows:

Axiom $\mathbf{8}^{\dagger\dagger}$ (Ordinality with Zero Comparability): For every $\boldsymbol{\omega} \in \Omega$ and $\boldsymbol{\omega}' \in \Omega$, if for each $\boldsymbol{d} \in \boldsymbol{D}$, there exists a zero-preserving positive affine transformation ρ_i such that $\boldsymbol{\omega}'_d = \rho_d \circ \boldsymbol{\omega}_d$, then $\boldsymbol{C}(\boldsymbol{\omega}) = \boldsymbol{C}(\boldsymbol{\omega}')$.[15]

List (2001) introduced a related axiom in social choice theory. Axiom $\mathbf{8}^{\dagger\dagger}$ permits an SSCR (SSCRE) to depend on which participants are eligible and on which criteria an agent is eligible for in a head-count manner. Despite this route out of the impossibility theorem, we still need more thorough information concerning the comparability among the criteria. In fact, if all participants are eligible for all criteria, the ethical dictatorship still holds in the same manner as in Theorem 6.

From this section's argument, we can conclude that the method of procedure is inferior to the method of aggregation from the viewpoint of effective use of ethical information, while the method of aggregation is inferior to the method of procedure from the viewpoint of limited informational basis.

6. Conclusion

This study demonstrates a new approach to social choice theory in the context of multi-slot assignments with a single-unit demand. We used multiple conflicting ethical criteria as basic information to help with social decisions. As an alternative to the standard aggregation method, we introduced the method of procedure for making convincing compromises between conflicting criteria. The method of procedure emphasizes a consistent respect for individual criteria across problems, while the method of aggregation emphasizes a consistent respect for individual agents across problems. We showed that the method of aggregation is superior to the method of procedure when we can utilize

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[15] Here, zero-preserving means $\rho_i(\mathbf{0}) = \mathbf{0}$.

detailed information concerning cardinality and comparability, while the method of procedure is superior to the method of aggregation when there are severe informational limitations.

We have shown that the two methods are incompatible with one another when applied simultaneously, even if each has its own advantages: only ethical dictatorships are induced by both methods. As an effective way to solve the dilemma of which method to use, we can propose integrating them in the following manner. Let us divide the multiple criteria into two distinct groups: the comparable group and the incomparable group. The former is the set of criteria that can be quantified and compared with each other in an easy way to understand. The latter is the set of the remaining criteria that are difficult to quantify and compare with each other. We combine the criteria in the comparable group into an artificially created criterion using the method of aggregation. We then adopt the method of procedure to configure an SSCR on the basis of this new criterion and the criteria in the incomparable group. This configuration can relieve the dilemma about which method to use.

This study is the first step toward a new research direction in social choice theory. What is important for future research is to extend our framework from specific problems to more general problems. For example, this study assumed homogeneous goods, single-unit demand, and in-kind assignments. By weakening these assumptions, we can more vividly and meaningfully consider issues that were not explicitly discussed in this study, such as stable match, role of willingness to pay, incentive for revelation, and possibility of pecuniary transfers within limited users and uses such as fake money in food banks (Prendergast, 2017).

Matsushima (2021), which is a companion paper to this study, investigated multi-slot assignments with ethical concerns, where the central planner is ex ante unaware to each agent's types. Matsushima showed that the central planner can design a multi-unit action format that can implement the social choice rule derived through the method of procedure in the strategy-proof manner.

Our framework has peculiarities in that each criterion is defined as a priority order over agents. To capture externalities or community merits more generally, it might be necessary to consider priorities over sets of agents who are assigned slots.

We analyzed the comprehensive model that has succeeded in capturing substances such as scarcity that are common to various issues. However, it is far from one-fit-all in social implementation. For example, in the issue of vaccination, not only its scarcity but also whether the vaccine can be administered on time without excess should be resolved. It is important to set a more detailed model according to the specific context. These challenges, however, are beyond the scope of this study.

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